Indian Journal of Science and Technology, *Vol 10(31), DOI: 10.17485/ijst/2017/v10i31/113886, August 2017*
ISSN (Print) : 0974-6846
ISSN (Online) : 0974-5645

Influence of Nuclear Charge Radius Parameterization on the Excitation Function of Nuclear Stopping

Sangeeta[1] and Varinderjit Kaur[2]

[1]School of Physics and Materials, Science, Thapar University, Patiala - 147004, Punjab, India; sangeeta.ar003@gmail.com

[2]Mata Gujri College, Fatehgarh Sahib - 140406, Punjab, India; drvarinderjit@gmail.com

Abstract

The initialization effects on the excitation function of nuclear stopping have been depicted through nuclear charge radii parameterizations within the framework of Isospin-dependent Quantum Molecular Dynamics (IQMD) model. The nuclear reactions of ${}^{58}_{28}Ni+{}^{58}_{28}Ni$ and ${}^{124}_{50}Sn+{}^{124}_{50}Sn$ have been simulated at incident energies between 30 to 1500 MeV/nucleon for two different nuclear charge radii parameterizations (isospin-independent and -dependent) by including as well as excluding the momentum dependent interactions (MDI). Our study reveals that role of change in radius parameterization on nuclear stopping is more emphasized around 400 MeV/nucleon. MDI affects the nuclear stopping at relatively low energy and its role diminishes with increase in energy. Moreover, the isospin-dependent radius parameterization along with MDI is able to reduce the gap between experimental findings of INDRA and ALADIN collaborations and theoretical calculations. **General Terms:** Models and Codes, Theory of Nuclear Reactions

Keywords: Heavy-Ion Collisions, Isospin-Dependent Nuclear Charge Radius, IQMD Model, Multifragmentation, Nuclear Reaction Simulation, Nuclear Stopping

1. Introduction

Physics of nuclear reactions at intermediate energies can be explained by knowing the relative motion of the fragments and the nuclear stopping[1,2]. Fruitful attempts have been made to extract the information about the properties of highly compressed nuclear matter (during the collision process) through nuclear stopping both experimentally as well as theoretically. The experimental data provided by INDRA and ALADIN collaborations[3,4] reveals negligible effect of isospin content of colliding nuclei on nuclear stopping. Liu et al.[5] studied the influence of symmetry energy and in-medium nucleon-nucleon cross-section for isospin asymmetric collisions and their study reveals that the nuclear stopping is more sensitive to in-medium nucleon-nucleon cross-section compared to symmetry energy. In another study by Li and Li[6], the excitation function of nuclear stopping (quadruple moment per nucleon and anisotropy ratio) is found to be sensitive towards the isospin dependence of medium correction of two-body cross-section. Moreover, weaker dependence of nuclear stopping on initial N/Z ratio as well as on symmetry potential has been reported. The systematic study carried out on the effect of isospin degree of freedom on nuclear stopping acknowledge that the magnitude of stopping depends strongly on isospin-dependent nucleon-nucleon cross-section and weakly on different forms of density-dependent symmetry energy[7-9].

So far, the nuclear stopping have been scrutinized via symmetry energy, in-medium nucleon-nucleon cross-section, initial N/Z ratio of colliding nuclei etc for a wide range of impact parameter (b), incident energy (E) and composite system mass ($A_{tot}=A_P+A_T$). Here A_P and A_T are mass number of projectile and target nucleus respectively. Nowadays, people have studied the structural/isospin/initialization effects through nuclear charge radii parameterizations. Various nuclear charge radii parameterizations exist in literature which are the conclusive

**Author for correspondence*

output of tremendous research made in low energy and low density nuclear dynamics[10-16]. These radii parameterizations are isospin-independent as well as -dependent (based on the adopted calculations). The isospin-dependent parameterizations are proposed principally to modify the isospin-independent nuclear charge radius parameterization obtained from the liquid drop model (LDM)[17] i.e.

$$R_{LDM} = r_o A^{1/3} \quad (3)$$

Where r_o = 1.12 fm and A is the mass number of nucleus. The recently used isospin parameter (I = (N-Z)/A) included expression is:

$$R_{RR} = 1.2332 A^{1/3} + \frac{2.8961}{A^{2/3}} - 0.18688 A^{1/3} I \quad (2)$$

This recently premised parametrization, proposed by Royer and Rousseau[16], can reproduce experimental root mean square (rms) radius of nucleus (N,Z > 8) accurately[18,19]. This radius parameterization has also been used by many authors and also incorporated with the theoretical models[20-22]. In theoretical models of heavy-ion collisions at intermediate energies, the radii parameterizations used to construct the nuclear matter are often isospin-independent. But one can observe from last few communications of this field that the change in radius (due to different parameterizations) at initial level are actually affecting the final stage production of nuclear reactions i.e. multifragmentation, nuclear flow and stopping[21-25]. The recent study proclaims that the increase in nuclear radius increases the transverse momentum as well as longitudinal momentum which enhance the multiplicity of fragments. However, the predominance of longitudinal momentum in larger radii reduces the nuclear stopping[24,25]. It is worth mentioning that, the above stated study of structural effects via nuclear radii parameterization on nuclear stopping has been carried out at a particular energy i.e. E = 50 MeV/nucleon. It is crucial to know that how the dominance of longitudinal momentum due to increase in radius changes with the entire range of intermediate energy regime. Therefore, the present exercise is carried out in order to answer this question. In addition to this, we will also focus on the impact of momentum dependent interactions (MDI). MDI also play crucial role to understand the relative interaction of the nuclear matter[26]. So, in the vicinity of above discussion, it would be interesting to explore the influence of nuclear charge radii parameterizations on the nuclear stopping along with MDI and see if it would help in better agreement of theoretical calculation with the experimental data.

The manuscript is organized as: Section 2. briefly explains the IQMD model. The results and discussion are presented in Section 3. followed by summary and acknowledgments in Section 4. and 5. respectively.

2. The Model

The Isospin-dependent Quantum Molecular Dynamics (IQMD)[27,28] model has been used successfully for the analysis of a large number of observables from low to relativistic energies. This model treats different charge states of nucleons, deltas and pions explicitly. In this model, baryons are represented by Gaussian-shaped density distributions,

$$f_i(r,p,t) = \frac{1}{\pi^2\hbar^2} e^{-\left(\vec{r}-\vec{r_i}(t)\right)^2 \frac{1}{2L}} e^{-\left(\vec{p}-\vec{p_i}(t)\right)^2 \frac{2L}{\hbar^2}} \quad (3)$$

Here, L is the Gaussian width regarded as the interaction range of the particle. The nucleons are primarily initialized in a sphere of radius in accordance with the LDM (as given in equation (1)) i.e. isospin-independent. In order to study the influence of nuclear charge radius, we have introduced the isospin-dependent parameterized forms of nuclear charge radius given in equation (2). The isospin degree of freedom enters into the calculations via. symmetry potential, cross sections and Coulomb interactions. The nucleons of the target and projectile interact by two- and three-body Skyrme forces, Yukawa potential and Coulomb interactions. A symmetry potential between protons and neutrons corresponding to the Bethe-Weizsacker mass formula has also been included. The hadrons propagate using Hamilton equations of motion:

$$\frac{dr_i}{dt} = \frac{\partial\langle H\rangle}{\partial p_i}; \; \frac{dp_i}{dt} = -\frac{\partial\langle H\rangle}{\partial r_i} \quad (4)$$

The baryon-baryon potential V^{ij}, in the above relation, reads as:

$$V^{ij}(\vec{r'}-\vec{r}) = V^{j}_{Skyrme} + V^{j}_{Yukawa} + V^{j}_{Coul} + V^{j}_{sym} + V^{j}_{MDI} \quad (5)$$

During collision, the two particles collide to each other if the minimum relative distance $\vec{r}$ of the centroids of their Gaussians during their motion (in their centre of mass frame) fulfils the requirement:

$$\left|\vec{r}_i - \vec{r}_j\right| \le \sqrt{\frac{\sigma_{tot}}{\pi}}\ , \sigma_{tot} = \sigma\left(\sqrt{s}, type\right) \tag{6}$$

Where "type" in above equation denotes the in going collision partners (N-N, N-Δ, N−π etc.). The total scattering cross section is:

$$\sigma_{tot} = \sigma_{el} + \sigma_{inel} \tag{7}$$

The total cross-section is the sum of the elastic and all inelastic cross-sections. The elastic and inelastic cross-sections for proton-proton (pp) and proton-neutron (pn) are used in IQMD model. The neutron-neutron (nn) cross-section is assumed to be equal to pp.

3. Results and Discussion

To study the excitation function of nuclear stopping, we simulated the reactions of ${}^{58}_{28}Ni + {}^{58}_{28}Ni$ and ${}^{124}_{50}Sn + {}^{124}_{50}Sn$ at an incident energy E = 400 MeV/nucleon for b=0.0 fm (i.e. central collisions). Figure 1 display the influence of nuclear charge radii parameterizations on the rapidity distribution (as a indicator for nuclear stopping) of protons (upper panels), neutrons (middle panels) and all nucleons (lower panels) for the reactions of ${}^{58}_{28}Ni + {}^{58}_{28}Ni$ (left panels) and ${}^{124}_{50}Sn + {}^{124}_{50}Sn$ (right panels) initialized with R_{LDM} (solid lines) and R_{RR} (dash lines). The doted and dash-dotted lines represent the calculation with R_{LDM} and R_{RR}, respectively, which includes the momentum dependence in the nuclear mean field (i.e. R_{LDM}+MDI and R_{RR}+MDI). The rapidity distribution is defined as Y = $Y_{c.m.}/Y_{beam}$, and $Y_{c.m.}$ is further defined as:

$$Y_{c.m.} = \frac{1}{2}\ln\frac{E(i) + p_z(i)c}{E(i) - p_z(i)c} \tag{8}$$

Where E(i) and $p_z(i)$ are the total energy and longitudinal momentum of the i^{th} particle. A sharp narrow Gaussian indicates better thermalization compared to broad one. From Figure 1, we see that the Gaussian distribution becomes broader as we switch from R_{LDM} to R_{RR}. This is because the calculated radii of ${}^{58}_{28}Ni$ and ${}^{124}_{50}Sn$ with R_{LDM} formula are lesser than the calculated radii with R_{RR} parameterization. Therefore the increase in radius shifts some of the nuclear matter from mid-rapidity zone to the projectile as well as target like rapidity zone in equal amount. This concludes that the compression in the participant zone is lesser in case of R_{RR} compared to R_{LDM} which signalizes the less number of binary collisions.

The inclusion of MDI damages the stability of nuclei[29], which further reduces the peak value of dN/dY. The percentage increment in radius of ${}^{58}_{28}Ni$ and ${}^{124}_{50}Sn$ nuclei are 13.98% and 8.85% respectively, while switching from R_{LDM} to R_{RR}. Corresponding to this increase in radius, the peak value of dN/dY reduces 11.1% (p), 12.5% (n) and 13.6% (all) for ${}^{58}_{28}Ni$ and 7 % (p), 7.6 % (n) and 9.65 % (all) for ${}^{124}_{50}Sn$ without MDI. On the other hand these percentages are 6.45 % (p), 8.8 % (n) and 9.6 % (all) for ${}^{58}_{28}Ni$ and 4.5, 6.6 and 5.6 for ${}^{124}_{50}Sn$ with inclusion of MDI. This information clearly indicates that the inclusion of MDI reduces the influence of nuclear charge radii parameterizations on the rapidity distribution. However, the peak value of rapidity distribution is more broaden for R_{RR} radius parameterization which includes MDI (i.e. R_{RR}+MDI). This quantifies both radius as well as MDI effect. To see how the radii parameterizations as well as the MDI affect the reaction dynamics throughout the energy range, we study the excitation function of nuclear stopping at incident energies between 30 MeV/nucleon to 1.5 GeV/nucleon. Figure 2 display the influence of nuclear charge radii parameterizations by including as well as excluding MDI for the reactions of ${}^{58}_{28}Ni + {}^{58}_{28}Ni$ (left panels) and ${}^{124}_{50}Sn + {}^{124}_{50}Sn$ (right panels) on the nuclear stopping of protons (as per the available experimental data). To estimate the stopping power we use momentum-based (R_p) anisotropic ratio, defined as[3-5]:

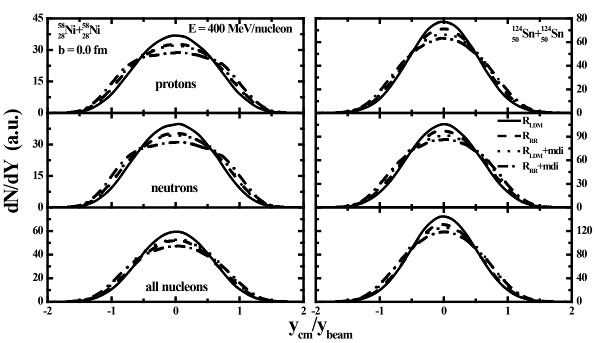

Figure 1. The rapidity distribution dN/dY for the reaction of ${}^{58}_{28}Ni + {}^{58}_{28}Ni$ (left panels) and ${}^{124}_{50}Sn + {}^{124}_{50}Sn$ (right panels) at incident energy of 400 MeV/nucleon. The top, middle and bottom panels are for protons, neutron and all nucleons respectively.

$$R_P = \frac{2}{\pi}\frac{\sum_{i=1}^{A_{tot}} p_\perp(i)}{\sum_{i=1}^{A_{tot}} p_\parallel(i)} \tag{9}$$

and energy-based anisotropic ratio (R_E), defined as:

$$R_E = \frac{1}{2}\frac{\sum_{i=1}^{A_{tot}} E_{\perp}(i)}{\sum_{i=1}^{A_{tot}} E_{\|}(i)} \quad (10)$$

Here, $E_{\perp}(i)$ ($E_{\|}(i)$) and $p_{\perp}(i)$ ($p_{\|}(i)$) are transverse (longitudinal) energy and momentum of the i^{th} particle respectively. The transverse and longitudinal momenta are $p_{\perp} = \sqrt{p_x^2(i) + p_y^2(i)}$ and $p_{\|} = p_z(i)$ respectively. The physical significance of R_P (or R_E) is the probability of transformation of longitudinal momentum (or energy) into the transverse momentum (or energy). For full nuclear stopping, R_P (and R_E) = 1. The value of R_P (and R_E) = <1 & >0, 0 and >1 conveys partial transparency, full transparency and super stopping. The value of nuclear stopping greater than unity can be explained by the preponderance of transverse momentum[30].

From Figure 2, one can see that the value of stopping is relatively small at low incident energies and the rising part is due to decreasing effect of Pauli-blocking. The maximum value of stopping has been obtained around 400 MeV/nucleon. Also, when energy exceeds 400 MeV/nucleon, the value of nuclear stopping falls gradually because of the ineffective role of Pauli-blocking. The influence of isospin-dependent nuclear charge radius on nuclear stopping is more around 400 MeV/nucleon. The nuclear charge radii parameterization starts affecting the magnitude of nuclear stopping observables at relatively high energies i.e. above 50 MeV/nucleon. This effect increases with increase in energy up to 800 MeV/nuceon and becomes constant above 800 MeV/nucleon. This reveals that, at relatively low energies, the predominance of longitudinal momentum (or energy) over transverse momentum (or energy) is very feeble. With increase in incident energy the longitudinal momentum (or energy) starts dominating the reaction dynamics.

The inclusion of MDI further reduces the values of R_P and R_E. The influence of MDI is more effective ay lower energies and vanishes as the energy increases. Hence, it is clear from the figure that the isospin-dependent radii parameterization along with MDI reduces the gaps between the theoretical calculations and experimental findings of INDRA and ALADIN[3,4] collaborations. Moreover, Figure 1 and 2 also verifies the previous observation[24,25] that the lighter nuclei are good probe to study the role of nuclear charge radii parameterizations on the nuclear stopping. Further it is important to note that, both isospin-independent and -dependent radii parameterizations behave similarly. There is no evidence of isospin effects on the nuclear stopping through isospin-dependent nuclear charge radii parameterization.

4. Summary

In summary, we study the role of change in nuclear radius through isospin-independent as well as dependent nuclear charge radii parameterizations along with momentum dependent interactions (MDI) using the IQMD model. We observe that the rapidity distribution of nuclear matter becomes broad for larger radii as well as by the inclusion of MDI. This clearly results in reducing the stopping values. The radii parameterization affects the nuclear stopping at relatively high energies and influence of MDI is at lower energies. Hence, the nuclear stopping calculations via isospin-dependent radius parameterization along with MDI are more close to the experimental findings of INDRA and ALADIN collaborations.

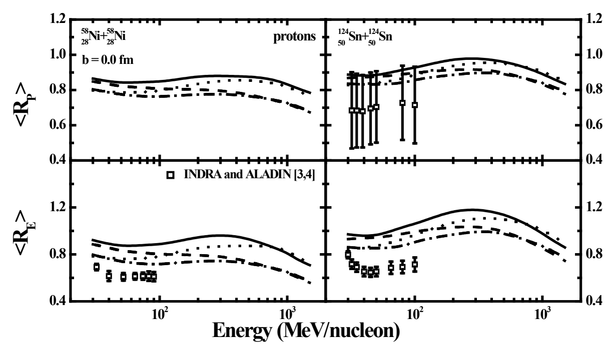

Figure 2. Incident energy dependence of R_P (upper panels) and R_E (lower panels) for the reactions of ${}^{58}_{28}Ni + {}^{58}_{28}Ni$ (left panels) and ${}^{124}_{50}Sn + {}^{124}_{50}Sn$ (right panels). The different lines have same meaning as in Figure 1.

5. Acknowledgments

The financial support from the Department of Science and Technology (DST), government of INDIA (New Delhi) in terms of INSPIRE Fellowship (grant no. DST/INSPIRE/03/2014/000234) and Young Scientist Award under the SERC Fast Track Scheme, wide letter no. SR/FTP/PS-020/2012 is gratefully acknowledged.

6. References

1. Bauer W. Nuclear stopping at intermediate beam energies. Physical Review Letters. 1988 Nov; 61:2534-7. Crossref. PMid:10039150.
2. Hong B. Nuclear stopping at sis energies by using isospin asymmetric nuclear collisions. Nuclear Physics A. 2003; 721:317c-20c. Crossref.
3. Lehaut G et al. Study of nuclear stopping in central collisions at intermediate energies. Physical Review Letters. 2010; 104:232701. Crossref. PMid:20867230.
4. Lopez O et al. In-medium effects for nuclear matter in the Fermi-energy domain. Phys. Rev. C 2014 Dec; 90:064602. Crossref.
5. Liu JY et al. Nuclear stopping as a probe for in-medium nucleon-nucleon cross sections in intermediate energy heavy ion collisions. Physical Review Letters. 2001 Feb; 86:975-8. Crossref PMid:11177988.
6. Li QF and Li ZX. Isospin effect on nuclear stopping in intermediate energy heavy ion collisions. Chinese Physics Letters. 2002; 19:321-3. Crossref, Crossref
7. Kumar S, Kumar S and Puri RK. Effect of symmetry energy on nuclear stopping and its relation to the production of light charged fragments. Physical Review C. 2010; 81:014601. Crossref.
8. Jain A, Kumar S and Puri RK. Influence of charge asymmetry and isospin-dependent cross section on nuclear stopping. Physical Review C. 2011 Nov; 84:057602. Crossref.
9. Vinayak KS and Kumar S. Effect of density-dependent symmetry energy on nuclear stopping. Journal of Physics G: Nuclear and Particle Physics. 2012 July; 39:095105.
10. Ngo H and Ngo Ch. Calculation of the real part of the interaction potential between two heavy ions in the sudden approximation. Nuclear Physics A. 1980 Apr; 348:140. Crossref.
11. Nerlo-Pomorska B and Pomorski K. Isospin dependence of nuclear radius. Zeitschrift fur Physik A. 1993 Dec; 344:359-61. Crossref.
12. Nerlo-Pomorska B and Pomorski K. Simple formula of nuclear charge radius. Zeitschrift fur Physik A. 1994 Jan; 348:169-72. Crossref
13. Myers WD and Swiatecki WJ. Nucleus-nucleus proximity potential and super heavy nuclei. Physical Review C. 2000 Sep; 62:044610. Crossref.
14. Zhan SQ et al. Isospin and Z1/3-dependene of the nuclear charge radii. The European Physical Journal A. 2002 Jan; 13:285-9. Crossref.
15. Denisov VY. Interaction potential between heavy ions. Physics Letters B. 2002 Feb; 526:315-21. Crossref.
16. Royer G and Rousseau R. On the liquid drop model mass formulae and charge radii. The European Physical Journal A. 2009 Mar; 42:541-5. Crossref.
17. Bohr A and Mottelson B. New York, Amsterdam: W. A. Benjamin Inc.: Nuclear Structure . 1969; 1:268.
18. Angeli I. A consistent set of nuclear rms charge radii: properties of the radius surface R (N, Z). Atomic Data and Nuclear Data Tables. 2004 May; 87:185-206. Crossref.
19. Angeli I and Marinova KP. Table of experimental nuclear ground state charge radii: An update. Atomic Data and Nuclear Data Tables. 2013; 99:69-95. Crossref.
20. Dutt I and Puri RK. Comparison of different proximity potentials for asymmetric colliding nuclei. Phys. Rev. C 2010 Jun; 81:064609. Crossref.
21. Bansal R, Gautam S, Puri RK and Aichelin J. Role of structural effects on the collective transverse flow and the energy of vanishing flow in nuclear collisions. Physical Review C. 2013 Jun; 87:061602(R). Crossref.
22. Gautum S. Effect of isospin dependence of radius on transverse flow and fragmentation in isobaric pairs. Physical Review C. 2013 Nov; 88:057603. Crossref.
23. Sangeeta, Jain A, Kumar S. Influence of isospin dependent nuclear charge radii on fragmentation in heavy ion collisions. Nuclear Physics A. 2014 May; 927:220-31. Crossref.
24. Sangeeta. Initialization effects via nuclear charge radii parameterizations on the nuclear stopping and++ its relation to distribution and production of light mass fragments. Acta Physica Polonica B. 2016 Mar; 47:991-6. Crossref.
25. Sangeeta. Structural effects through nuclear charge radius in mass asymmetric collisions. Acta Physica Polonica B. 2016 Mar; 48, in press.
26. Kumar S and Kumar S. Systematic study on system size dependence of global stopping: Role of momentum-dependent interactions and symmetry energy. Chinese Physics Letters. 2010; 27(6):062504.
27. Hartnack C et al. Modelling the many-body dynamics of heavy ion collisions: Present status and future perspective. The European Physical Journal A. 1998; 1:151-69. Crossref.
28. Hartnack C et al. Strangeness production close to the threshold in proton-nucleus and heavy-ion collisions. Physics Reports. 2012; 510:119-200. Crossref.
29. Singh J, Kumar S and Puri RK. Momentum dependent interactions and the asymmetry of the reaction: Multifragmentation as an example. Physical Review C. 2001 Apr; 63:054603. Crossref.
30. Renfordt RE. Stopping power and collective flow of nuclear matter in the reaction Ar+Pb at 0.8 GeV/u. 1984.