

# Compensative Retinal Response to Departures from Unit Contrast and Unit Spot-Size Ratio

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## Abstract

The objective of this work is to show how the retina responds to the loss of luminance efficiency due to increasing departure from either unit contrast or unit spot-size ratio in a compensative fashion. The methods adopted are two-wave interference and the single mode wave guiding of a photoreceptor cone. A correlation is established between loss of luminance efficiency due to oblique incidence and contrast in the interference pattern. Similarly, a relation between the fraction of the power that is not able to be coupled to a cone due to the peripheral entry of light and the spot-size ratio departure is developed. The findings are that due to maximum departure from unit contrast, the retinal response takes the traditional Stiles-Crawford route, but with no departure the Stiles-Crawford Effect of the first kind (SCE I) becomes totally irrelevant for the retina and for intermediate situations (between unit and zero contrast) the retinal response is controlled by a modified SCE I weakened proportionate to the contrast of the interference pattern on the retina. Likewise, when the incident spot-size and the waveguide mode spot-size match, the spot-size ratio is unity, departure is zero, the coupling of power is 100% and visibility loss is zero. But with gradual enhancement in the spot-size ratio departure, the visibility loss also increases. Again, a visibility loss of 90 % corresponds to a departure of either 6-fold or 1/6-fold (equivalent to a pupil entry point of 4 mm). This suggests that a pupil entry point of 3.5 mm may point to a loss of 70-80 %, a result reached with departure from contrast also. This way of correlating the retinal response to either contrast or spot-size ratio has the advantage of employing them as potential biomarkers for early detection of diseases affecting the photoreceptors.

**Keywords:** Biomarker, Contrast, Retinal Response, Spot-size Ratio, Stiles-Crawford Effect, Visibility Loss

## 1. Introduction

The Stiles-Crawford Effect of the first kind (SCE I) where a peripheral entry of a beam of light results in loss of visibility is shown to be linked to the directional sensitivity of the photoreceptor cones<sup>1</sup>. Though it is retinal in origin, pupil apodization of the effect has been modelled successfully to account for the reduction of visibility due to the peripheral entry of light<sup>2,3</sup>. Similarly, modelling a photoreceptor cone as a waveguide supporting a fundamental mode, the retinal response is evaluated using polymethyl methacrylate and crown glass waveguide<sup>4</sup>. Bacteriorhodopsin thin film spatial light modulator is also proposed for the first time to study the contrast-controlled retinal response<sup>5,6</sup>.

Works based upon different waveguides<sup>7</sup>, photonic crystal waveguide<sup>8</sup>, related to retinal identification<sup>9</sup>

and primate retinal system for better computational modelling<sup>10</sup> take a different approach. Apart from the pupil apodization and wave guiding approaches, recently a new approach of understanding SCE I from the perspective of departure<sup>11</sup> has the potential of using pupil apodization, contrast elevation and spot-size ratio as biomarkers<sup>12</sup> for early detection of glaucoma<sup>13</sup> and retinitis pigmentosa<sup>14</sup>.

## 2. Theoretical Modelling

The two-wave interference is adopted to evaluate the retinal response. The approach is to vary the contrast in the interference pattern to obtain the retinal response. Next, a photoreceptor cone is modelled as a waveguide supporting a single fundamental mode. The approach is to evaluate the retinal response on the basis of the fraction of the power that is not able to be coupled which is shown to

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be governed by the spot-size ratio departure, that is, to the extent by which the incident spot-size and the waveguide mode spot-size differs from each other.

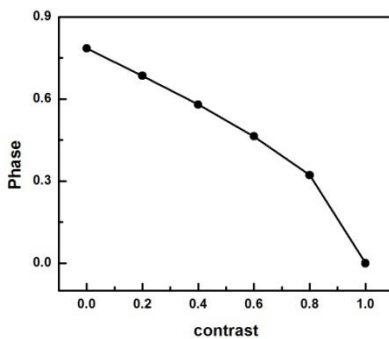
First, the mathematical exploration of the two-wave interference pattern on the retina would lead to a contrast-controlled retinal response. Secondly, the waveguide modelling of a photoreceptor cone would lead to an uncoupled power (interpreted as visibility loss) dependent on the spot-size ratio departure.

## 2.1 Interference Pattern on the Retina

When two plane light waves capable of interference and coming symmetrically from two opposite edges of the pupil meet at a point on the retina, an interference pattern of intensity ( $I$ ) and phase ( $\varphi$ ) is formed with a contrast ( $m$ ) dependent both on the intensity of the individual light waves ( $I_1, I_2$ ) and the underlying phase difference [ $kx - (-kx) = 2kx$ ] between them where  $k$  denotes the magnitude of the incident wave vector component projected onto the retina<sup>6,15</sup>.

$$\begin{aligned} \frac{I}{I_1 + I_2} &= 1 + m \cos(2kx) \\ \tan \varphi &= - \left[ \frac{(m-1) - \sqrt{1-m^2}}{(m+1) + \sqrt{1-m^2}} \right] (\tan kx) \\ m &= \frac{2\sqrt{I_1 I_2}}{I_1 + I_2} \end{aligned} \quad (1)$$

As seen from Eq. 1, for unit contrast ( $m = 1$ ) the wave front slope contained in the phase  $\varphi$  is zero (for  $I_1 = I_2$ ). But, more the intensities of individual beams are unbalanced, more is the departure of contrast from unity leading to a gradual enhancement of wave front slope at the retina as illustrated in Figure 1.



**Figure 1.** Variation of phase with contrast. More the intensities of individual beams are unbalanced, more is the departure of contrast from unity leading to a gradual enhancement of wave front slope at the retina.

Hence the enhancement in wave front slope ( $\varphi = 0$  for axial entry and  $\varphi$  is  $\frac{\pi}{4}$  for peripheral entry for a fixed retinal location, say,  $x = \frac{\lambda}{8}$ ) with the gradual shifting of the light beam's entry into the eye from the centre to the edge of the pupil modifies the pupil entry point ( $r$ ) with contrast as follows<sup>6,15</sup>.

$$r_{eff} = - \left[ \frac{(m-1) - \sqrt{1-m^2}}{(m+1) + \sqrt{1-m^2}} \right] r \quad (2)$$

Where ' $r$ ' is the pupil entrance point from the peak of the visibility.

In the absence of the interference pattern, the traditional Stiles-Crawford Effect of the first kind<sup>1</sup> (SCE I) [1] visibility ( $\eta$ ) is given as<sup>16</sup>

$$\eta(r) = e^{-0.115r^2} \quad (3)$$

Where the conventional SCE parameter is 0.115/mm<sup>2</sup>. The replacement of  $r$  with  $r_{eff}$  gives the modified visibility (in presence of interference) as<sup>6</sup>

$$\eta(r, m) = e^{-0.115r_{eff}^2} = e^{-0.115 \left( \frac{1-m}{1+m} \right)^2 r^2} \quad (4)$$

Eq. 4 shows the dependence of visibility on the contrast in addition to the pupil entry point. This is the required contrast-controlled retinal response function.

## 2.2 Wave guiding of Light

The entire theoretical approach of the model can be outlined as follows: A collimated and spatially filtered He-Ne laser of wavelength 632.8 nm is to be used as a source. This sends out a Gaussian beam whose intensity drops to 14% of its peak value at distance  $\omega_0$  on either side of the on-axis value. This is then focused on the retina to a spot-size of<sup>11,17</sup>.

$$\omega_r = \frac{\lambda f_{eye}}{\pi n_{eye} \omega_0} \quad (5)$$

Where the eye parameters used (in the absence of aberrations) are those of the reduced eye, i.e., a focal length  $f_{eye} = 22.2$  mm and a constant index of refraction  $n_{eye} = 1.33$ . The focused beam couples its power at the retina to the guided modes of a photoreceptor. And a small number of modes share among them the total power carried forward by the photoreceptor. When the entering beam is Gaussian and matches to the location and width of the photoreceptor  $n = \frac{\omega_r}{\omega_m} = 1$  ( $\omega_r$ : incident spot size,  $\omega_m$ : waveguide mode spot size) perfectly, the coupling to the fundamental mode (LP<sub>01</sub>) becomes the largest. The actual number of possible modes is found

from the V number of the waveguide defined by<sup>4</sup>.

$$V = \frac{2\pi r_i}{\lambda} \sqrt{n_i^2 - n_s^2} \quad (6)$$

Where  $n_i$  and  $n_r$  are the indices of refraction of the inner segment (assumed to be uniform) and the surrounding cladding respectively with  $r_i$  being the radius of the photoreceptor cone. For fundamental mode, V must be less than  $V_0 = 2.405$  which happens if the foveal cones are less than  $2.218 \lambda$  in size and indeed it is so with  $r_i$  having values between<sup>18</sup> 1 to  $1.4 \mu\text{m}$ .

In this model as the incident beam couples light only to the fundamental mode represented by a Gaussian function of width  $2\omega_m$  the fraction of power transmitted to the photoreceptor if the incident with its peak value at the photoreceptor axis can be found as<sup>11,17</sup>

$$T(\theta) = \left[ \frac{2\omega_r\omega_m}{\omega_r^2 + \omega_m^2} \right]^2 \exp \left[ \frac{-2(\pi n_{eye} \omega_r \omega_m)^2 \theta^2}{\lambda^2 (\omega_r^2 + \omega_m^2)} \right]$$

$$\left[ \frac{2\omega_r\omega_m}{\omega_r^2 + \omega_m^2} \right]^2 \exp \left[ \frac{-2(\pi n_{eye} \omega_r \omega_m)^2 r^2}{\lambda^2 (\omega_r^2 + \omega_m^2) f_{eye}^2} \right] \quad (7)$$

Writing Eq. 7 in terms of the spot size ratio  $\left(\frac{\omega_r}{\omega_m}\right)$  (where r is the distance from the on-axis value)

$$T\left(\frac{\omega_r}{\omega_m}\right) = \left[ \frac{2\frac{\omega_r}{\omega_m}}{1 + \left(\frac{\omega_r}{\omega_m}\right)^2} \right]^2 \exp \left[ -2 \left( \frac{\pi n_{eye}}{\lambda f_{eye}} \right)^2 \left( \frac{\omega_r^2}{1 + \left(\frac{\omega_r}{\omega_m}\right)^2} \right) r^2 \right] \quad (8)$$

Where  $\theta = \frac{r}{f_{eye}}$ .

So,

$$T\left(\frac{\omega_r}{\omega_m}\right) = \left[ \frac{2\frac{\omega_r}{\omega_m}}{1 + \left(\frac{\omega_r}{\omega_m}\right)^2} \right]^2 \quad (9)$$

Thus the power that is not able to be coupled can be expressed as

$$1 - T\left(\frac{\omega_r}{\omega_m}\right) = 1 - \left[ \frac{2\frac{\omega_r}{\omega_m}}{1 + \left(\frac{\omega_r}{\omega_m}\right)^2} \right]^2 = \left( \frac{n^2 - 1}{n^2 + 1} \right)^2 \quad (10)$$

Where  $= \frac{\omega_r}{\omega_m}$ . This is the required spotsizes ratio controlled retinal response<sup>11,14</sup>.

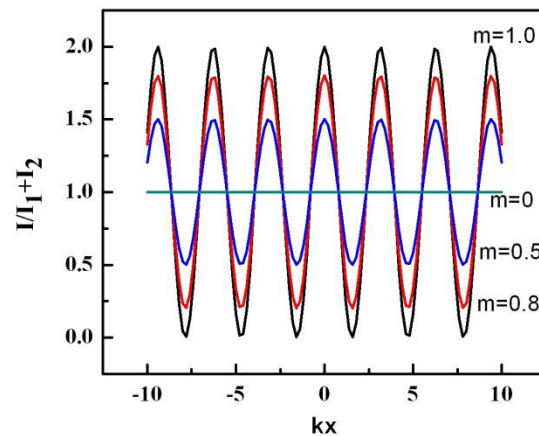
### 3. Results and Discussion

The response of the retina to departure from the unit contrast in an interference pattern formed on the retina

will be discussed from the perspective of Eq. 4. Likewise, how the departure from the perfect matching of the spot-sizes governs the retinal response will be ascertained from Eq. 10.

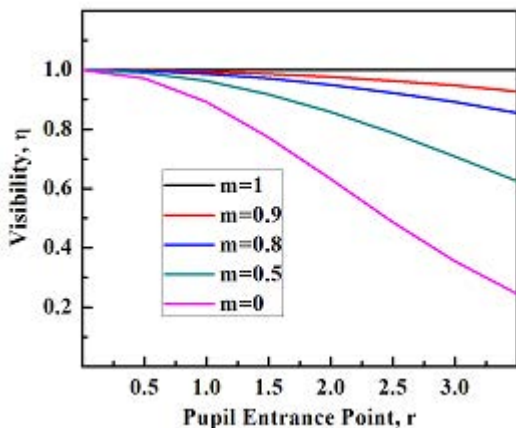
#### 3.1 Interference Pattern on the Retina

The intensity distribution for nil departure from unit contrast ( $m=1$ ), maximum departure ( $m=0$ ) and for intermediate departures (here,  $m=0.5$  and  $m=0.8$  are considered) is graphically presented in Figure 2 by using Eq. 1.



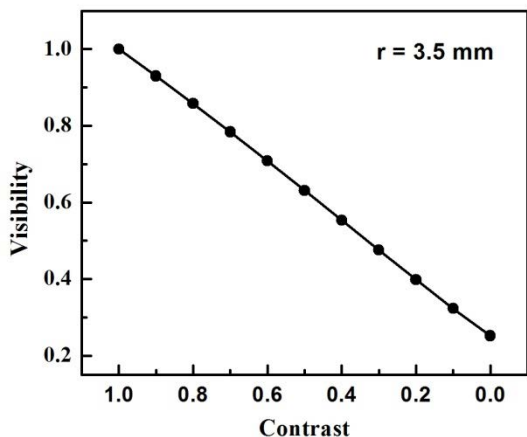
**Figure 2.** The intensity distribution for nil departure from unit contrast ( $m = 1$ ), maximum departure for  $m = 0$ , and intermediate departures for  $m = 0.5$  and  $m = 0.8$ .

From Eq. 4,  $\eta(r, m) = e^{-0.115 \left( \frac{1-m}{1+m} \right)^2 r^2}$ , it is seen that the visibility depends both on i) to what extent the beams are displaced from the centre of the pupil ( $r$ ) and ii) the contrast of the interference pattern ( $m$ ) formed on the retina by the coherent beams. So when visibility,  $\eta$ , is plotted versus the distance of the beam from the centre of the pupil ( $r$ ) for different values of the contrast (from  $m=1$  to  $m=0$  through  $m=0.9, 0.8$  and  $0.5$ ) three interesting observations are noted. First, with no departure from unit contrast the visibility or the retinal response does not depend at all on how the beams are entering through the pupil. Secondly, for maximum departure ( $m = 0$ ) occurring with the disappearance of the interference pattern from the retina, the retinal response follows the traditional SCE I. And finally, with the gradual enhancement of the departure from zero to maximum the visibility does decrease, but not that quickly with the increase of  $r$  compared to the traditional SCE I as illustrated in Figure 3.



**Figure 3.** Variation of visibility with pupil entry point for different departures. Nil departure ( $m = 1$ ) is independent of pupil entry point, other departures ( $m \neq 1$ ) are pupil entry point dependent with maximum departure ( $m = 0$ ) showing maximum response in the form of more visibility loss.

This we can see more explicitly in Figure 4 by keeping the pupil location fixed at  $r = 3.5$  mm, and gradually increasing the departure from zero to maximum (i.e.,  $m = 1$  to  $m = 0$ ). The visibility responds accordingly by decreasing to almost 24 % of its maximum value.



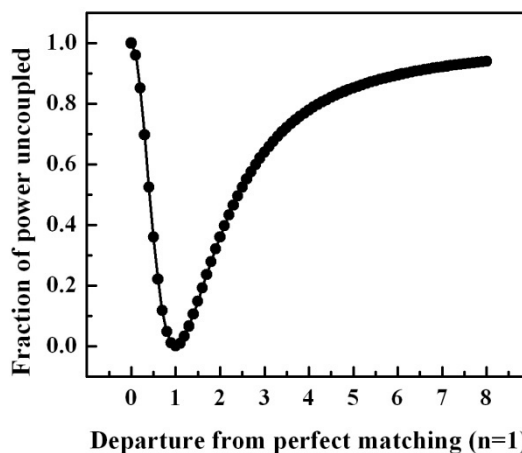
**Figure 4.** Variation of visibility with contrast. Gradual enhancement of the departure from zero to maximum (i.e.,  $m = 1$  to  $m = 0$ ) the visibility drops to 24 % of its peak value.

With the maximum departure from unit contrast the retinal response takes the traditional SCE route, but with no departure, the SCE I becomes totally irrelevant for the retina. For intermediate situations (between  $m = 1$  to  $m = 0$ ) the retinal response is controlled by a modified SCE I

weakened proportionate to the contrast of the interference pattern on the retina.

### 3.2 Wave guiding of Light

Using Eq. 10, the fraction of uncoupled power is plotted against spot-size ratio departure (i.e., for  $n = \omega_r / \omega_m = 1$ , departure is 0 in Figure 5. For perfect matching, ( $\frac{\omega_r}{\omega_m} = 1$ ), that is, when the departure is zero, the fraction of uncoupled power is also zero. With the increase of the departure the uncoupled power increases and for six-fold departure the uncoupled power becomes 90 % of the peak power achieved for nil departure (or axial entry of the beam).



**Figure 5.** Response of uncoupled power to departure from perfect matching. At perfect matching condition ( $n = \omega_r / \omega_m = 1$ ), the uncoupled power is zero. The uncoupled power increases with increase of departure in both the direction. With  $n = 6$  or  $n = \frac{1}{6}$  the uncoupled power drops to 90 % of the peak value.

## 4. Conclusion

Retina’s response to various departures, be it from unit contrast or from the perfect matching of unit spot-size ratio has an underlying symmetry as discussed in the conclusion below and summarised at the end.

### 4.1 Interference Pattern on the Retina

This becomes more evident from the presence of an interference pattern, an attribute of coherence on the retina. With zero departure from unit contrast, the visibility does not diminish. But once the departure starts

increasing, the visibility also starts decreasing. Thus the retina responds to departure from unit contrast. And the visibility drops to 24 % of its zero departure value for a fixed pupil entry point of 0.35 cm.

## 4.2 Wave guiding of Light

When the incident spot size and the waveguide mode spot size matches, the spots size ratio is one, departure is zero, the coupling of power is 100 % and visibility loss is zero. But with gradual enhancement in the spot-size ratio departure, the fraction of power unable to be coupled (or the visibility loss) also increases. Again, a visibility loss of 90 % corresponds to a departure of either 6-fold or 1/6-fold (equivalent to a pupil entry point of 4 mm). This suggests that a pupil entry point of 3.5 mm may point to a loss of 70-80 %, a result reached earlier with departure from contrast.

Thus, to summarise: The retina's response to departures be it due to contrast, or mismatch in coupling (spot-size ratio) is to always diminish the luminance efficiency, primarily to improve vision quality in photopic vision.

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