

# Optimization of Surface Roughness and Metal Removal Rate in End Milling using Taguchi Grey Relational Analysis

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## Abstract

**Objectives:** The objective of the present work is to assess the effects of machining parameters in milling on surface roughness and metal removal rate of AISI H11 steel. It is used in the manufacturing industries for making die casting moulds, extrusion dies, moulds for glass industry, punches, piercing tools, mandrels etc. **Methods/Statistical Analysis:** The objective of the study was carried out by machining experiments which were conducted on CNC vertical milling machine whose maximum speed was 6000 RPM. Design of experiments based on Taguchi grey relational analysis with three independent factors (cutting speed, feed rate and depth of cut), three levels L27 orthogonal array has used to develop relationships for predicting surface roughness and metal removal rate. **Findings:** The surface roughness was measured using surface roughness tester (Mitutoyo surf-test-4) and the averages were calculated to obtain the surface roughness of the samples. Metal removal rate was calculated using the formula in terms of width of cut, depth of cut and table feed rate. **Improvements:** Model significance tests were conducted using ANOVA table and effects of various parameters were investigated.

**Keywords:** AISI H11, ANOVA, Metal Removal Rate, Surface Roughness

## 1. Introduction

End milling process is one of the common metal cutting operations used for machining parts in manufacturing industry. It is usually performed at the final stage in manufacturing a product and surface roughness of the produced job plays an important role. In general, the surface roughness affects wear resistance, ductility, tensile, fatigue strength, etc., for machined parts and cannot be neglected in design.<sup>1</sup>

In End Milling, the cutter, called end mill, has a diameter less than the work piece width. The cutter generally rotates on an axis vertical to the work piece. It can be tilted to machine tapered surfaces. Cutting teeth are located on both the end face of the cutter and the periphery of the cutter body. End mills with flat ends (so called squire-end mills) are used to produce pockets, closed or end key slots, etc. The depth of the feature may be machined in a

single pass or may be reached by machining at a smaller axial depth of cut and making multiple passes.

End milling is one of the important machining operations, widely used in most of the manufacturing industries due to its capability of producing complex geometric surfaces with reasonable accuracy and surface finish. However, with the invention of CNC milling machine, the flexibility has been adopted along with versatility in end milling process. In order to build up a bridge between quality and productivity and to achieve the same in an economic way, the present study highlights optimization of CNC end milling process parameters to provide good surface finish and high Material Removal Rate (MRR).

The surface finish of the machined surface has been identified as quality attribute whereas MRR has been treated as performance index directly related to productivity. Attempt has been made to optimize quality and productivity in a manner that these multi-criterions

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could be fulfilled simultaneously up to the expected level. Multi-objectives related to quality and productivity has been accumulated to evaluate an equivalent single quality index (called grey relational grade); which has been optimized finally by Taguchi based Grey relational method.<sup>4</sup>

## 2. Methodology

### 2.1 Taguchi Grey Relational Analysis

Traditional experimental design methods are very complicated and difficult to use. Additionally, these methods also require a large number of experiments as the number of process parameters increases. Experiments are designed using Taguchi method so that effect of all the parameters could be studied with minimum possible number of experiments. Using Taguchi method, appropriate Orthogonal Array has been chosen and experiments have been performed as per the set of experiments designed in the orthogonal array. Signal to Noise ratios are also calculated for analyzing the effect of machining parameters more accurately. There are 2 Signal-to-Noise ratios of common interest for optimization of static problems used in present study as are:

(1) Smaller-The Better

$$\eta = -10 \log \frac{1}{n} \sum_{i=1}^n y_i^2 \quad (1)$$

(2) Larger-The Better

$$\eta = -10 \log \sum_{i=1}^n \frac{1}{y_i^2} \quad (2)$$

Where,  $\eta$ - Signal to Noise (S/N) Ratio,

$y_i$  – ith observed value of the response,

$n$  - Number of observations in a trial,

$y$  - Average of observed values (responses)

Regardless of the category of the performance characteristics, the higher S/N ratio corresponds to a better performance. Therefore, the optimal level of the process parameters is the level with the highest S/N value. The statistical analysis of the data is performed by Analysis of Variance (ANOVA) to study the contribution of the various factors and interactions and to explore the effects of each process on the observed values.

The use of Taguchi method with grey relational analysis to optimize the end milling operations with multiple performance characteristics includes the following steps:

1. Identify the performance characteristics and cutting parameters to be evaluated.
2. Determine the number of levels for the process parameters.
3. Select the appropriate orthogonal array and assign the cutting parameters to the orthogonal array.
4. Conduct the experiments based on the arrangement of the orthogonal array.
5. Normalize the experiment results of surface roughness and metal removal rate.
6. Perform the grey relational generating and calculate the grey relational coefficient.
7. Calculate the grey relational grade by averaging the grey relational coefficient.
8. Analyze the experimental results using the grey relational grade and statistical ANOVA.
9. Select the optimal levels of cutting parameters.

#### 2.1.1 Data Pre Processing

In grey relational analysis, the data pre-processing is the first step performed to normalize the random grey data with different measurement units to transform them to dimensionless parameters. Thus, data pre-processing converts the original sequences to a set of comparable sequences. Different methods are employed to pre-process grey data depending upon the quality characteristics of the original data.

Experimental data i.e. measured features of quality characteristics of the product are first normalized ranging from zero to one. This process is known as grey relational generation. In grey relational generation, the normalized data corresponding to Lower-the-Better (LB) criterion can be expressed as:

$$x_i = \frac{\max y_i(k) - y_i(k)}{\max y_i(k) - \min y_i(k)} \quad (3)$$

For higher-the-better (HB) criterion, the normalized data can be expressed as:

$$x_i = \frac{y_i(k) - \min y_i(k)}{\max y_i(k) - \min y_i(k)} \quad (4)$$

Here  $x_i(k)$  is the value after the grey relational generation,  $\min y_i(k)$  is the smallest value of  $y_i(k)$  for the kth response, and  $\max y_i(k)$  is the largest value of  $y_i(k)$  for the kth response. An ideal sequence  $x_0(k)$  is for the responses. The purpose of grey relational grade is to reveal the degrees of relation between the sequences say,  $[x_0(k) \text{ and } x_i(k), i = 1, 2, 3, \dots, 27]$ .

### 2.1.2 Grey Relational Coefficient and Grey Relational Grade

Next, based on normalized experimental data, grey relational coefficient is calculated to represent the correlation between the desired and actual experimental data. Then overall grey relational grade is determined by averaging the grey relational coefficient corresponding to selected responses. The overall performance characteristic of the multiple response process depends on the calculated grey relational grade. This approach converts a multiple-response process optimization problem into a single response optimization situation; the single objective function is the overall grey relational grade. The optimal parametric combination is then evaluated by maximizing the overall grey relational grade.

The grey relational coefficient  $\xi_i(k)$  can be calculated as:

$$\xi_i(k) = \frac{\Delta \min(k) + \Psi \Delta \max}{\Delta O_i(k) + \Psi \Delta \max} \quad (5)$$

Here deviation sequence,  $\Delta O_i(k) = \|x_0(k) - x_i(k)\|$  (6)

is difference of the absolute value  $x_0(k)$  and  $x_i(k)$ ;  $\Psi$  is the distinguishing coefficient  $0 \leq \Psi \leq 1$ ;  $\Delta \min$  = the smallest value of  $\Delta O_i$ ; and  $\Delta \max$  = largest value of  $\Delta O_i$ . After averaging the grey relational coefficients, the grey relational grade  $\gamma_i$  can be computed as:

$$\gamma_i = \frac{1}{n} \sum_{k=1}^n \xi_i(k) \quad (7)$$

Here  $n$  = number of process responses. The higher value of grey relational grade corresponds to intense relational degree between the reference sequence  $x_0(k)$  and the given sequence  $x_i(k)$ .

The reference sequence  $x_0(k)$  represents the best process sequence. Therefore, higher grey relational grade means that the corresponding parameter combination is closer to the optimal. In the aforesaid study, it has been assumed that all quality features are equally important. But in practical case, it may not be so. Depending on the area of application, different response may have different preference and thereby, different tolerance limit. For example, the surface roughness and the MRR; both may be or may not be of equal importance. It depends on the decision maker. Therefore, different weightages have to be assigned to different responses. If different priority weightages have been assigned to different responses, the equation for calculating overall grey relational grade becomes:

$$\gamma_i = \frac{\sum_{k=1}^n W_k \xi_i(k)}{\sum_{k=1}^n W_k} \quad (8)$$

Here,  $\gamma_i$  is the overall grey relational grade for  $i$ th experiment.  $\xi_i(k)$  is the grey relational coefficient of  $k$ th response in  $i$ th experiment and  $w_k$  is the weightage assigned to the  $i$ th response. So basically in this study, a TGRA has been used to establish a correlation between the independent variables and the performance characteristics; therefore, the experiments were performed according to a Taguchi design of experiments.

## 3. Experimental Study

In the present study, Taguchi grey relational analysis is applied to determine the optimal milling parameters to achieve better surface roughness and good metal removal rate for AISI H11 steel under varying machining conditions. Following important issues will be addressed in the study.

- The relationships between the milling parameters (cutting speed, feed rate, depth of cut) and the response factors (surface roughness and metal removal rate).
- The optimal conditions of milling parameters for optimum combination of better surface finish and good metal removal rate.

**Table 1.** Chemical Composition of AISI H11, % weight

Element	C	Mn	Si	Cr	Mo	V	P	S
%age	0.44	0.30	1.040	5.14	1.20	0.87	0.016	0.012

### 3.1 Design of Experiments

Number of experiments required for any experimentation work, mainly depends on the approach adopted for design of experiment. Thus it important to have a well-designed experiment so that no. of experiments required can be minimized. In the present study, the design suggested by TGRA has been implemented to analyze the effect of three independent variables for milling i.e. cutting speed, feed rate and depth of cut on surface roughness and metal removal rate. The process control parameters and their levels are given in Table 2.

Complete design layout for performing experimentation is summarized in Table 3 and corresponding experimental results are summarized in Table 4. This demonstrates a total number of 27 experiments for the complete experimentation.

**Table 2.** Process control parameters and their levels according to TGRA

Parameter	Symbol	Level 1	Level 2	Level 3
Speed (rpm)	A	300	700	1100
Feed (mm/tooth)	B	0.12	0.20	0.30
Depth of Cut (mm)	C	0.20	0.40	0.60

**Table 3.** Complete design layout

Expt. No.	A: Speed (rpm)	B: Feed (mm/tooth)	C: Depth of Cut (mm)
1	300	0.12	0.2
2	300	0.12	0.4
3	300	0.12	0.6
4	300	0.20	0.2
5	300	0.20	0.4
6	300	0.20	0.6
7	300	0.30	0.2
8	300	0.30	0.4
9	300	0.30	0.6
10	700	0.12	0.2
11	700	0.12	0.4
12	700	0.12	0.6
13	700	0.20	0.2
14	700	0.20	0.4
15	700	0.20	0.6
16	700	0.30	0.2
17	700	0.30	0.4
18	700	0.30	0.6
19	1100	0.12	0.2
20	1100	0.12	0.4
21	1100	0.12	0.6
22	1100	0.20	0.2
23	1100	0.20	0.4
24	1100	0.20	0.6
25	1100	0.30	0.2
26	1100	0.30	0.4
27	1100	0.30	0.6

**Table 4.** Experimental results

Expt. No.	Surface roughness (µm)	Metal removal rate (mm <sup>3</sup> / sec)
1	4.83	3.12
2	4.11	6.24
3	4.84	9.36
4	5.7	5.2
5	4.94	10.4
6	6.58	15.6
7	6.84	7.8
8	6.89	15.6
9	5.61	23.4
10	2.29	7.28
11	1.97	14.56
12	1.88	21.84
13	1.75	12.13
14	2.22	24.26
15	2.61	36.4
16	2.51	18.2
17	3.16	36.4
18	3.17	54.6
19	2.36	11.44
20	2.36	22.88
21	2.33	34.32
22	2.48	19.06
23	2.47	38.13
24	2.2	57.2
25	2.9	28.6
26	2.22	57.2
27	2.28	85.8

### 3.2 Surface Roughness Measurement

Surface roughness is defined as the finer irregularities of the surface texture that usually result from the inherent action of the material condition or the machining process. Although there are many parameters that are related to surface roughness in literature but the most accepted parameter is Centerline Average (CLA) surface Roughness Value (Ra). Mathematically, Ra is the arithmetic value of the departure of the profile from centerline along sam-

pling length. In the present study, surface roughness of the work pieces after milling was measured by using surface roughness tester (Mitutoyo surfest - 4).

### 3.3 Metal Removal Rate Calculation

The basic schematics according to which the response material removal rate is calculated and the terms used are:

$N$  = RPM of Cutter

$n$  = Number of Teeth on Cutter

$W$  = Width of cut

$T$  = Depth of cutter

$V$  = Cutting speed

$L$  = Length of pass or cut

$f_m$  = Table (machine) Feed

$f_t$  = Feed/tooth of cutter

$D$  = Cutter Diameter

Table Feed:

$f_m = f_t \times N \times n$

Cutting time:  $CT = L/f_m$

Metal Removal Rate for end milling is calculated as follows:

$$MRR = \frac{\text{Volume Removed}}{\text{Cutting Time}} = \frac{L \times W \times t}{CT} = W \times t \times f_m$$

## 4. Results and Discussion

### 4.1 Optimization of Surface Roughness and Material Removal Rate

The complete results of the 27 experiments performed as per the experimental plan are given in Table 5. The cutting speed, feed and depth of cut have been taken as input parameters. The surface roughness and material removal rate have been taken as the response.

**Table 5.** Experimental design and collected response data

ExNo.	Parametric combination (Design of experiment)			Response features	
	Speed (A) (rpm)	Feed (B) (mm/ tooth)	Depth of cut (C) (mm)	Ra ( $\mu\text{m}$ )	MRR ( $\text{mm}^3/\text{sec}$ )
1	300	0.12	0.2	4.83	3.12
2	300	0.12	0.4	4.11	6.24
3	300	0.12	0.6	4.84	9.36
4	300	0.20	0.2	5.7	5.2
5	300	0.20	0.4	4.94	10.4
6	300	0.20	0.6	6.58	15.6
7	300	0.30	0.2	6.84	7.8
8	300	0.30	0.4	6.89	15.6
9	300	0.30	0.6	5.61	23.4
10	700	0.12	0.2	2.29	7.28
11	700	0.12	0.4	1.97	14.56
12	700	0.12	0.6	1.88	21.84
13	700	0.20	0.2	1.75	12.13
14	700	0.20	0.4	2.22	24.26
15	700	0.20	0.6	2.61	36.4
16	700	0.30	0.2	2.51	18.2
17	700	0.30	0.4	3.16	36.4
18	700	0.30	0.6	3.17	54.6
19	1100	0.12	0.2	2.36	11.44
20	1100	0.12	0.4	2.36	22.88
21	1100	0.12	0.6	2.33	34.32
22	1100	0.20	0.2	2.48	19.06
23	1100	0.20	0.4	2.47	38.13
24	1100	0.20	0.6	2.2	57.2
25	1100	0.30	0.2	2.9	28.6
26	1100	0.30	0.4	2.22	57.2
27	1100	0.30	0.6	2.28	85.8

### 4.2 Optimal Solution of Bi-Objective Optimization (Surface Roughness and Material Removal Rate taken Together)

Taguchi grey relational analysis uses S/N ratio rather than mean value of data. The term signal represents the

mean value and the term noise represents the undesirable value for the output characteristic. There are several S/N calculation methods available depending on type of characteristic; Lower is Better (LB), Nominal is Best (NB) and Higher is Better (HB). For surface roughness (LB) and for MRR (HB) criterion has been used (Equation 1 and 2 respectively). The computed S/N ratios for each quality characteristic are presented in Table 6.

**Table 6.** S/N ratio calculation for Ra and MRR

	Ra (µm)	S/N ratio	MRR (mm <sup>3</sup> /sec)	S/N ratio
1	4.83	-13.67894262	3.12	09.88309188
2	4.11	-12.27683644	6.24	15.90369179
3	4.84	-13.69690723	9.36	19.42551697
4	5.7	-15.11749711	5.2	14.32006687
5	4.94	-13.87453898	10.4	20.34066679
6	6.58	-16.36451787	15.6	23.86249197
7	6.84	-16.70112203	7.8	17.84189205
8	6.89	-16.76438444	15.6	23.86249197
9	5.61	-14.97925723	23.4	27.38431715
10	2.29	-7.196709647	7.28	17.24262759
11	1.97	-5.889324523	14.56	23.26322750
12	1.88	-5.483156985	21.84	26.78505268
13	1.75	-4.860760974	12.13	21.67721602
14	2.22	-6.927059489	24.26	27.69781593
15	2.61	-8.332810147	36.4	31.22202767
16	2.51	-8.232394119	18.2	25.20142776
17	3.16	-9.993741652	36.4	31.22202767
18	3.17	-10.02118524	54.6	34.74385285
19	2.36	-7.458240059	11.44	21.16852049
20	2.36	-7.458240059	22.88	27.18912040
21	2.33	-7.347118421	34.32	30.71094558
22	2.48	-7.889033617	19.06	25.60245793
23	2.47	-7.853939065	38.13	31.62533611
24	2.2	-6.848453616	57.2	35.14792058
25	2.9	-9.247959958	28.6	29.12732066
26	2.22	-6.927059489	57.2	35.14792058
27	2.28	-7.158696940	85.8	38.66974576

It is necessary to normalize the original data before analyzing it with the Grey relation theory or any other methodology. An appropriate value is deducted from the values in the same array to make the value of this array approximately equal to 1. Normalize S/N ratio values by using Equation (3) for surface roughness and Equation

(4) for material removal rate. The results are given in above Table 7.

**Table 7.** Data pre-processing results

Expt. No.	Response values (normalized)	
	Ra	MRR
1	0.740798100	0.000000000
2	0.623009917	0.209145528
3	0.742307272	0.331487818
4	0.861648234	0.154133058
5	0.757229766	0.363278586
6	0.966407996	0.485620876
7	0.994685450	0.276475349
8	1.000000000	0.485620876
9	0.850034973	0.607963167
10	0.196238455	0.255657908
11	0.086407601	0.464803436
12	0.052286265	0.587145726
13	0.000000000	0.409708061
14	0.173585675	0.618853588
15	0.291680024	0.741278784
16	0.283244271	0.532133257
17	0.431211613	0.741278784
18	0.433517096	0.863621075
19	0.218209110	0.392036833
20	0.218209110	0.601182360
21	0.208874000	0.723524651
22	0.254399230	0.546064371
23	0.251451006	0.755289042
24	0.166982150	0.877657709
25	0.368559960	0.668512182
26	0.173585675	0.877657709
27	0.193045082	1.000000000

**Table 8.** Deviation sequence

Expt. No.	Deviation sequence	
	Ra 1.0000	MRR 1.0000
1	0.259201900	1.000000000
2	0.376990083	0.790854472
3	0.257692728	0.668512182
4	0.138351766	0.845866942
5	0.242770234	0.636721414

6	0.033592004	0.514379124
7	0.005314550	0.723524651
8	0.000000000	0.514379124
9	0.149965027	0.392036833
10	0.803761545	0.744342092
11	0.913592399	0.535196564
12	0.947713735	0.412854274
13	1.000000000	0.590291939
14	0.826414325	0.381146412
15	0.708319976	0.258721216\
16	0.716755729	0.467866743
17	0.568788387	0.258721216
18	0.566482904	0.136378925
19	0.781790890	0.607963167
20	0.781790890	0.398817640
21	0.791126000	0.276475349
22	0.745600770	0.453935629
23	0.748548994	0.244710958
24	0.833017850	0.122342291
25	0.631440040	0.331487818
26	0.826414325	0.122342291
27	0.806954918	0.000000000

The deviation sequence is the absolute difference between the reference sequence and the comparability sequence after normalization. It is determined using Equation (6) and summarized in Table 8.

### 4.3 Calculated Grey Relational Coefficients and Grey Relational Grades

#### 4.3.1 Case A: General Machining $W1 = W2 = 0.5$

The Grey relational analysis is performed from the data in Table 7 and Table 8, by calculating the Grey relational co-efficient for the normalized S/N ratio values using Equation (5).

The value for  $\xi$  is taken as 0.5. Both response viz surface roughness and material removal rate are assumed to have equal weightage considering general machining conditions. The results are given in Table 9.

Overall Grey Relational Grade (GRG) is determined by averaging the Grey relational coefficient corresponding to selected responses using Equation (7). The overall performance characteristic of the multiple response process depends on the calculated GRG. Thus Grey relational

approach converts a multiple response optimization problem into a single response optimization problem.

The optimal parametric combination is carried out to obtain the highest Grey relational grade. The higher value of the GRG corresponds to a relational degree between the reference sequence and the given sequence. The reference sequence represents the best process sequence. Therefore, a higher GRG means that the corresponding parameter combination is closer to the optimal. The mean response for the GRG and the mean effect plot of the GRG are very important to obtain the optimal process condition.

### 4.4 Mean Effects on Overall Grey Relational Grades

Mean effect analysis of variables in orthogonal array of TGRA is very simple. It's enough to calculate average value of GRGs at the desired level to determine the effect of any parameter.

For example the mean effect of A at level 1 is obtained from averaging all 1-9 test runs. The mean effect of other variables at each level can be computed in the same manner.

**Table 9.** Mean effects on overall grey relational grades for  $w1 = w2 = 0.5$

Level	Speed (A) (rpm)	Feed (B) (mm/tooth)	Depth of cut (C) (mm)
1	0.6086540	0.465069	0.492210
2	0.4796190	0.537934	0.537219
3	0.5333908	0.618661	0.592235
Average grade	0.5405546	0.5405546	0.5405546
Max. – Min.	0.129035	0.153592	0.100026
Percent of Deviation	33.7211	40.1387	26.1401
Rank	2	1	3

Mean effect of parameters are computed and listed in Table 9. As large value of mean GRG is favorable, the optimum set of parameters is 'A at first level, B at third level and C at third level respectively (A1 B3 C3)'.

The second last row in Table 9 presents percent of deviation for each parameter. Thus the most significant process parameter is the feed rate followed by cutting

speed and depth of cut that affect the optimization of multiple performance characteristics.

Figure 1 show that the GRG values decreases from first level to second level and subsequently increases to third level with speed. Whereas with increase in feed and depth of cut, the GRG values increases throughout.

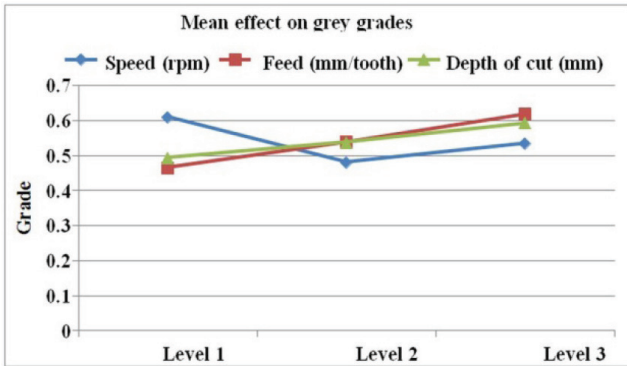


Figure 1. Grey relational grades with varying input parameters for w1 = w2 = 0.5.

### 4.5 Analysis of Variance (ANOVA)

Table 10 shows the summarized results of ANOVA for general machining. It shows that all the three input factors i.e. speed; feed and depth of cut are significant factors. The percentage contribution of speed, feed and depth of cut are 29.13 %, 40.93 % and 17.4 % respectively.

Predicted optimum condition

The predicted values of GRG at the optimal levels are calculated by using the relation:

$$n = nm + \sum_{i=1}^o (nim - nm)$$

Where n = Predicted value after optimization

nm = Total mean value of quality characteristic

nim = Mean value of quality characteristic at optimum level of each parameter

o = Number of main machining parameters that effects the response parameters

The expected mean at the optimal settings ( $\mu$ ) is calculated by using the following formula

$$\mu = A1 + B3 + C3 - 2 \times Tgg \tag{10}$$

Where A1, B3 and C3 are the mean values of the grey relational grade with the parameters at optimum levels and Tgg is the overall mean of grey grade (using Table 9).

$$\mu = 0.6086 + 0.6186 + 0.5922 - 2 \times 0.5405$$

$$= 0.738$$

Confidence interval (CI) is calculated as

$$C.I = \sqrt{Fa(1, f_e) \times V_e / N_e} \tag{11}$$

Where,  $Fa(1, f_e)$  = F value from the F table at a required confidence level and at DOF 1 and

error DOF  $f_e = 8$  (in present case) and we get = 5.3177

$V_e$  is the error mean square = 0.002812207 using Table 11,

$N_e$  = Effective number of replications

$$= 27 / 1 + 6$$

$$C.I = 0.0621$$

Therefore 95% confidence interval of the predicted optimum condition is given by the following equation, where the Grey relational grade value after conducting the confirmation experiments with optimal setting point, i.e. A1 B3 C3

$$(0.7380 - 0.0621) < \mu < (0.7380 + 0.0621)$$

$$(0.6759) < \mu < (0.8001).$$

#### 4.5.1 Case B: Rough Machining W1 = 0.2, W2 = 0.8

Both response viz surface roughness and material removal rate are assumed to have unequal weightage. The 20% weightage for response surface roughness and 80% for material removal rate 0.8 is assumed for considering the rough machining conditions. Overall Grey relational grade (GRG) is determined by using Equation (8). The results are given in Table 11.

A higher GRG means that the corresponding parameter combination is closer to the optimal value. The mean response for the GRG and the mean effect plot of the GRG are very important to obtain the optimal process condition.

Mean effects on overall grey relational grades

The mean effect of parameters are computed and listed in Table 11. As large value of mean GRG is favorable, the optimum set of parameters is 'A at third level, B at third level and C at third level respectively (A3 B3 C3)'.

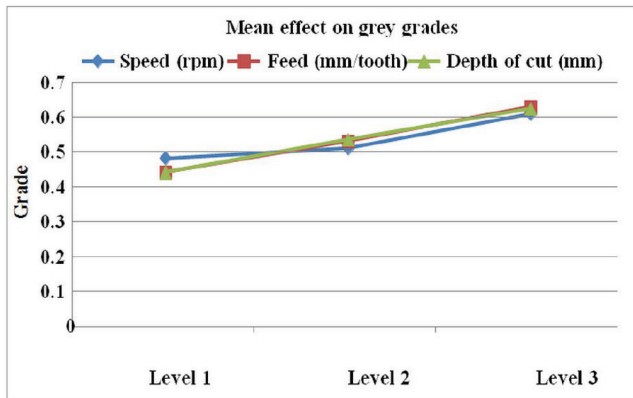
The second last row in Table 11 presents percent of deviation for each parameter. Thus the most significant process parameter is the feed rate followed by depth of cut and cutting speed that affects the optimization of multiple performance characteristics.



**Table 11.** Mean effects on overall grey relational grades for W1 = 0.2, W2 = 0.8

Level	Speed (A) (rpm)	Feed (B) (mm/tooth)	Depth of cut (C) (mm)
1	0.5044571	0.4682022	0.468035
2	0.5304402	0.5477256	0.552265
3	0.617037	0.6360065	0.631635
Average	0.5506448	0.5506448	0.550645
Max -min	0.1125799	0.1678043	0.1636
Percent of Deviation	25.3393	37.7828	36.8778
Rank	3	1	2

Figure 2 shows the grey relational grade values with respect to input parameters. It is clear that the GRG values increase throughout with all three input parameters, speed, feed and depth of cut. The rate of increase corresponding to feed and depth of cut seems to be faster than speed.



**Figure 2.** Grey relational grades with varying input parameters for W1= 0.2, W2 = 0.8.

**4.5.2 Analysis of Variance (ANOVA)**

Table 13 shows that all three input factors i.e. speed, feed and depth of cut are significant factors. The percentage contribution of speed, feed and depth of cut are 18.48 %, 37.49 % and 35.61 % respectively.

Predicted optimum condition

The expected mean at optimum setting using Table 13 is

$$\mu = A1 + B3 + C3 - 2 \times T_{gg} \tag{12}$$

$$= 0.6170 + 0.6360 + 0.6316 - 2 \times 0.5506$$

$$= 0.7834$$

Confidence interval (CI) is calculated as

$$C.I = \sqrt{F\alpha(1, f_e) \times V_e / N_e} \tag{13}$$

Where,  $F\alpha(1, f_e) = 5.3177$

$V_e = 0.001984065$  using Table 14,  $C.I = 0.0512$

The predicted optimum condition is given by

$$(0.7834 - 0.0512) < \mu < (0.7834 + 0.0512)$$

$$(0.7322) < \mu < (0.8346).$$

**4.5.3 Case C: Finish Machining W1 = 0.8, W2 = 0.2**

Both response viz surface roughness and material removal rate are assumed to have unequal weightage. The 80% weightage for response surface roughness and 20% for material removal rate is assumed for considering the finish machining conditions. Overall Grey relational grade (GRG) is determined by using Equation (8). The results are given in Table 14.

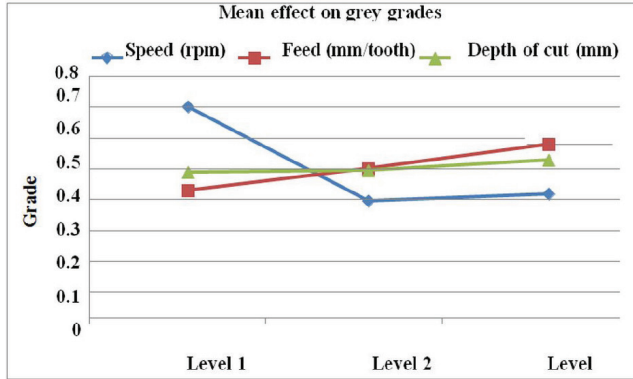
Mean effects on overall grey relational grades

The mean effect of parameters are computed and listed in Table 15. As large value of mean GRG is favorable, the optimum set of parameters is 'A at first level, B at third level and C at third level respectively (A1 B3 C3)'.

The second last row in Table 16 presents percent of deviation for each parameter. Thus the most significant process parameter is the cutting speed followed by feed rate and depth of cut that affect the optimization of multiple performance characteristics.

**Table 15.** Mean effects on overall grey relational grades for W1 = 0.8, W2 = 0.2

Level	Speed (A) (rpm)	Feed (B) (mm/tooth)	Depth of cut (C) (mm)
1	0.7128509	0.461936	0.516385
2	0.4287978	0.528142	0.522172
3	0.4497445	0.601316	0.552836
Average	0.5304644	0.530464	0.530464
Max -min	0.2840531	0.139381	0.036452
Percent of deviation	61.8736	30.2832	07.8431
Rank	1	2	3



**Figure 3.** Grey relational grades with varying input parameters for  $W1 = 0.8, W2 = 0.2$ .

Figure 3 shows the grey relational grade values with respect to input parameters. It is clear that the GRG values decreases from first level to second level and subsequently increases to third level with speed. Whereas with increase in feed and depth of cut, the GRG values increases throughout.

**4.5.4 Analysis of Variance (ANOVA)**

Table 16 shows that speed and feed are significant factors. The percentage contribution of speed, feed and depth of cut are 71.55 %, 13.88 % and 1.09 % respectively.

Predicted optimum condition

The expected mean at optimum setting using Table 16 is

$$\mu = A1 + B3 + C3 - 2 \times Tgg \tag{14}$$

$$= 0.7128 + 0.6013 + 0.5528 - 2 \times 0.5304$$

$$= 0.8061$$

Confidence interval (CI) is calculated as

$$C.I = \sqrt{F\alpha(1, fe) \times V_e / N_e} = \tag{15}$$

Where,  $F\alpha(1, fe) = 5.3177$

$V_e = 0.007403037$  using Table 17,

$$C.I = 0.1010$$

The predicted optimum condition is given by

$$(0.8061 - 0.1010) < \mu < (0.8061 + 0.1010)$$

$$(0.7051) < \mu < (0.9071).$$

**4.6 Confirmation Experiment**

The confirmation experiment is conducted at the optimum settings to verify the quality characteristics for milling of AISI H11 steel. The optimum combinations for the predicted milling parameters were set, and two trials were conducted. In order to assess the closeness of the observed value with that of the predicted value, the Confidence Interval (CI) value for the optimum factor level combination at a 95% confidence level is determined and presented in Table 18. Hence the TGRA method for the optimization of the multi response problems is a very useful tool for the milling of AISI H11 steel.

**Table 10.** Results of ANOVA for general machining  $w1 = w2 = 0.5$

FACTOR	D.F	SUM OF SQUARES	MEAN SQUARES	F-RATIO	PERCENT CONTRIBUTION	F > F Table
SPEED (S)	2	0.075617968	0.037808984	13.444596	0.291365667	Significant
FEED (F)	2	0.106250547	0.053125273	18.890955	0.409396896	Significant
DOC (D)	2	0.045173456	0.022586728	8.0316736	0.17405908	Significant
S x F	4	0.001814859	0.000453715	0.1613376	0.006992882	
F x D	4	0.005770794	0.001442699	0.5130129	0.022235605	
S x D	4	0.002404161	0.00060104	0.2137255	0.009263538	
ERROR	8	0.022497655	0.002812207		0.086686332	
TOTAL	26	0.25952944	0.118830646		1	
F0.05(2,8)	4.459					
F0.05(4,8)	3.8378					

**Table 12.** Calculated grey relational coefficients and grey relational grades for W1 = 0.2 and W2 = 0.8

Exp. No	Orthogonal array L27			Grey relational coefficient		Grade for W1 = W2 = 0.5	Grade order
	A	B	C	Ra	MRR		
1	1	1	1	0.658586339	0.333333333	0.398383934	26
2	1	1	2	0.570131874	0.387340332	0.423898641	25
3	1	1	3	0.659898111	0.427894555	0.474295266	20
4	1	2	1	0.783267200	0.371507750	0.453859640	22
5	1	2	2	0.673155677	0.439861512	0.486520345	19
6	1	2	3	0.937045526	0.492912352	0.581738987	11
7	1	3	1	0.989482689	0.408655436	0.524820887	14
8	1	3	2	1.000000000	0.492912354	0.594329882	9
9	1	3	3	0.769272160	0.560514972	0.602266409	8
10	2	1	1	0.383505712	0.401818763	0.398156153	27
11	2	1	2	0.353708750	0.483000057	0.457141796	21
12	2	1	3	0.345372146	0.547732551	0.507260470	16
13	2	2	1	0.333333333	0.45859277	0.433540882	24
14	2	2	2	0.376956122	0.567442588	0.529345295	13
15	2	2	3	0.413797678	0.659003584	0.609962402	7
16	2	3	1	0.410928823	0.516600042	0.495465798	18
17	2	3	2	0.467819454	0.659003584	0.620766758	5
18	2	3	3	0.468830769	0.78569541	0.722322482	2
19	3	1	1	0.390079227	0.45127854	0.439038677	23
20	3	1	2	0.390079227	0.556286368	0.523044940	15
21	3	1	3	0.387258873	0.643935446	0.592600131	10
22	3	2	1	0.401412726	0.524144381	0.499598050	17
23	3	2	2	0.400464861	0.671401427	0.617214114	6
24	3	2	3	0.375088751	0.803416395	0.717750866	4
25	3	3	1	0.441914712	0.60133172	0.569448318	12
26	3	3	2	0.376956122	0.803416395	0.718124341	3
27	3	3	3	0.382568666	1.00000000	0.876513733	1

**Table 13.** Results of ANOVA for rough machining W1 = 0.2, W2 = 0.8

FACTOR	D.F	SUM OF SQUARES	MEAN SQUARES	F-RATIO	PERCENT CONTRIBUTION	F > F table
SPEED (S)	2	0.062545088	0.031272544	15.761856	0.184883290	significant
FEED (F)	2	0.126827298	0.063413649	31.961480	0.374901510	significant
DOC (D)	2	0.120477504	0.060238752	30.3612820	0.356131518	significant
S x F	4	0.002250265	0.000562566	0.28354230	0.006651785	
F x D	4	0.004616138	0.001154035	0.58165170	0.013645306	
S x D	4	0.005706164	0.001426541	0.71899920	0.016867423	
ERROR	8	0.015872519	0.001984065		0.046919168	
TOTAL	26	0.338294977	0.160052152		1.000000000	
F0.05(2,8)	4.4590					
F0.05(4,8)	3.8378					

**Table 14.** Calculated grey relational coefficients and grey relational grades for W1 = 0.8, W2 = 0.2

Exp. No	Orthogonal array L27			Grey relational coefficient		Grade for W1 = 0.8, W2 = 0.2	Grade order
	A	B	C	Ra	MRR		
1	1	1	1	0.658586339	0.333333333	0.593535738	8
2	1	1	2	0.570131874	0.387340332	0.533573566	9
3	1	1	3	0.659898111	0.427894555	0.613497400	7
4	1	2	1	0.783267200	0.371507750	0.700915310	5
5	1	2	2	0.673155677	0.439861512	0.626496844	6
6	1	2	3	0.937045526	0.492912352	0.848218891	3
7	1	3	1	0.989482689	0.408655436	0.873317239	2
8	1	3	2	1.000000000	0.492912354	0.898582470	1
9	1	3	3	0.769272160	0.560514972	0.727520722	4
10	2	1	1	0.383505712	0.401818763	0.387168322	24
11	2	1	2	0.353708750	0.483000057	0.379567011	26
12	2	1	3	0.345372146	0.547732551	0.385844227	25
13	2	2	1	0.333333333	0.45859277	0.358385221	27
14	2	2	2	0.376956122	0.567442588	0.415053415	22
15	2	2	3	0.413797678	0.659003584	0.462838859	14
16	2	3	1	0.410928823	0.516600042	0.432063067	19
17	2	3	2	0.467819454	0.659003584	0.506056280	11
18	2	3	3	0.468830769	0.78569541	0.532203697	10
19	3	1	1	0.390079227	0.45127854	0.402319090	23
20	3	1	2	0.390079227	0.556286368	0.423320655	21
21	3	1	3	0.387258873	0.643935446	0.438594188	18
22	3	2	1	0.401412726	0.524144381	0.425959057	20
23	3	2	2	0.400464861	0.671401427	0.454652175	17
24	3	2	3	0.375088751	0.803416395	0.460754280	16
25	3	3	1	0.441914712	0.60133172	0.473798114	13
26	3	3	2	0.376956122	0.803416395	0.462248177	15
27	3	3	3	0.382568666	1.00000000	0.506054933	12

**Table 16.** Results of ANOVA for rough machining W2 = 0.2, W1 = 0.8.

FACTOR	D.F	SUM OF SQUARES	MEAN SQUARES	F-RATIO	PERCENT CONTRIBUTION	F > F table
SPEED (S)	2	0.451049739	0.22552487	30.463831	0.71555403	significant
FEED (F)	2	0.087493998	0.043746999	5.9093314	0.138802171	significant
DOC (D)	2	0.006907545	0.003453773	0.4665346	0.010958263	significant
S x F	4	0.011134634	0.002783658	0.3760157	0.017664198	
F x D	4	0.011092311	0.002773078	0.3745865	0.017597057	
S x D	4	0.003447835	0.000861959	0.1164331	0.005469713	
ERROR	8	0.059224296	0.007403037		0.093954568	
TOTAL	26	0.630350358	0.286547373		1.000000000	
F0.05(2,8)	4.4590					
F0.05(4,8)	3.8378					

## 5. Conclusion

The present work has successfully demonstrated the application of Taguchi based grey relational analysis for multi response optimization of process parameters in End milling of AISI H11 steel. The important conclusions drawn from the present work are summarized as follows:

- 1) Out of three parameters considered feed rate is identified as the most significant and influential machining parameter followed by cutting speed. Whereas depth of cut has the least influence on surface roughness and MRR for general machining conditions.
- 2) For finish machining conditions the significant parameters are cutting speed and feed rate.
- 3) An increase in the value of predicted weighted GRG confirms the improvement in the performance of milling process using optimal values of process parameters.
- 4) The optimal combination of the cutting parameters obtained for maximizing MRR is the set with A3, B3 and C3.
- 5) Taguchi grey relational analysis does not involve any complicated mathematical theory or computation and thus can be employed by the engineers without a strong statistical background.

## 6. References

1. Anjan KK, Chandrasekaran M, Mandal A, Singh AK. Prediction of optimum cutting parameters to obtain desired surface in finish pass end milling of aluminium alloy with carbide tool using artificial neural network. *World Academy of Science Engineering and Technology*. 2011; 57:751–7.
2. Kolahan F, Gomezerji R, Moghaddam MA. Application of Taguchi method grey analysis and ANOVA in optimization of titanium alloys milling. 3rd International Conference on Manufacturing Engineering (ICME); Tehran, Iran; 2011.
3. Lu HS, Chang CK, Hwang NC, Chung CT. Grey relational analysis coupled with principal component analysis for optimization design of the cutting parameters in high-speed end milling. *Journal of Materials Processing Technology*. 2009; 209:3808–17. Crossref.
4. Moshat S, Datta S, Bandyopadhyay A, Pal PK. Parametric optimization of CNC end milling using entropy measurement technique combined with grey-Taguchi method. *International Journal of Engineering Science and Technology*. 2010; 2(2):1–12. Crossref.
5. Sreenivasulu R, Rao CS. Design of Experiments based Grey Relational Analysis in Various Machining Processes - A Review. *Research Journal of Engineering Sciences*. 2013; 2(1):21–6.
6. Zain AM, Haron H, Sharif S. Application of GA to optimize cutting conditions for minimizing surface roughness in end milling machining process. *Expert Systems with Applications*. 2010; 37:4650–9. Crossref.