

# The Effect of Nonlinear Loads on MMF Wave of a Synchronous Generator

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## Abstract

**Objectives:** In this paper, the effect of harmonics made by nonlinear loads on some parameters of a synchronous generator such as Magneto Motive Force (MMF) will be analyzed. **Methods/Statistical Analysis:** The MMF equations are derived, for two cases of linear and nonlinear loads based on the winding function theory and analytical method. All spatial and time harmonic components in this derivation will be considered. Also, the space vector variations of this field with respect to time harmonics will be analyzed. **Findings:** Nonlinear loads with time harmonics injection in the synchronous generator can disturb the magnetic field and the output voltage. These harmonics have other important effects such as increasing the vibration and noise, thermal loading, reduction of efficiency, etc. Also it will be shown that the space vector of magnetic field can be used to introduce effects of time harmonics of load in synchronous generator. **Application/Improvements:** This paper can be used to reach the harmonic models of synchronous generator.

**Keywords:** MMF, Nonlinear Load, Synchronous Generator, Space Vector, Winding Function

## 1. Introduction

Generating pure sinusoidal voltage by a synchronous generator is almost impossible. Some unavoidable features such as non-sinusoidal winding distribution, core saturation, teeth and slot effects, etc. inject spatial harmonics into magnetic flux and cause a disturbance in the output voltage waveforms.

Nonlinear loads are one of the major challenges in power networks and many equipments<sup>1</sup>. Nowadays, technology advances and in particular, using power electronic devices, have highlighted the role and effects of these loads.

One of the most important effects of them is time harmonics injection in power systems and equipment. These components can decrease the efficiency and load margins of power network<sup>2</sup>. Although spatial harmonics may be modified by various methods such as optimized design, the injected time harmonics increase with the diversity of loads and filtering or modifying them is hard and costly.

The analysis of nonlinear load and harmonics in power systems and protection devices and also their effects have been studied by many papers<sup>3-6</sup>.

Since the synchronous generator (SG) is one of the main equipment in the power network, effects of harmonics on its parameters, behavior, and operation is very important. The studies related to SG and harmonic components of loads are done in different areas. The procedure design of a generator for feeding nonlinear loads is one of the subjects in this area. The optimized design of a salient pole SG to minimize the excess losses and voltage disturbances when feeding a nonlinear load has been presented in<sup>7</sup>.

The effect of harmonic components on modeling, parameter calculation and operation of synchronous generator has also been interesting for researchers. For example, the effect of harmonics on load angle of SG and winding inductances has been presented in<sup>8,9</sup> respectively.

Modeling and controlling a SG in presence of an active DC load has been done in<sup>10</sup>.

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Among various parameters of SG, noise and vibration has contributed significantly due to time harmonics. The magnetic noise of a salient pole SG has been calculated in<sup>11,12</sup>.

An analytical method to calculate and analyze the unbalance magnetic pulls has been introduced in<sup>13,14</sup>. Oliviera have analyzed the effects of nonlinear and unbalanced loads on voltage and electromagnetic torque of a generator<sup>15</sup>. In<sup>16</sup> the vibration of rotor and stator has been studied. The magnetic force under static eccentricity and harmonics in<sup>17</sup> and under eddy losses conditions in<sup>18</sup> has been studied.

In this paper, the magneto motive force (MMF) of a turbo generator considering all time and spatial harmonics is calculated analytically. This equation can be used to analyze and model a SG or to calculate other parameters such as linkage flux and inductances.

The paper is organized as follows. At first, the analytical terms of MMF for linear and nonlinear loads will be derived. Then, the effect of time harmonics on the space vector of MMF is introduced. Finally, the conclusion will be presented.

## 2. Magneto Motive Force (MMF)

One method to calculate the MMF distribution in the electric machines is the winding function theory<sup>19</sup>. In this theory, the electric machine is assumed to be a linear system. The MMF distribution of a winding carrying the current  $i(t)$  is introduced as below.

$$f(\varphi_e, t) = N(\varphi_e) \cdot i(t) \tag{1}$$

Where  $\varphi_e$  and  $N(\varphi_e)$  are the displacement angle and the winding function, respectively.  $N(\varphi_e)$  is an even function and consists of only odd-order harmonics as follows.

$$N(\varphi_e) = N_{c1} \cos \varphi_e + N_{c3} \cos 3\varphi_e + N_{c5} \cos 5\varphi_e + \dots \tag{2}$$

By considering the above equation, MMF waveforms for linear and nonlinear loads will be derived in the following.

### 2.1 The MMF Distribution for Pure Sinusoidal Currents

To calculate the magnetic field in presence of linear loads, it is assumed that the stator currents are only composed of fundamental components. These currents are introduced in equation 3.

$$\begin{aligned} i_a(t) &= I_m \cos(\omega t + \theta_a) \\ i_b(t) &= I_m \cos\left(\omega t + \theta_a - \frac{2\pi}{3}\right) \\ i_c(t) &= I_m \cos\left(\omega t + \theta_a + \frac{2\pi}{3}\right) \end{aligned} \tag{3}$$

The winding function of phase  $a$  is according to equation 2. For two other phases i.e.  $b$  and  $c$ , this function can be written as below.

$$N_b(\varphi_e) = N_{c1} \cos(\varphi_e - \frac{2\pi}{3}) + N_{c3} \cos 3(\varphi_e - \frac{2\pi}{3}) + N_{c5} \cos 5(\varphi_e - \frac{2\pi}{3}) + \dots \tag{4}$$

$$N_c(\varphi_e) = N_{c1} \cos(\varphi_e + \frac{2\pi}{3}) + N_{c3} \cos 3(\varphi_e + \frac{2\pi}{3}) + N_{c5} \cos 5(\varphi_e + \frac{2\pi}{3}) + \dots \tag{5}$$

By multiplying the equations 2 and 3, the MMF of phase  $a$  can be achieved as below equation.

$$\begin{aligned} f_a(\varphi_e, t) &= N_a(\varphi_e) \cdot i_a(t) = \\ &N_{c1} I_m \cos \varphi_e \cos(\omega t + \theta_a) \\ &+ N_{c3} I_m \cos 3\varphi_e \cos(\omega t + \theta_a) \\ &+ N_{c5} I_m \cos 5\varphi_e \cos(\omega t + \theta_a) + \dots \end{aligned} \tag{6}$$

According to the following trigonometric union:

$$\cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha + \beta) + \cos(\alpha - \beta)] \tag{7}$$

the equation 6 can be simplified and rewritten as follow.

$$\begin{aligned} f_a(\varphi_e, t) &= \frac{I_m}{2} \{ N_{c1} [\cos(\varphi_e + \omega t + \theta_a) + \cos(\varphi_e - \omega t - \theta_a)] \\ &+ N_{c3} [\cos(3\varphi_e + \omega t + \theta_a) + \cos(3\varphi_e - \omega t - \theta_a)] \\ &+ N_{c5} [\cos(5\varphi_e + \omega t + \theta_a) + \cos(5\varphi_e - \omega t - \theta_a)] + \dots \end{aligned} \tag{8}$$

In the same manner, for phases  $b$  and  $c$ , the following equations can be organized.

$$\begin{aligned} f_b(\varphi_e, t) &= \frac{I_m}{2} \{ N_{c1} \left[ \cos\left(\varphi_e + \omega t + \theta_a - \frac{4\pi}{3}\right) + \cos(\varphi_e - \omega t - \theta_a) \right] \\ &+ N_{c3} \left[ \cos\left(3\varphi_e + \omega t + \theta_a - \frac{2\pi}{3}\right) + \cos\left(3\varphi_e - \omega t - \theta_a + \frac{2\pi}{3}\right) \right] \\ &+ N_{c5} \left[ \cos\left(5\varphi_e + \omega t + \theta_a - 4\pi\right) + \cos\left(5\varphi_e - \omega t - \theta_a - \frac{8\pi}{3}\right) \right] + \dots \end{aligned} \tag{9}$$

$$\begin{aligned} f_c(\varphi_e, t) &= \frac{I_m}{2} \{ N_{c1} \left[ \cos\left(\varphi_e + \omega t + \theta_a + \frac{4\pi}{3}\right) + \cos(\varphi_e - \omega t - \theta_a) \right] \\ &+ N_{c3} \left[ \cos\left(3\varphi_e + \omega t + \theta_a + \frac{2\pi}{3}\right) + \cos\left(3\varphi_e - \omega t - \theta_a - \frac{2\pi}{3}\right) \right] \\ &+ N_{c5} \left[ \cos\left(5\varphi_e + \omega t + \theta_a + 4\pi\right) + \cos\left(5\varphi_e - \omega t - \theta_a + \frac{8\pi}{3}\right) \right] + \dots \end{aligned} \tag{10}$$

The resultant MMF wave is generated by summing the MMF of three phases. After math calculations and some simplifications, the resultant MMF wave can be summarized as equation 11.

$$\begin{aligned}
 f_{total}(\varphi_e, t) &= f_a(\varphi_e, t) + f_b(\varphi_e, t) \\
 + f_c(\varphi_e, t) &= \frac{3I_m}{2} [N_{c1} \cos(\varphi_e - \omega t - \theta_a) \\
 + N_{c5} \cos(5\varphi_e + \omega t + \theta_a) + N_{c7} \cos(7\varphi_e - \omega t - \theta_a) + \dots
 \end{aligned} \tag{11}$$

Two important points may be observed in the equation 11. First, there is no component of third order and its multipliers in the resultant MMF. Second, the fifth component of MMF rotates reversely. Third, fifth and seventh components are usually considered in studies and higher harmonics are neglected due to their negligible magnitude. According to equation 11, the variation of MMF waveform of a synchronous generator with 4 poles, 60HZ, 32 slots in stator, full pitch angle winding and 35 turns per coil has been calculated and depicted in Figure 1.

### 2.1 The MMF Distribution with Nonlinear Load

If the generator feeds nonlinear loads, other harmonic components appear in its stator currents, in addition to the fundamental components. These time harmonics cause disturbance in the magnetic field of the generator. One of the famous nonlinear loads is the power electronic devices, as shown in Figure 2. In this system, a SG is feeding a resistor via a diode rectifier. The set of resistors and rectifiers acts as a nonlinear load for the generator. All sub harmonics that may be generated by this load will be neglected.

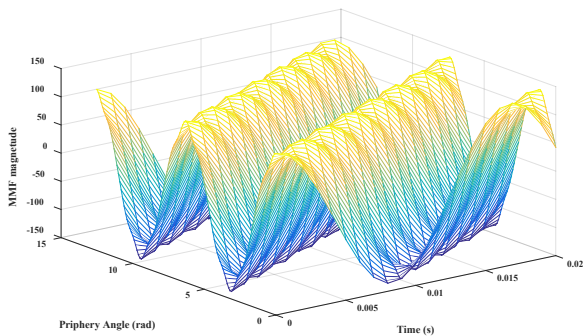


Figure 1. Distribution of generator’s MMF with a linear load.

In this section, the impact of harmonics made by nonlinear loads in the magnetic field of the generator

is considered. For this purpose, the load currents are assumed as below. These currents may include all harmonic components, in addition to fundamental ones.

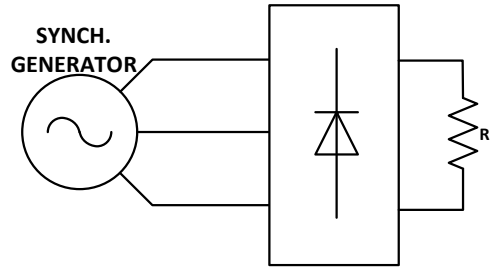


Figure 2. A nonlinear load and generator.

$$\begin{aligned}
 i_a(t) &= \sum_{n=1}^{\infty} I_n \cos n\omega t \\
 i_b(t) &= \sum_{n=1}^{\infty} I_n \cos n\left(\omega t - \frac{2\pi}{3}\right) \\
 i_c(t) &= \sum_{n=1}^{\infty} I_n \cos n\left(\omega t + \frac{2\pi}{3}\right)
 \end{aligned} \tag{12}$$

According to the above equations, MMF produced by winding of phase a is as follows.

$$\begin{aligned}
 f_a(\varphi_e, t) &= \sum_{h=1,3,\dots}^{\infty} N_{ch} \cosh \varphi_e \cdot \sum_{n=1}^{\infty} I_n \cos n\omega t \\
 &= \sum_{h=1,3,\dots}^{\infty} \sum_{n=1}^{\infty} N_{ch} I_n \cosh \varphi_e \cos n\omega t \\
 &= \frac{1}{2} \sum_{h=1,3,\dots}^{\infty} \sum_{n=1}^{\infty} N_{ch} I_n [\cos(h\varphi_e + n\omega t) + \cos(h\varphi_e - n\omega t)]
 \end{aligned} \tag{13}$$

Winding function and also currents of windings of phase b and c have 120o phase difference with respect to phase a. Regarding these facts, the MMF of phase b and c can be calculated in the same way as phase a. After calculating the MMF of phases b and c in a similar manner, the resultant MMF can be derived by adding three MMF waveforms. After some manual calculations and simplifications, the final expression will be written as 14.

$$\begin{aligned}
 f_{total}(\varphi_e, t) &= \frac{1}{2} \sum_{h=1,3,\dots}^{\infty} \sum_{n=1}^{\infty} N_{ch} I_n \{ [\cos(h\varphi_e + n\omega t) + \cos(h\varphi_e - n\omega t)] \\
 &+ [\cos\left(h\left(\varphi_e - \frac{2\pi}{3}\right) + n\left(\omega t - \frac{2\pi}{3}\right)\right) + \cos\left(h\left(\varphi_e - \frac{2\pi}{3}\right) - n\left(\omega t - \frac{2\pi}{3}\right)\right)] \\
 &+ [\cos\left(h\left(\varphi_e + \frac{2\pi}{3}\right) + n\left(\omega t + \frac{2\pi}{3}\right)\right) + \cos\left(h\left(\varphi_e + \frac{2\pi}{3}\right) - n\left(\omega t + \frac{2\pi}{3}\right)\right)] \}
 \end{aligned} \tag{14}$$

This is a comprehensive equation including all spatial and time harmonics. Usually, first, fifth and seventh spatial harmonic components exist in the electric machines. In almost nonlinear load currents, these components are dominant. Hence, considering only dominant harmonics, the equation 14 can be summarized as 15.

$$\begin{aligned}
 f_{total}(\varphi_e, t) = & \frac{3}{2} \{ N_{c1} I_1 \cos(\varphi_e + \omega t) + N_{c1} I_5 \cos(\varphi_e + 5\omega t) \\
 & + N_{c1} I_7 \cos(\varphi_e - 7\omega t) + N_{c5} I_1 \cos(5\varphi_e + \omega t) + N_{c5} I_5 \cos(5\varphi_e - 5\omega t) \\
 & + N_{c5} I_7 \cos(5\varphi_e + 7\omega t) + N_{c7} I_1 \cos(7\varphi_e - \omega t) \\
 & + N_{c7} I_5 \cos(7\varphi_e + 5\omega t) + N_{c7} I_7 \cos(7\varphi_e - 7\omega t) \} \quad (15)
 \end{aligned}$$

The current harmonics magnitude of a typically load is shown in Figure 3. As it is observed, dominant components are 1st, fifth and seventh harmonics and other components can be ignored. The MMF waveform of the generator, calculated by equation 15, is shown in Figure 4 in terms of time and position. By comparing Figures 2 and 4, the effect of the load current is evident.

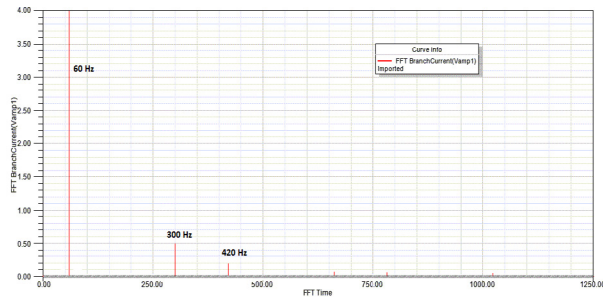


Figure 3. Harmonic components of load current.

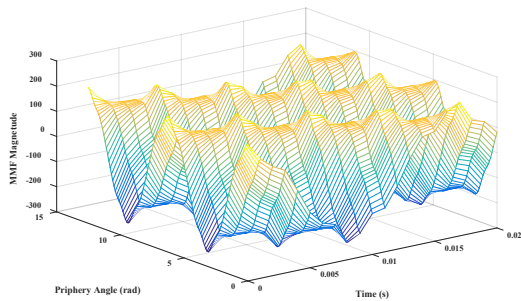


Figure 4. Distribution of generator's MMF with a linear load.

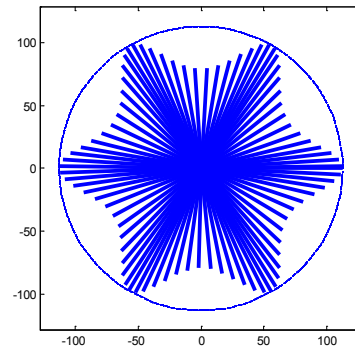
### 3. The Space Vector Method to Analyze the Nonlinear Load Harmonics

In this section a method to analyze the effect of harmonic components of load is presented. This method has been used to analyze some parameters of electric machines such as broken bars in induction machines<sup>19</sup>.

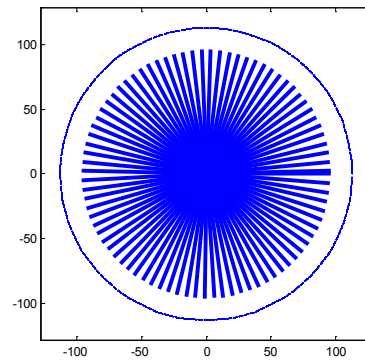
It is seems that the time harmonics made by nonlinear loads can effect on the space vector of resultant MMF wave. For this purpose the space vector of MMF wave is defined as in equation 16.

$$\vec{F} = f_a(\varphi_e, t) + a f_b(\varphi_e, t) + a^2 f_c(\varphi_e, t) \quad (16)$$

In this equation,  $a$  is a complex number and defined as  $a = 1e^{j2\pi/3}$ . Variations of the MMF space vector corresponding to pervious used loads is depicted in Figure 5. As seen in this figure, with load harmonics, the space vector of MMF swings with a frequency which is 6 times higher than the fundamental frequency.



(a)



(b)

Figure 5. Variations of Space vector of MMF. a): linear load, b): nonlinear load.

### 4. Conclusion

Nonlinear loads can inject harmonic components and disturb the MMF waveform significantly. In addition to spatial and time distribution of MMF wave, the space vector of MMF regarding all harmonics is also derived. This equation can be used to analyze the effects of loads on the magnitude and the ripple of torque in a synchronous generator. To continue the research, it may be analyzed the effects of harmonics of load on generator's parameters

such as resistors, inductance, heating, etc. Also the modeling of generator can be affected by this harmonics.

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