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Assessment of Performance of RC Beams Strengthened with Hybrid FRP Laminates using Multivariate Linear Regression Modeling

J. Karthick*, K. Suguna and P. N. Raghunath

Department of Civil and Structural Engineering, Annamalai University, Chidambaram - 608002, Tamil Nadu, India;
karthickjaisankar16@gmail.com, sugunaraghunath@gmail.com, pnr_ks@yahoo.com

Abstract

Objectives: An attempt has been made to suggest regression model for evaluating concrete beams with externally bonded hybrid FRP reinforcement using OriginPro-8.5 software. **Methods:** Fourteen beam specimens were subjected to monotonic and cyclic loading. The mechanical and geometrical properties of constituent materials such as thickness, tensile strength and elasticity modulus of FRP laminates, and characteristic strength of concrete and characteristic strength of steel were considered as independent variables. The performance parameters such as strength, deformation, ductility, crack history and number of cycles were considered as dependent variables. **Findings:** After conducting the multivariate linear regression analysis, a fitness of 0.192 to 0.850 and a root mean square error of 0.201 to 8.311 were observed while predicting the performance parameters of test beams under static loading condition. A fitness of 0.477 and a root mean square error of 1.702 were observed while evaluating the test beam specimens under cyclic loading condition. This observation clearly shows the validity of the proposed regression equations for evaluating the test beam specimens under monotonic and cyclic loading. The predicted results and the test results were compared through scatter plots. The predicted results have also been validated with other researcher's experimental data. **Applications:** By using this regression equation, the performance of the both unstrengthened and strengthened beams can be assessed without any experimental work.

Keywords: GFRP; Hybrid FRP, Linear Regression, RC Beams, Strengthening

1. Introduction

FRP composites find broad spectrum of applications in civil engineering. Upgradation of structures becomes mandatory due to changes in loading and occupancy, ageing and degeneration as a result of environmental effects. FRP stands out as a promising player for the purpose of structural hardening. Current literature available on the performance of FRP strengthened concrete beams provides a practical insight into their strength and ductility. In[1] several investigations have been carried out on concrete beams with externally bonded mono and hybrid FRP. It was observed that the strength increased by 114% for the upgraded beams compared to control beams. In[2] studied the response of concrete beams with surface mounted mono and hybrid FRP. The authors reported an increase of 30-98% in load capacity based on the proportion of carbon and glass fibres in hybrid FRP. In[3–5] also investigated the response of RC beams with surface mounted mono and hybrid FRP. Meanwhile, in recent years, computational tools such as Artificial Neural Networks (ANN), Adaptive Neuro-Fuzzy Inference Systems (ANFIS), Genetic Algorithms (GA) and regression modeling have been used in various civil engineering applications. In[4] conducted regression analysis for HSC beams with external GFRP subjected to monotonic and cyclic loading. The authors observed a good correlation between the predicted and test results.

1.1 Research Significance

In the existing literature, evaluation of concrete beams with externally bonded FRP reinforcement using soft computing technique was not much attempted and only limited efforts have been made in developing a regression

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** Author for correspondence*

model for the problem. The present study focuses on assessing the static and cyclic behaviour of concrete beams with surface mounted FRP reinforcement using multi-variate regression model. The predicted results and the test results were compared through scatter plots. The predicted results have also been validated with other researcher's experimental data.

2. Materials and Methods

2.1 Regression Modeling

Regression is the attempt to explain the variation in a dependent variable using the variation in independent variables. Linear regression provides additional statistical information about the relationship between two quantitative variables. The regression equation is the formula for the trend or fit line which enables us to predict the dependent variable for any given value of the independent variable. Normally, the regression equation has two parts - the intercept and the slope. The intercept is the point on the vertical axis where the regression line crosses. Generally, it does not offer a few valuable statistics. The slope is the change in the dependent variable for unit change in the independent variable. The slope tells us the direction and magnitude of the change.

2.2 Estimation of Performance of Regression Model

The accuracy of regression model can be ascertained through the estimation of the following errors:

2.2.1 Legendre's Principle of Least Squared Error

Legendre's principle of least squared is the widely used technique for solving linear regression problems. Linear least squares regression also gets its name from the way the estimates of the unknown parameters are computed. In the least squares method the unknown parameters are estimated by minimizing the sum of the squared deviations between the data and the model. The minimization process reduces the over determined system of equations.

2.2.2 Karl Pearson's Coefficient of Correlation

Karl Pearson's Coefficient of Correlation is the most widely used method of measuring the degree of relationship between two variables. Correlation co-efficient between two variables x and y, usually denoted by r(x,y) of r_{xy} is a numerical measure of linear relationship between them and is defined as

$$r_{xy} = \frac{E(xy) - E(x)E(y)}{\sigma_x \sigma_y} = \frac{\operatorname{cov}(x,y)}{\sigma_x \sigma_y}$$

2.2.3 Regression Coefficient

The equation to the line of regression of y on x is $y - \bar{y} = r_{xy}\frac{\sigma_y}{\sigma_x}(x - \bar{x})$ and the equation to the line of regression of x on y is $x - \bar{x} = r_{xy}\frac{\sigma_x}{\sigma_y}(y - \bar{y})$

$r_{xy}\frac{\sigma_y}{\sigma_x}$ is called the regression coefficient of y on x and is denoted by b_{yx}.

$r_{xy}\frac{\sigma_x}{\sigma_y}$ is called the regression coefficient of x on y and is denoted by b_{xy}.

2.2.4 Sum of Squared Errors

Error sum of squares SSE is the sum of squared differences between each observation and its group's mean. SSE is defined as

$$SSE = \sum_{i=1}^{n} (x_i - E(x))^2$$

2.2.5 Mean Squared Error

The mean squared error of an estimator measures the average of the squares of the errors, the MSE is defined as

$$MSE = \frac{\sum_{i=1}^{n} \left(x_i - E(x)\right)^2}{n}$$

2.2.6 Root Mean Squared Error

Root Mean Squared Error (RMSE) is calculated by taking the square root of Mean Squared Error. The mathematical expression for Root Mean Squared Error (RMSE) is as follows:

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{n} (x_i - E(x))^2}{n}}$$

2.2.7 Root Mean Squared Percentage Error (RMSPE)

Root Mean Squared Percentage Error (RMSPE) is the square root of the sum of squared values of percentage error in prediction divided by the number of samples. The accuracy of prediction can be assessed through calculating this error. Thus, higher values of this error denote higher error levels.

$$\text{RMSPE}=\sqrt{\frac{\sum_{i=1}^{n}(x_i - E(x))^2}{n}}\times 100$$

2.3 Multivariate Linear Regression

This analysis is appropriate when the researcher has a one dependent variable which is presumed to be a function of two or more independent variables. The objective of this analysis is to make a prediction about the dependent variable based on its covariance with all the concerned independent variables. The fundamental equation of this analysis is,

$$\left\{\begin{bmatrix} \dfrac{\partial}{\partial a_0} \\ \dfrac{\partial}{\partial a_1} \\ \dfrac{\partial}{\partial a_2} \\ \dfrac{\partial}{\partial a_3} \\ \vdots \\ \dfrac{\partial}{\partial a_n} \end{bmatrix}\right\} \sum_{i=1}^{K}\left(P_i - \left(a_0 + a_1 x_{1\mathrm{i}} + a_2 x_{2\mathrm{i}} a_3 x_{3\mathrm{i}} + \cdots + a_n x_{ni}\right)\right) = \left\{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}\right\}$$

where, $a_0 \cdots a_n$ are the constants to be evaluated, $x_1 \cdots x_n$ are the independent variables, P is the dependent variable or the actual result value for the set of i^{th} input data and K is the number data sets available for regression. On executing the partial derivative operators, above equation reduces to,

$$\sum_{i=1}^{K}\begin{bmatrix} 1 & x_{1\mathrm{i}} & x_{2\mathrm{i}} & x_{3\mathrm{i}} & \cdots & x_{ni} \\ x_{1\mathrm{i}} & x_{1\mathrm{i}}^2 & x_{1\mathrm{i}}\, x_{2\mathrm{i}} & x_{1\mathrm{i}}\, x_{3\mathrm{i}} & \cdots & x_{1\mathrm{i}}\, x_{ni} \\ x_{2\mathrm{i}} & x_{2\mathrm{i}}\, x_{1\mathrm{i}} & x_{2\mathrm{i}}^2 & x_{2\mathrm{i}}\, x_{3\mathrm{i}} & \cdots & x_{2\mathrm{i}}\, x_{ni} \\ x_{3\mathrm{i}} & x_{3\mathrm{i}}\, x_{1\mathrm{i}} & x_{3\mathrm{i}}\, x_{2\mathrm{i}} & x_{3\mathrm{i}}^2 & \cdots & x_{3\mathrm{i}}\, x_{ni} \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ x_{ni} & x_{ni}\, x_{1\mathrm{i}} & x_{ni}\, x_{2\mathrm{i}} & x_{ni}\, x_{3\mathrm{i}} & \cdots & x_{ni}\, x_{ni} \end{bmatrix}\left\{\begin{matrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_n \end{matrix}\right\} = \sum_{i=1}^{K}\left\{\begin{matrix} P_i \\ P_1 P_i \\ P_2 P_i \\ P_3 P_i \\ \vdots \\ P_n P_i \end{matrix}\right\}$$

The above equation can be resolved by summing up the values of independent and dependent variables after carrying out the essential operations. The logic presented in above equation was executed using Orginpro-8.5 software to estimate the regression coefficients.

3. Results and Discussion

3.1 Regression Equations

Regression equations have been proposed for evaluating concrete beams with surface mounted FRP reinforcement under static and cyclic loading. The parameters considered for the study were load at first crack stage, deflection at first crack stage, load at yield stage, deflection at yield stage, load at service stage, deflection at service stage, load at ultimate stage, deflection at ultimate stage, crack width at ultimate stage, number of cracks, average spacing of cracks, deflection ductility and deflection ductility ratio in static loading condition and number of cycles in cyclic loading condition. The nomenclature of all the test beams is presented in Table 1. The experimental results used for regression analysis are presented in Table 2. The proposed regression equations of tested beams are presented in Table 3. The results predicted through regression analysis are presented in Table 4. The proposed regression equation for number of cycles is presented in Table 5. The number of cycles predicted through regression analysis and the experimental data used for regression analysis are presented in Table 6.

Table 1. Nomenclature

Test beam	Description
S1-1/S2-1	Control beams
S1-2/ S2-2	Beams strengthened with three layers of GFRP sheets
S1-3/ S2-3	Beams strengthened with five layers of GFRP sheets
S1-4/ S2-4	Beams strengthened with one layer of CFRP and two layers of GFRP sheets
S1-5/ S2-5	Beams strengthened with two layers of CFRP and two layers of GFRP sheets
S1-6/ S2-6	Beams strengthened with three layers of CFRP and two layers of GFRP sheets
S1-7/ S2-7	Beams strengthened with five layers of CFRP and two layers of GFRP sheets

Table 2. Experimental results of tested beams used in regression analysis

Beam Designation	S1-1	S1-2	S1-3	S1-4	S1-5	S1-6	S1-7
Thickness of FRP Laminates (mm)	0	3.00	5.00	3.52	3.96	4.39	5.26
Tensile Strength of FRP Laminates (MPa)	0	446.90	451.50	382.89	401.81	440.32	463.05
Elasticity Modulus of FRP Laminates (GPa)	0	13.90	17.40	26.80	28.20	30.80	32.40
Compressive Strength of concrete (MPa)	25.60	25.60	25.60	25.60	25.60	25.60	25.60
Yield Strength of Steel (MPa)	572.29	572.29	572.29	572.29	572.29	572.29	572.29
First Crack Load (kN)	12.50	27.50	37.50	20.00	27.50	30.00	40.00
Deflection at First Crack Load (mm)	0.95	1.16	1.28	2.90	2.50	1.60	1.10
Yield Load (kN)	32.00	70.00	82.50	56.00	62.00	70.00	80.00
Deflection at Yield Load (mm)	5.00	4.02	4.33	7.50	7.00	6.50	5.80
Ultimate Load (kN)	70.00	112.50	117.50	145.00	162.50	170.00	190.00
Deflection at Ultimate Load (mm)	15.80	16.50	18.60	29.30	26.20	25.30	23.60
Crack Width (mm)	0.26	0.26	0.26	0.16	0.20	0.20	0.22
No. of Cracks	16.00	27.00	31.00	41.00	40.00	36.00	33.00
Average Spacing of Cracks(mm)	145.00	58.00	48.00	32.00	47.00	58.00	72.00
Deflection Ductility	3.16	4.10	4.30	3.91	3.74	3.89	4.07
Deflection Ductility Ratio	1.00	1.47	1.54	1.24	1.18	1.23	1.29
Energy Ductility	6.57	12.72	13.45	9.39	10.54	7.83	6.98
Energy Ductility Ratio	1.00	2.25	2.65	1.43	1.61	1.19	1.06

Table 3. Regression equations for tested beams

Sl. No	Predicted parameter	Regression Equation	Fitness	RMSE
1	First Crack Load (kN)	$3.611E\text{-}5 + 2.681t_{FRP} + 0.032f_{FRP} - 0.288E_{frp} + 9.245f_{ck} + 0.021f_{y}$	0.365	4.292
2	Deflection at First Crack load (mm)	$3.076E\text{-}5 - 0.295t_{FRP} + 1.582E\text{-}6f_{FRP} + 0.087E_{frp} + 7.874E\text{-}5f_{ck} + 0.002f_{y}$	0.365	0.650
3	Yield Load(kN)	$159.360 + 9.808t_{FRP} + 0.043f_{FRP} - 0.786E_{frp} - 0.661f_{ck} - 0.197f_{y}$	0.850	2.836
4	Deflection at Yield load(mm)	$-5.343 + 0.175t_{FRP} + 0.004f_{FRP} - 0.076E_{frp} + 0.135f_{ck} + 0.012f_{y}$	0.307	1.701
5	Service Load(kN)	$-147.245 + 10.899t_{FRP} + 0.029f_{FRP} - 0.099E_{frp} + 0.322f_{ck} + 0.318f_{y}$	0.696	7.698
6	Deflection at Service Load (mm)	$-9.272 + 1.533t_{FRP} + 0.009f_{FRP} - 0.208E_{frp} + 0.337f_{ck} + 0.018f_{y}$	0.171	4.089
7	Ultimate Load(kN)	$-220.846 + 16.350t_{FRP} + 0.043f_{FRP} - 0.150E_{frp} + 0.483f_{ck} + 0.476f_{y}$	0.696	8.311
8	Deflection at Ultimate Load (mm)	$34.685 + 2.058t_{FRP} + 0.019f_{FRP} - 0.386E_{frp} + 0.449f_{ck} - 0.064f_{y}$	0.551	2.996
9	Crack Width at Ultimate Stage (mm)	$2.126E\text{-}6 + 0.051t_{FRP} - 9.201E\text{-}4f_{FRP} - 0.005E_{frp} + 5.441E\text{-}5f_{ck} + 0.001f_{y}$	0.563	0.225
10	Number of Cracks	$5.425E\text{-}5 - 0.470t_{FRP} + 0.007f_{FRP} + 0.633E_{frp} + 0.001f_{ck} + 0.031f_{y}$	0.799	0.201
11	Average spacing of cracks (mm)	$4.532E\text{-}4 + 2.633t_{FRP} - 0.220f_{FRP} - 0.422E_{frp} + 0.012f_{ck} + 0.259f_{y}$	0.810	0.119
12	Deflection Ductility	$1.362 + 0.378t_{FRP} + 1.216E\text{-}4f_{FRP} - 0.024E_{frp} + 0.047f_{ck} + 5.629E\text{-}4f_{y}$	0.192	3.541
13	Deflection Ductility Ratio	$-4.558 + 0.113t_{FRP} + 3.104E\text{-}4f_{FRP} - 0.011E_{frp} + 0.023f_{ck} + 0.009f_{y}$	0.793	5.790

Note: tFRP-Thickness of FRP laminate (mm) ; fFRP- Tensile Strength of FRP laminate (MPa) ; Efrp- Elasticity Modulus of FRP laminate (GPa) ; fck- Compressive Strength of Concrete at 28 days (MPa) ; fy-Yield strength of tensile Steel (MPa).

Table 4. Results predicted through regression analysis

Sl. No.	Test Beam	First Crack Load (kN)	Deflection at First Crack Load (mm)	Yield Load (kN)	Deflection at Yield Load (mm)	Service Load (kN)	Deflection at Service Load (mm)	Ultimate Load (kN)	Deflection at Ultimate Load (mm)	Crack Width at Ultimate Stage (mm)	No. of Cracks	Average Spacing of Cracks (mm)	Deflection Ductility	Deflection Ductility Ratio
1	S1-1	11.50	1.10	29.70	4.98	42.99	9.66	63.92	9.56	0.57	18	148.53	2.86	1.18
2	S1-2	29.80	1.50	67.42	6.24	84.27	15.39	120.11	18.86	0.25	28	52.25	3.71	1.50
3	S1-3	34.30	1.20	84.48	6.33	88.86	17.77	122.48	21.71	0.33	30	55.02	4.39	1.69
4	S1-4	25.40	2.40	59.62	5.09	89.81	12.93	133.92	13.73	0.27	36	36.54	3.59	1.40
5	S1-5	26.80	2.40	63.65	5.14	105.01	13.48	151.72	14.46	0.26	37	52.66	3.73	1.44
6	S1-6	28.50	2.50	67.48	5.17	103.56	13.94	160.02	15.07	0.24	38	50.22	3.83	1.47
7	S1-7	31.10	2.00	75.73	5.29	119.54	15.15	184.98	16.67	0.25	39	66.84	4.13	1.56

Table 5. Proposed regression equation for number of cycles

Sl. No	Predicted parameter	Regression Equation	Fit-ness	RMSE
1	Number of Cycles	$-0.484 + 0.080B - 0.034D - 1.522\rho + 0.218f_{ck} + 0.649t_{frp}$	0.477	1.702

Table 6. Number of cycles predicted through regression analysis

Sl. No	Test Beam	B (mm)	D (mm)	ρ (%)	f_{ck} (MPa)	t_{FRP} (MPa)	No. of Cycles	
							Exp.	Reg.
1	S2-1	150	250	0.603	25.6	0.00	5	8
2	S2-2	150	250	0.603	25.6	3.00	11	10
3	S2-3	150	250	0.603	25.6	5.00	12	11
4	S2-4	150	250	0.603	25.6	3.52	10	10
5	S2-5	150	250	0.603	25.6	3.96	10	10
6	S2-6	150	250	0.603	25.6	4.39	9	11
7	S2-7	150	250	0.603	25.6	5.26	9	11

Note: B - Breath of the section (mm) ; D - Depth of the section (mm) ; ρ- Reinforcement Ratio ; fck- Compressive Strength of Concrete at 28 days (MPa) ; tFRP-Thickness of FRP laminate (mm).

3.2 Validation of Predicted Results

The experimental results pertinent to this problem from other researchers published work were used for the purpose of validating the model. Data such as breath of the section (B), depth of the section (D), reinforcement ratio (ρ), thickness of FRP laminates (t_{frp}), tensile strength of FRP laminates (f_{frp}), modulus of elasticity of FRP laminates (E_{frp}), characteristic strength of concrete at 28 days (f_{ck}) and yield strength of steel (f_y) taken from the research works[1–3,5–11] are presented in Tables 7 to 9. Orginpro-8.5 software has been used for formulating the proposed regression equation. The proposed equation has been validated against the test data reported in the published literature. The results predicted through regression modeling are presented through Figure 1 (a) to (n) in the form of scatter plots. The convergence of predicted results through regression modeling has been evaluated using statistical indicators and presented in Table 10. Good convergence was observed between the experimental results and those predicted through regression modeling.

Table 7. Source data used for validation of predicted results (independent variables)

Author	Beam Designation	Thickness of FRP laminates (mm)	Tensile Strength of FRP (MPa)	Elasticity Modulus of FRP (GPa)	Compressive Strength of Concrete (MPa)	Yield Strength of Steel (MPa)
Attari et al, (2012)	PA1	1.50	403.00	43.50	39.00	500.00
	PA2	2.00	325.00	19.20	39.00	500.00
	PA3	1.60	400.00	28.00	39.00	500.00
	PB4	2.00	218.00	27.00	39.00	500.00
	PB5	2.00	218.00	27.00	39.00	500.00
	PB6	2.00	218.00	27.00	39.00	500.00
Kim et al, (2011)	GG	0.34	648.51	7.06	31.93	531.16
	CG	0.45	648.51	9.36	31.93	531.16
	GC	0.45	648.51	9.39	31.93	531.16
	CCG	0.56	648.51	11.66	31.93	531.16
	GCG	0.79	648.51	16.42	31.93	531.16
	GCC	0.56	648.51	11.66	31.93	531.16
Hawilleh et al,(2014)	BG	0.35	786.50	34.13	50.00	540.00
Demakos et al, (2013)	B1/GFRP	1.30	550.00	27.50	20.00	550.00
	B1/GFRP/CMAS	1.30	550.00	27.50	20.00	550.00
Shanmugavelu et al, (2015)	SCB	0.00	0.00	0.00	24.00	456.00
	SCSM3	3.00	126.20	7.47	24.00	456.00
	SCSM5	5.00	156.00	11.39	24.00	456.00
	SWR3	3.00	147.40	6.86	24.00	456.00
	SWR5	5.00	178.09	8.99	24.00	456.00

Table 8. Source data used for validation of predicted results (dependent variables)

Author	Test Beam	First Crack Load (kN)	Deflection at First Crack Load (mm)	Yield Load (kN)	Deflection at Yield Load (mm)	Service Load (kN)	Deflection at Service Load (mm)	Ultimate Load (kN)	Deflection at Ultimate Load (mm)	Crack Width at Ultimate Stage (mm)	No. of Cracks	Average Spacing of Cracks (mm)	Deflection Ductility	Deflection Ductility Ratio
Attari et al, (2012)	PA1	-	-	43.60	5.20	51.84	14.04	77.76	21.06	-	-	-	4.05	0.74
	PA2	-	-	45.76	5.17	52.63	15.10	78.95	22.65	-	-	-	4.38	0.8
	PA3	-	-	43.59	5.15	57.65	16.59	86.47	24.88	-	-	-	4.83	0.89
	PB4	-	-	41.71	5.10	51.09	11.22	76.64	16.83	-	-	-	3.30	0.61
	PB5	-	-	43.28	5.12	45.47	10.72	68.20	16.08	-	-	-	3.14	0.58
	PB6	-	-	41.20	5.45	36.63	6.87	54.94	10.30	-	-	-	1.89	0.35
Kim et al, (2011)	GG	-	-	57.24	8.64	53.51	17.34	80.26	26.01	-	-	-	3.01	0.85
	CG	-	-	57.45	7.79	55.59	17.30	83.38	25.95	-	-	-	3.33	0.94
	GC	-	-	58.81	7.22	55.59	16.85	83.38	25.28	-	-	-	3.5	0.99
	CCG	-	-	54.82	9.31	63.63	19.06	95.45	28.59	-	-	-	3.07	0.87
	GCG	-	-	62.86	8.24	66.64	15.49	99.96	23.23	-	-	-	2.82	0.79
	GCC	-	-	68.97	7.65	69.45	15.75	104.17	23.63	-	-	-	3.09	0.88
Rami Hawilleh et al, (2014)	BG	-	-	25.34	7.40	51.23	13.79	76.84	20.69	-	-	-	3.03	1.09
Demakos et al, (2013)	B1/GFRP	-	-	45.00	2.83	45.00	4.00	67.50	6.00	-	-	-	2.12	0.59
	B1/GFRP/ CMAS	-	-	49.00	3.01	49.67	4.40	74.50	6.60	-	-	-	2.19	0.61
Shanmugavelu et al, (2015)	SCB	40	1.1	40.00	3.30	43.33	6.13	65.00	9.20	0.48	33	72	2.79	1.00
	SCSM3	12.5	0.95	45.00	3.55	65.00	8.97	97.50	13.45	0.12	9	145	3.79	1.36
	SCSM5	15	0.97	50.00	3.65	68.33	9.60	102.50	14.40	0.16	14	110	3.95	1.41
	SWR3	17.5	1.02	52.50	3.70	66.67	9.40	100.00	14.10	0.24	18	86	3.81	1.37
	SWR5	20	1.1	60.00	3.88	71.67	10.33	107.5	15.50	0.30	22	72	3.99	1.43

Table 9. Source data used for validation of number of cycles

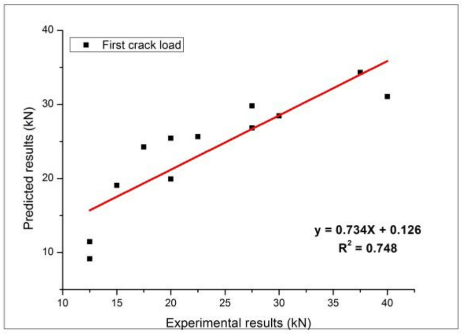
Author	Beam Designation	B (mm)	D (mm)	ρ (%)	f_{ck} (MPa)	t_{FRP} (MPa)	No. of Cycles	
							Exp.	Reg.
Shanmugavelu et al, (2016)	S-CB	150	250	0.603	21.2	0	6	7
	S-CSM3	150	250	0.603	22.4	3	8	9
	S-CSM5	150	250	0.603	22.5	5	9	10
	S-WR3	150	250	0.603	22.8	3	9	9
	S-WR5	150	250	0.603	23.2	5	10	10
	S-UDC3	150	250	0.603	23.6	3	11	9
	S-UDC5	150	250	0.603	23.8	5	15	11
Wight and Sozen (2011)	1	149.86	256.5	1.47	26.2	0	7	6
	2	152.4	256.5	1.47	26.2	3	7	8
	3	157.48	256.5	1.47	26.2	0	9	7
	4	154.94	256.5	1.47	26.2	3	9	9
Scribner and Wright (2010)	1	203.2	218.4	1.27	19.2	0	12	11
	2	203.2	218.4	1.27	19.2	3	12	13
	3	203.2	256.4	1.63	19.2	0	7	9
	4	203.2	256.4	1.63	19.2	3	8	11
	5	254	307.34	2.62	19.2	0	10	10
	6	254	307.34	2.62	19.2	3	12	12
DarvinandNmai(1986)	F-1	190.5	387.35	1.03	22.1	0	5	5
	F-2	190.5	390.65	1.02	20.1	0	2	4
	F-3	190.5	387.35	1.02	23.2	3	6	7
	F-4	190.5	390.65	0.69	24.2	0	9	6
	F-5	190.5	387.35	0.69	23.8	3	8	8
Thandavamoorthy (1999)	FB 1-1	150	400	0.565	57.43	0	8	10
	FB 1-2	150	400	0.565	57.48	0	12	10
	FB 2-1	150	400	0.392	51.27	0	6	8
	FB 2-2	150	400	0.392	52.79	0	9	9

Table 10. Estimation of convergence of predicted results using statistical indicators

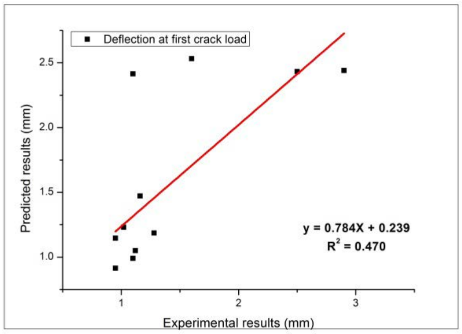
Parameter	Predicted Results through Regression Equations		
	Standard Error	R^2	r
First Crack Load (kN)	0.126	0.748	0.878
Deflection at First Crack load (mm)	0.310	0.415	0.684
Yield Load(kN)	0.071	0.880	0.941
Deflection at Yield load(mm)	0.108	0.445	0.687
Service Load(kN)	0.095	0.769	0.883
Deflection at Service Load (mm)	0.108	0.337	0.607
Ultimate Load(kN)	0.094	0.757	0.877
Deflection at Ultimate Load (mm)	0.106	0.641	0.811
Crack Width at Ultimate Stage (mm)	0.131	0.713	0.860
Number of Cracks	0.106	0.852	0.931
Average spacing of cracks (mm)	0.128	0.796	0.903
Deflection Ductility	0.108	0.353	0.620
Deflection Ductility Ratio	0.084	0.834	0.918
Number of Cycles	0.089	0.545	0.748

3.3 Observations on the Regression Equations

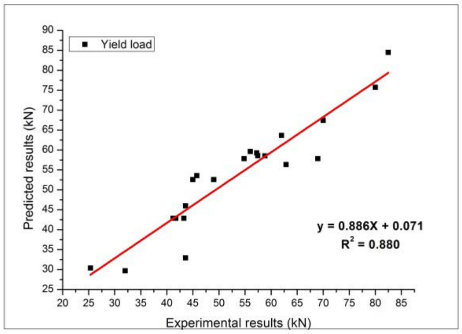
After conducting the multivariate linear regression analysis, a fitness of 0.192 to 0.850 and a root mean square error of 0.201 to 8.311 were observed while predicting the performance parameters of test beams under static loading condition. A fitness of 0.477 and a root mean square error of 1.702 were observed while predicting the performance parameters of test beams under cyclic loading condition. The above observations clearly show the validity of the proposed regression equations for evaluating the performance parameters of both unstrengthened and strengthened beams under monotonic and cyclic loading. Also, good convergence was observed between the experimental results and those obtained from multivariate linear regression modeling (Table 9).


First crack load
y = 0.734X + 0.126
R² = 0.748
Predicted results (kN)
Experimental results (kN)
10
20
30
40
10
15
20
25
30
35
40

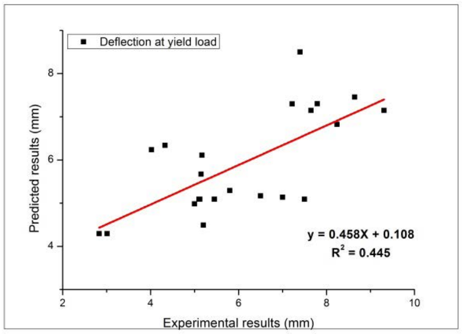
(a)


Deflection at first crack load
y = 0.784X + 0.239
R² = 0.470
Predicted results (mm)
Experimental results (mm)
1.0
1.5
2.0
2.5
1
2
3

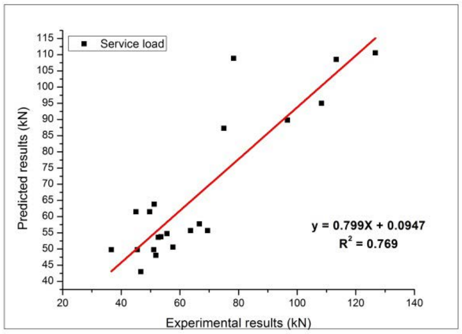
(b)


Yield load
y = 0.886X + 0.071
R² = 0.880
Predicted results (kN)
Experimental results (kN)
25
30
35
40
45
50
55
60
65
70
75
80
85
90
20
25
30
35
40
45
50
55
60
65
70
75
80
85

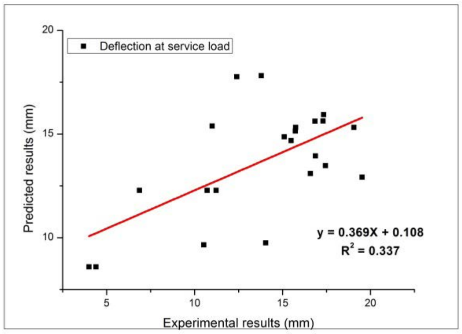
(c)


Deflection at yield load
y = 0.458X + 0.108
R² = 0.445
Predicted results (mm)
Experimental results (mm)
4
6
8
2
4
6
8
10

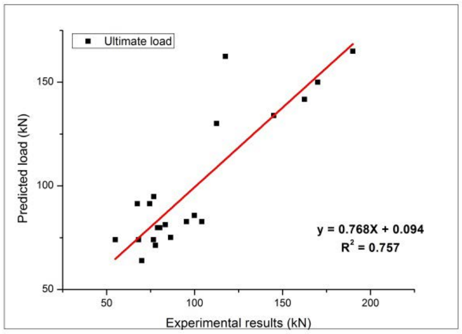
(d)


Service load
y = 0.799X + 0.0947
R² = 0.769
Predicted results (kN)
Experimental results (kN)
40
45
50
55
60
65
70
75
80
85
90
95
100
105
110
115
20
40
60
80
100
120
140

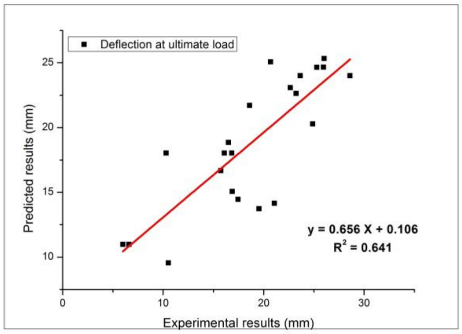
(e)


Deflection at service load
y = 0.369X + 0.108
R² = 0.337
Predicted results (mm)
Experimental results (mm)
10
15
20
5
10
15
20

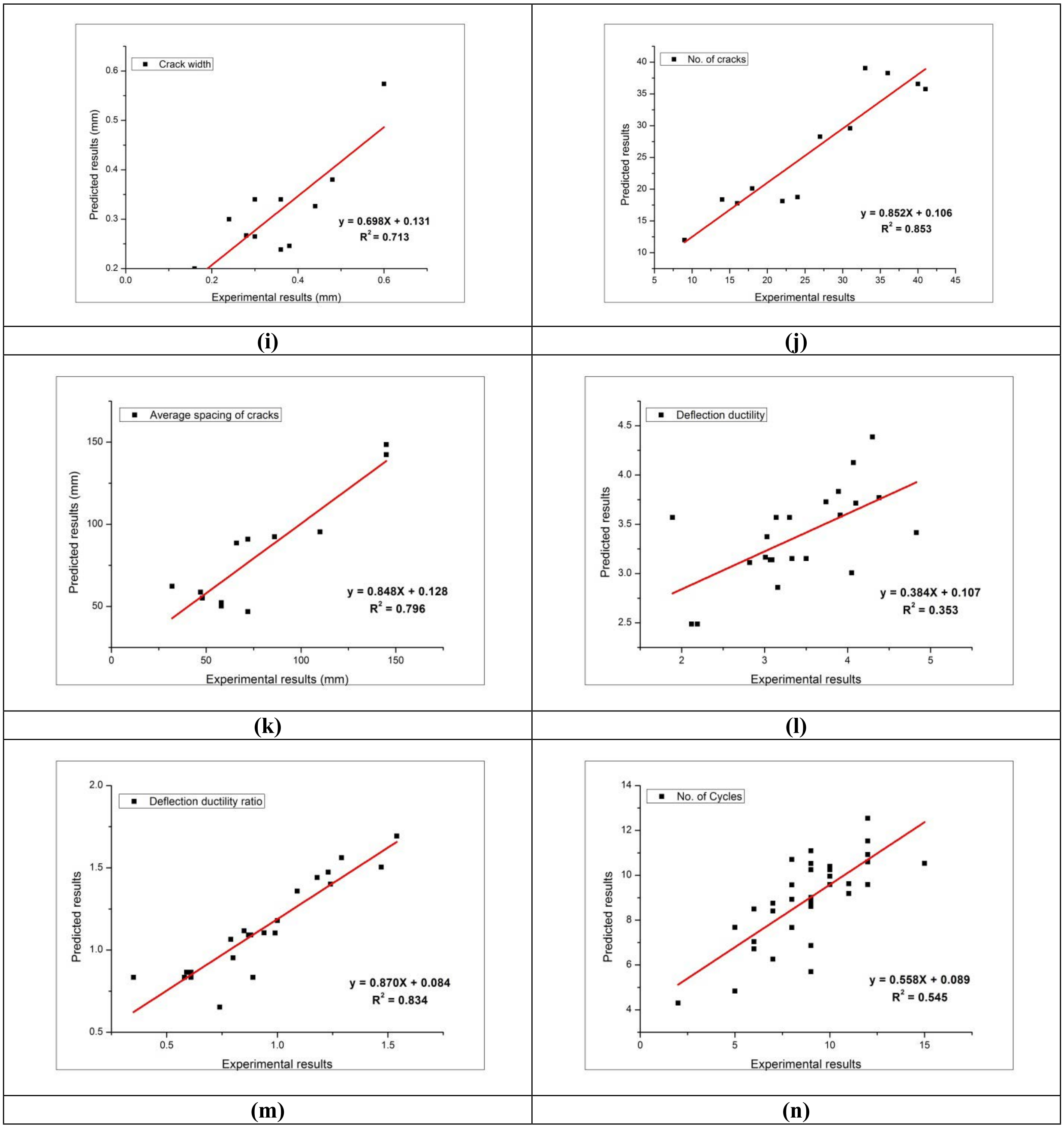
(f)


Ultimate load
y = 0.768X + 0.094
R² = 0.757
Predicted load (kN)
Experimental results (kN)
50
100
150
50
100
150
200

(g)


Deflection at ultimate load
y = 0.656 X + 0.106
R² = 0.641
Predicted results (mm)
Experimental results (mm)
10
15
20
25
0
10
20
30

(h)



(i)

(j)

(k)

(l)

(m)

(n)

Figure 1. Scatter plots of experimental versus regression results. (a) First crack load. (b) Deflection at first crack load. (c) Yield load. (d) Deflection at yield load. (e) Service load. (f) Deflection at service load. (g) Ultimate load. (h) Deflection at ultimate load. (i) Crack width at ultimate stage. (j) Number of cracks. (k) Average spacing of cracks. (l) Deflection ductility. (m) Deflection ductility ratio. (n) Number of cycles.

4. Conclusions

A multivariate linear regression analysis has been conducted for evaluating concrete beams with surface mounted FRP reinforcement under static and cyclic loading conditions. The results predicted through regression have been validated using other researcher's results. Good convergence was observed between the experimental results and those through regression analysis. It can be intended from the analysis conducted that multivariate linear regression could be used to estimate the performance of FRP strengthened beams under monotonic and cyclic loading with reasonable levels of accuracy.

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