Design and Analysis of Numerical Interleaver for IDMA Schemes with Iterative Multi-user Detection

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Abstract
Objective: This work provides a comprehensive study of a novel orthogonal interleaver set based on particular numerical equation for generation of user specific interleavers. Methods/Analysis: Performance analysis is done based upon the computer simulations in various scenarios with IDMA system and SC-FDMA-IDMA system, under different channel conditions including Additive White Gaussian Noise (AWGN) and Under Water Acoustic (UWA) channel to compare proposed interleaver with other interleavers in literature. The performance of different interleavers is compared on the basis of bit error rate (BER) results and also by hardware and bandwidth requirements. Findings: Compared to other interleavers discussed in literature, the proposed interleaver set is easy to generate, more-over this work has presented a method which doesn’t require common master random interleaving pattern to develop other interleaving patterns. The analysis confirms that the proposed method of interleaver generation outperforms various other interleavers in terms of BER performance and bandwidth requirement, while providing considerably lower complexity. Applications: Applications of orthogonal interleavers include all non-orthogonal multiple access schemes based on Multi-User Detection (MUD) strategy including Interleave Division Multiple Access (IDMA), Orthogonal Frequency Division Multiplexing-Interleave Division Multiple Access (OFDM-IDMA) and Single Carrier Frequency Division Multiple Access-Interleave Division Multiple Access (SC-FDMA-IDMA) scheme, employing user specific interleavers for user separation.

Keywords: IDMA, Numerical Interleaver, Random Interleaver, SC-FDMA-IDMA, Tree based Interleaver, UWA

1. Introduction
Multiple access is a fundamental need in wireless cellular systems. Multiple access is possible through various orthogonal approaches or non-orthogonal approaches. Orthogonal multiple access in Time Division Multiple Access (TDMA), Frequency Division Multiple Access (FDMA), Orthogonal Frequency Division Multiple Access (OFDMA) and SC-FDMA doesn’t allow users to access whole spectrum simultaneously. Therefore, orthogonal multiple access schemes cannot achieve theoretical spectral and power efficiency limits. It has been reported in that non-orthogonal approaches can achieve spectral and power efficiency limits to fully exploit the advantage of multi-user gain, in delay-sensitive and fading environments like UWA environment. Generally speaking, Code Division Multiple Access (CDMA) technique is used as non-orthogonal multiple access technique in wireless cellular systems. Multiple Access Interference (MAI) imposes the main problem in CDMA performance and single user detection also outcomes in reduced performance. MUD has been suggested in to alleviate this problem. However, the gain of MUD in CDMA is realized, practically at the cost of increased receiver complexity. To make MUD advantage possible in practice, the low cost Chip By Chip (CBC) iterative MUD technique for IDMA scheme has been suggested in.

Orthogonal interleavers are the essential part of the IDMA and associated hybrid schemes including OFDM-IDMA and SC-FDMA-IDMA. The significance of multiple orthogonal interleavers as the sole resource for user distinction has been recognized as a primary
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component in IDMA systems. Since multiple users are distinguished by separate interleavers, so an interleaver design should include that,

The interleaver set generation method is easy to implement, i.e. minimal information exchange between mobile station and base station is required. And the generation algorithm should be less complex also.

And the most important point is that the cross-correlation among interleavers should be least for avoiding the interference with each other.

At the receiver side CBC-MUD algorithm is used in IDMA systems. In case of MUD strategy, data has to be interleaved and de-interleaved for the scheduled number of iterations, so interleaver set design should be such that, it consumes minimum memory also with less complexity to generate interleaver sequence.

The focus of this work is to design and analyze an optimum orthogonal interleaver set for IDMA related systems that can work well in band-limited underwater acoustic environment also. User specific interleavers are designated from the interleaving arrangements available from total sum of chips. In random interleavers significant amount of memory is required to store individual pattern of each user at the transmitter and receiver sides. Many more interleavers are available in literature but most of them are based on the approach of selection of user-specific interleavers from the available set of random interleavers. Still, the interleaving and de-interleaving mechanism complexity remains same; with that of high bandwidth and hardware requirements also. In this work, an optimum interleaver named as numerical interleaver is proposed for solution of the above problems related to computational complexity, bandwidth and hardware requirements while providing better BER performance in AWGN channel conditions and in band-limited UWA channels as well.

The description of this work is given in the following sections. Coming section describes in brief the considered system model for IDMA scheme and SC-FDMA-IDMA scheme, followed by the description of different interleavers available, in the same context of research. Followed by the need to generate new interleaver set in section 3, section 4 describes the detailed algorithm of proposed numerical interleaver set. Performance evaluation results can be found in Section 5, where comparisons with different interleavers are presented. Final remarks can be found in Section 6.

2. Various Interleavers in IDMA based Schemes

To analyze the performance of the proposed interleaver with K simultaneous users, IDMA system model in AWGN environment and SC-FDMA-IDMA system model in UWA channel environment have been con-
sidered in this work. IDMA transmitter model is shown in Figure 1 for K simultaneous users. Convolutionally encoded data sequence is spreaded and re-arranged by a user specific interleaver. In IDMA and related multiple access schemes interleavers have been used to differentiate different users. After user specific interleaving the correlated adjacent chips are converted into uncorrelated chips, which enables the simple CBC detection at the receiver side. An iterative sub-optimal receiver as has been approved for IDMA system is also shown in Figure 1. A Primary Signal Estimator (PSE) and K single user a-posteriori probability decoders (DECs) iteratively decode the information sequence. Multiple access issues are handled by PSE, while in DECs coding operations are handled.

After partially resolving the multiple access issues in PSE, the information sequence of users is de-interleaved individually. Further improvement in estimation is done in the decoder unit. The decoder outputs are fed back to the PSE to further improve the performance in next iteration. Many of such iterations are repeated to get a certain level of performance. Last iteration will take the hard decision on information bits.

To evaluate SC-FDMA-IDMA system performance, considered transmitter and receiver model has been shown in Figure 2. At the transmitter side each user’s convolutionally encoded data is spreaded and interleaved with user specific interleaver. After this M, binary phase shift keying (BPSK) modulated symbol sequence is linearly pre-coded by an M by M Fast Fourier Transform (FFT) matrix. Then localized sub-carrier mapping is used to map FFT output to one of the N (N>M) orthogonal sub-carrier. Time domain signals are transmitted through transmitter after processing through Inverse Fast Fourier Transform (IFFT) block.

At the receiver side signal is de-sub-carrier mapped after N-point FFT. The user with highest Signal to Noise Ratio (SNR) is detected first and so on other users also, using successive Multi-user Interference Canceller (MIC), Minimum Mean Square Error equalization (MMSE) and decoding process iteratively.

This is obvious till this discussion that the performance of any IDMA based system depends upon the design of orthogonal interleaver set. Interleavers remove burst errors, but also provide a means to de-correlate various users. A good orthogonal interleaver set directly affects the complexity of IDMA based systems. Low correlation among users ensures that less number of iterations will be required for decoding the information sequence to attain a certain level of satisfaction.

Figure 2. SC-FDMA-IDMA transmitter and receiver structure.
Interleavers generated according to the orthogonality criteria having minimum numbers of collisions are accepted as part of the IDMA based systems. It is an important criterion in generating the orthogonal interleavers. Otherwise as the number of user increases in a cellular system the correlation among them increases and this correlation increase among the different users will degrade BER performance and BER inferior to a certain limit is unacceptable. In the same context of research some commonly used interleavers in IDMA related schemes are explained below in brief.

2.1 Random Interleaver (RI)
Random interleaver generates dissimilar random patterns arbitrarily. These arbitrary patterns are generated according to a random combination.

2.2 Power Interleaver (PI)
According to power interleaver generation algorithm one common master random interleaver pattern, π is generated and is allotted to the first user. Interleaving pattern for the second user is computed using the same common master random interleaver two times in sequence like this, π(π). The same process of interleaver generation continues as the number of users increases. The count number of a particular user decides the number of cycles needed to generate its specific interleaver.

2.3 Tree based Interleaver (TBI)
In tree based interleaver generation, two master random interleavers are selected for first two users. These two master random interleavers follow tree branch structure to develop interleaving masks for other users.

For the purpose of an illustration, π₁ and π₂ have been taken as two master random interleavers. Following the binary tree format as described in, other users’ interleaving patterns are calculated. As per the tree format, interleaving pattern of 3rd and 4th users’ are calculated using π₃ = (π₁(π₁)) and π₄ = (π₁(π₂)) respectively, same way interleaving pattern of 5th and 6th users’ are calculated using π₅ = (π₁(π₁)) and π₆ = (π₂(π₂)) respectively. And interleaving pattern for 14th user is calculated using the interleaving pattern of 2nd user, as follows. π₁₄ = π₂ (π₂(π₂)). It is clear that only two clock cycles are needed to obtain interleaving pattern of 14th user in tree based algorithm as compared to 13 clock cycles in case of power interleaver.

3. Motivation for Numerical Interleaver
As we know that if patterns of interleaving are not random, then MAI among users increases considerably and the performance of MUD at the receiver side doesn’t converge in to a satisfactory value. But the main problem associated with random interleaver is that huge amount of memory space is needed to store user specific random pattern at each point of communication system. For large user count this is not a practical solution.

Figure 3. Algorithm for numerical interleaver structure.

Power interleaver as suggested resolves the above stated problem of random interleaver. According to this scheme a common master random interleaver is used to generate different patterns of multiple users. The only pattern needed to be transmitted during transmission is common master random interleaving pattern and the user count. In this way, the problem of extra bandwidth and memory consumption is resolved at different points of communication system. However, computational complexity increases considerably as the user count goes up. There is still a need to minimize bandwidth and memory requirement and see options for better BER performance.

To summarize here we can say that in tree based interleaver we need to transfer first two rows of inter-
leaving sequence, as receiver does not know them. In random interleaver also, there is need to transfer all interleaved patterns. Power interleaver also needs to transfer interleaved pattern of first user or master pattern. All this results in high bandwidth consumption, which is not acceptable, especially in band-limited environments like UWA environment.

All the problems can be solved if interleaver is such that the transmitted pattern is changed by changing only one or two values, also, which does not require any interleaved pattern to be transmitted with data sequence. Thus, the interleaver design is such that it solves the bandwidth requirement problem along with reduced BER and hardware requirements.

In this work a novel interleaver named as numerical interleaver has been proposed, which can resolve the above-stated problems of existing interleavers in literature.

### 4. Numerical Interleaver Generation Algorithm

As mentioned above that after interleaving the order of a data sequence is re-arranged in a deterministic format. In random interleaver scrambling patterns for different users are arbitrary. But in numerical interleaver data scrambling patterns for different users are deterministic using certain mathematical expressions. That’s why this interleaver is named as numerical interleaver, because the sequence is being generated using some mathematical equation.

To describe the proposed scheme conveniently, total number of users in a cellular system is taken as K, and M is the total number of chips transferred by each user. The mapping of each chip in a sequence is such that it depends upon the mapping of the previous chip according to following numerical equation.

\[ X_n = A \cdot X_{n-1} + C \cdot m \quad 1 \leq n \leq K \times M \]  

(1)

Here variable \( X_o \) is used to map chip position after interleaving as described further, and, A and C are defined as interleaver constants. First value of \( X_o \) can be any integer value. The value of m is changed sequentially according to the chip number of a particular user from 1, 2, 3, … up to M. According to the proposed scheme for each user interleaving sequence pattern vector, p [ ] is obtained sequentially for each chip according to the following steps.

Initialization: \( X_o = 1, \ L = 100, \ A \ and \ C \) can be initialized as any prime numbers.
While \( n \leq K \times M, \) for \( \forall \ n, \)
For \( \forall \ m \)
\[ \text{obtain value of} \ X_n \]
compute \( X_n \cdot M, \) store remainder as r
\[ \text{swap chip position} \ p[m] \text{ with} \ p[r] \]
End for

Store interleaving pattern vector p [ ] of current user
Re-initialize \( m = 1, \) for next user
End while

Note that while moving to the next user, value of \( X_n \)
needs not to be re-initialized. As per this algorithm first value of \( X_n \), for the next user is obtained from previous user’s last value \( X_{n-1} \), and so on the process continues for all the users.

It should be pointed out that the value of \( X_n \) goes on increasing while moving from one user to the next user. However for too large values of \( X_n \) (values out of system specification) it can be re-initialized to a smaller value again. In this work MATLAB environment has been used to simulate and analyze the performance of the proposed scheme, so the upper limit has been set as \( 10^{14} \). Beyond this value \( X_n \) is re-initialized to a value equal to the value of remainder, when \( X_n \) is divided by a limit value \( L \). \( L \) is an arbitrary smaller number chosen to give some appropriate value to re-initialize \( X_n \).

Patterns of interleaving are more correlated in other interleavers described in literature, since they use one or two master random interleavers to generate other user’s interleavers. The proposed numerical interleaver generates more random patterns in comparison to power interleaver and tree-based interleavers. Obviously more randomness among users has a significant influence on MAI cancellation. It should be pointed out that the values of A and C are prime values to create more randomness among users. Any odd or even number or 2 and 5 values are not suitable for A and C to generate random number, if A and C are 2 or even, then result will always be even, similarly if A and C are 5, then result will always contain 5 or 0 in the end. That creates less randomness or more cross correlation. The number generated out of this whole process should be more random rather than always even or odd or always 5 or 0 in the end, therefore, prime values of A and C except 2 and 5 can be used.

Different values of A and C results in different interleaving patterns. This feature can be exploited to choose
an interleaver set according to particular channel conditions.

The detailed algorithm for the proposed method is shown in Figure 3.

For the purpose of an illustration, consider a multiple access system involving 4 users with the chip length per user taken as 4. Constant values $A$ and $C$ have been set as 3 and 5 respectively. The upper limit for $X_n$ is used as $10^{14}$. For $X_n$ initialized as 1, and limit value $L$ as 100, the output patterns following the numerical interleaver algorithm for 3 user are as follows. Before interleaving the sequence is,

1 2 3 4

After interleaving,

3 1 4 2
3 4 1 2
3 2 1 4
1 4 3 2

This whole interleaver matrix can be re-generated at the receiver also, provided the values of $A$ and $C$ have been transmitted. The proposed method requires less information exchange between transmitter and receiver end, as compared to random interleaver. Memory requirement is also reduced at the receiver side while performing MUD. After detecting the signal for first user, the interleaver pattern for second user can be updated using the last variable value $X^M_n$. With this method, at any detection stage, only the current user's interleaver pattern and its variable $X^M_n$ are required to be stored.

In the presented method first order expression is chosen to generate multiple users’ interleaver patterns. If some higher order expression like

$$X_n = A \left(X_{n-1}\right)^2 + B \left(X_{n-1}\right) + C \times m$$

is selected, then frequency of crossing upper limit of $X_n$ will increase. The value of $X_n$ will be re-initialized to a pre-determined lower value, more frequently, resulting in a condition with more cross-correlated interleaver patterns.

5. Performance Evaluation

This section demonstrates the performance of the numerical interleaver through various numerical simulations. The existing interleavers in literature are compared with the proposed interleaver by considering important attributes such as BER performance, hardware requirement and bandwidth occupancy.

5.1 BER Performance

To analyze BER performance of the proposed interleaver in multiple access environment with different channel conditions IDMA scheme has been simulated for AWGN conditions and SC-FDMA-IDMA scheme has been simulated for UWA channel conditions. MATLAB simulations have been performed for each user’s data, convolutionally encoded with rate 1/2. Spreading length is taken as 16 before interleaving with user specific interleaver. BPSK modulation has been applied on 128 bit data for both IDMA and SC-FDMA-IDMA systems.

![Figure 4](image-url) BER performance of Numerical interleaver (A=3, C=5) with other interleavers for 16 users with IDMA in AWGN.

![Figure 5](image-url) BER performance of Numerical interleaver (A=3, C=7) with other interleavers for 32 users with IDMA in AWGN.

Further to evaluate SC-FDMA-IDMA scheme performance, $N = 512$ number of sub-carrier system is simulated
for M=128 complex modulated symbol sequence block. For UWA channel performance shallow water channel model[2] has been considered.

At the receiver side while performing MUD, BPSK modulated 128 bit data is decoded iteratively. RI, PI and TBI have been also simulated to evaluate and compare the performance of numerical interleaver. After a series and loops of test of numerical interleaver with different values of interleaver constants $A$ and $C$, it has been verified that, its BER performance is always better than tree based interleaver and power interleaver and comparable (sometimes better) to random interleaver under same conditions.

Numerical interleaver has been simulated with multiple values of interleaver constants $A$ and $C$ as shown in Figures 4, 5 and 6 for IDMA system in AWGN condition and in Figures 7 and 8 for SC-FDMA-IDMA system in UWA channel condition. As can be seen from Figure 4 and 7 that when total number of users are 16, results for all four interleavers are comparable. Figure 5, 6 and 8 show the performance of various interleavers with 32 users and 64 users also. The performance of tree based interleaver deteriorates as the number of users increase due to high degree of correlation among users. We observe from these Figures that performance of numerical interleaver is still comparable or sometimes better than random interleaver and power interleaver for large number of users as well. So the proposed numerical interleaver can completely substitute the random interleaver in IDMA based systems.

Table 1. Summary of hardware

<table>
<thead>
<tr>
<th>Components</th>
<th>RI</th>
<th>PI</th>
<th>TBI</th>
<th>NI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adder/subtractor</td>
<td>272</td>
<td>1152</td>
<td>590</td>
<td>416</td>
</tr>
<tr>
<td>Comparator</td>
<td>128</td>
<td>576</td>
<td>280</td>
<td>64</td>
</tr>
<tr>
<td>D Flip-Flop</td>
<td>832</td>
<td>3520</td>
<td>1600</td>
<td>96</td>
</tr>
<tr>
<td>Multiplexer</td>
<td>576</td>
<td>2592</td>
<td>1164</td>
<td>----</td>
</tr>
<tr>
<td>Multiplier</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>120</td>
</tr>
</tbody>
</table>

5.2 Hardware Requirement and Bandwidth Requirements

Field Programmable Gate Array (FPGA) implementation is an effective tool in analyzing and comparing the hard-
ware requirements of different algorithm for circuits. All the interleavers including RI, PI, TBI and NI have been implemented on FPGA in XILINX environment for 8 users with 8 chips/ user. The results obtained are as follows.

5.2.1 Hardware Requirement

Here the comparison of RI, PI, TBI and NI is performed based on hardware requirements for generation of user-specific interleaver at transmitter side. For the comparison purpose, the parameters selected are D-flip flop, comparator, adder/subtractor and multiplexer.

As shown by Table 1, it is evident that the requirement of hardware components for the numerical interleaver is at minimum level in comparison to those for others. Power interleavers require highest number of hardware components, at transmitter end. Hardware requirement increases successively for each next user as the number of user increases in a system, due to number of loops required to generate an interleaver also increases. While in case of random interleaver generation mechanism, hardware requirement does not increase significantly with increment in user count.

5.2.2 Final Register Requirement

Table 2. Final register report

<table>
<thead>
<tr>
<th>Components</th>
<th>RI</th>
<th>PI</th>
<th>TBI</th>
<th>NI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Register</td>
<td>850</td>
<td>3528</td>
<td>1600</td>
<td>96</td>
</tr>
<tr>
<td>Flip-Flop</td>
<td>850</td>
<td>3528</td>
<td>1600</td>
<td>96</td>
</tr>
</tbody>
</table>

It is evident from Table 2 that the requirement of registers and flip-flops for power interleaver is at the highest level in comparison to that for RI, TBI and NI. The numerical interleaver requires least amount of components in final register report due to requirement of lesser components for the storage of intermediate interleaving sequences occurring during the calculation of user-specific interleavers.

5.2.3 Device Utilization Report

As can be seen through Table 3, the summary of device utilization is presented for RI, PI, TBI and NI. The parameters for comparison are chosen to be count of slices, Look-Up Tables (LUTs), and inputs/ output (I/O) ports, required for structuring the interleavers. LUTs are arrangement of data in the form of an array. Use of LUTs reduces the processing time required for complex computation to a simple array indexing process.

<table>
<thead>
<tr>
<th>Components</th>
<th>RI</th>
<th>PI</th>
<th>TBI</th>
<th>NI</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of slices</td>
<td>9381</td>
<td>48245</td>
<td>22176</td>
<td>13808</td>
</tr>
<tr>
<td>No. of slice Flip-Flops</td>
<td>847</td>
<td>3528</td>
<td>1600</td>
<td>96</td>
</tr>
<tr>
<td>No. of 4 input LUTs</td>
<td>18180</td>
<td>94799</td>
<td>43708</td>
<td>26565</td>
</tr>
<tr>
<td>No. of Ios</td>
<td>129</td>
<td>129</td>
<td>129</td>
<td>129</td>
</tr>
<tr>
<td>No. of bonded IOBs</td>
<td>129</td>
<td>129</td>
<td>129</td>
<td>121</td>
</tr>
</tbody>
</table>

In power interleaver, more LUTs are needed in order to store intermediate calculations due to looping operation. However, in random interleaver, significant count of LUTs and I/O ports are required to store the generated user-specific interleavers. Comparatively lesser amount of LUTs are needed in tree based interleaver for storage of data as compared to power interleaver. In numerical interleaver, LUTs are needed only to store the value of variable $X_n$ computed for last user.

The requirement of I/O ports is dependent on deployment of respective LUTs. Therefore, due to lesser requirement of LUTs, the numerical interleaver needs least amount of I/O devices. From the above stated comparison, it is apparent that the numerical interleaver requires least amount of devices while highest numbers of components are needed to construct power interleaver, for 8 users with data of 8 chips/ user.

5.2.3 Bandwidth Requirements

This section provides analysis to assess the bandwidth consumption of the four interleavers discussed in the previous sections. Data format required for exchanging information between base station and mobile station regarding interleavers is shown in Figure 9. It is assumed that the system has 100 users and one block of data transmitted by each user is 256 bits long (after coding and spreading). Note that 8 bits are required to represent sequence number for 256 bits data and 7 bits are needed to denote user number.

It can be easily understood that random interleaver will consume the maximum bandwidth due to requirement of transmission of the entire user-specific interleaving matrix. While tree based interleaver will consume comparatively more bandwidth in comparison to power interleaver, due to deployment of two master interleavers in place of only one master interleaver deployed in case of power interleaver. However, bandwidth requirement of numerical interleaver is very less as compared to
RI, PI and TBI, as we need to transmit only two integer values of $A$ and $C$ to the receiver. As discussed in previous sections that bandwidth is a critical resource in UWA communications, so the presented numerical interleaver is a good choice for UWA communications.

6. Conclusion

In this communication we have proposed an interleaver and its comprehensive comparison with other interleavers including RI, PI, and TBI. We performed computer simulations in various scenarios with IDMA system and SC-FDMA-IDMA systems under different channel conditions including AWGN and UWA. It is pointed out that by changing the values of $A$ and $C$ all interleaving sequences can be changed. This feature can be exploited to make whole interleaver matrix more channel-dependent, and we can use this interleaver in acoustic environment also. It is observed that BER performance of numerical interleaver is comparable to random interleaver and power interleaver, and better than tree based interleaver. This makes numerical interleaver a potential candidate for next-generation wireless communication systems based on IDMA schemes. FPGA implementation of all the concerned interleavers has been carried out in this paper. After the analysis, it has been observed that proposed numerical interleaver requires least hardware as compared to other interleavers discussed in this work. The bandwidth requirement of proposed numerical interleaver is found to be considerably less than that of RI, PI and TBI. Hence, the numerical interleaver mechanism is ideally best suited for IDMA based multiple access sys-

Figure 9. Data format for (a) random interleaver (b) power interleaver c) tree based interleaver (d) numerical interleaver.
tems in band-limited acoustic environment also, as its integral part.

7. Acknowledgement

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8. References