

# Understanding of Quantum Tomography for Determining the Superposed and Entangled States in Quantum Computing

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## Abstract

In this brief review paper aims at the understanding of experimental and theoretical work done so far on the quantum tomography and its relevance to quantum computing. In order to determine ortho normal and entangled states the procedure adopted is explained and understanding of two superposition states in a qubit is defined. The concept of the Bloch sphere to understand the reconstruction of quantum states is also discussed. Attempt has been made to understand various mathematical techniques used by other authors to determine state superposition by efficient algorithms such as Shore's and Grover's algorithms. The basic experimental technique used in quantum tomography to experimentally determine the orthogonal and superposition states uses homodyne balanced detection which is discussed and described in detail here. Also explain a simple procedure for writing and reading a qubit. A complete characterization procedure is discussed for any quantum state of two orthogonally polarized states and the paper gives a systematic procedure to collect the necessary data.

**Keywords:** Bloch Sphere, Homodyne Tomography, Quantum Entanglement, Quantum Superposition, Quantum Tomography

## 1. Introduction

The fundamental concept of classical computing is based on a binary bit. Quantum computing is also based on a similar concept as quantum bit (Qubit). In classical computing all higher and lower states are represented by a string of ones & zeros, while in the case of qubit it is a linear superposition all possible states<sup>1</sup>.

Ever since it was proposed by Feynman and its subsequent discovery, quantum entanglement has taken a central role in broad context in the use of quantum hypothesis. Quantum entanglement is a natural effect for any multi wave system in which at least two sub systems cannot be separated. If we observe in our world and assume the overall environment as a system then many of the states are probably entangled<sup>2</sup>. A popular notion is that in two subsystems the states are more separable than non-separable, which is not true.

The Poincare (Bloch) sphere is used as a geometric representation of the qubit state space as a point in the three dimensional unit spheres. The surface of the Bloch sphere is representing the pure qubit state and inside the sphere are the mixed states and maximally mixed states lie at the center of the sphere<sup>3</sup>. The states which superimpose are the only stable states which can be measured and accessed.

We can construct a quantum state by an optical pulse which contains a single photon by using quantum homodyne tomography<sup>4</sup>. Homodyne tomography is a reliable technique of reconstructing desired quantum state by spatial and temporal based excitation using the phase switching.

The density matrix  $\rho$  is a complete description of a quantum state<sup>5</sup>. Quantum state tomography is a method for determining the density matrix of a quantum system by providing many copies of available quantum system.

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## 2. Quantum Superposition

The heart of quantum mechanics is the superposition principle<sup>6</sup>. According to superposition principle two or more states may be superposed and give a new state<sup>7</sup>. We can also define the quantum superposition of a quantum system as it can be any linear superposition of those two and more states.

Superposition can be represented mathematically in the following form as –

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \quad (1)$$

Where for example,

$\alpha$  is amplitude of state  $|0\rangle$

$\beta$  is amplitude of state  $|1\rangle$

$\alpha$  and  $\beta$  are complex number and  $|\alpha|^2 + |\beta|^2 = 1$  (State of Normalization).

If there is a k-level quantum system then superposition state is defined as –

$$|\psi\rangle = a_0|0\rangle + \dots + a_{k-1}|k-1\rangle, \quad (2)$$

Where,  $\sum_{j=0}^{k-1} |a_j|^2 = 1$

A superposition is the basic unit of information in quantum computers. It is known as a Qubit.

## 3. How the Measurement of a Superposition State is Done

We are interested in measuring the quantum state  $|\psi\rangle$ . We cannot know about a superposition state itself, but we can see classical states with their probability<sup>8</sup>. If we measure a Qubit, then the state of the Qubit after measurement will be  $|0\rangle$  with probability  $|\alpha|^2$ , and  $|1\rangle$  with probability  $|\beta|^2$  as shown in Figure 1.

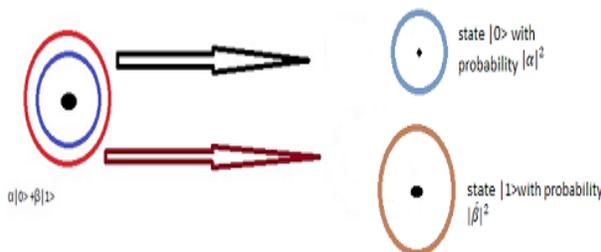


Figure 1. Measurement of superposition state.

## 4. Quantum Entanglement

The multiple particles are linked together in a way when we measure of one particle's quantum state and then we can determine the possible quantum state of the other wave functions. This phenomenon is called quantum entanglement.

The most basic entangled quantum system is a pair of Qubits. Suppose we have two Qubits they are given by,  $a_0|0\rangle + a_1|1\rangle$ , and  $\beta_0|0\rangle + \beta_1|1\rangle$ . Then joint state of two Qubits is given by the tensor product and is represented as  $a_0\beta_0|00\rangle + a_0\beta_1|01\rangle + a_1\beta_0|10\rangle + a_1\beta_1|11\rangle$ . Generally two Qubits are entangled and they cannot be decomposed into individual Qubits. Quantum entanglement state is generally represented by –

$$|\psi(x_1, x_2)\rangle = \int_{i=1}^{\infty} a_i |\psi(x_i)\rangle_i |\phi(x_2)\rangle_i \quad (3)$$

There are some examples of entanglement states viz. Bell state, Greenberger-Horne- Zeilinger (GHZ) state (Figure 2(a)) and W-state (Figure 2(b)).

An entangle quantum state of  $N > 2$  subsystem is called GHZ state<sup>9</sup>. In GHZ state each of subsystem is a two dimensional system and it is given by  $|GHZ\rangle = \frac{|0\rangle^{\otimes N} + |1\rangle^{\otimes N}}{\sqrt{2}}$ .

GHZ state is a form of quantum superposition of state  $|0\rangle$  and  $|1\rangle$ . For example we can write a GHZ state for 3 qubits  $|GHZ\rangle = \frac{|000\rangle + |111\rangle}{\sqrt{2}}$ .

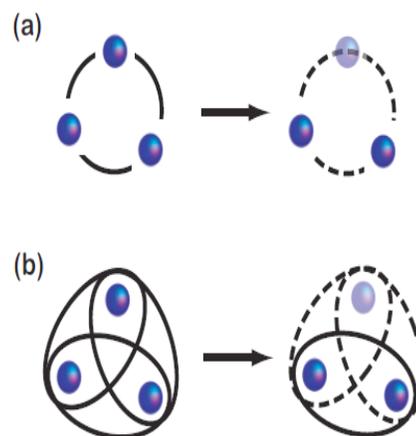


Figure 2. (a) graphically representation of GHZ state, (b) Graphically representation of W. state. <sup>21</sup>Bell state is also an example of the entangled state of a qubit. There are four entangled two qubit Bell state defined as<sup>11</sup>-

An entangle quantum state of three qubit is called the W-state<sup>10</sup> and it is written as -  $|W\rangle = \frac{|001\rangle + |010\rangle + |100\rangle}{\sqrt{3}}$ .

This is graphically represented as follows:

$$|\varnothing^2\rangle = \frac{1}{\sqrt{2}}(|0\rangle_1 \otimes |0\rangle_2 + |1\rangle_1 \otimes |1\rangle_2) \quad (4)$$

$$|\psi^+\rangle = \frac{1}{\sqrt{2}}(|0\rangle_1 \otimes |0\rangle_2 + |1\rangle_1 \otimes |1\rangle_2) \quad (5)$$

$$|\psi^+\rangle = \frac{1}{\sqrt{2}}(|0\rangle_1 \otimes |0\rangle_2 + |1\rangle_1 \otimes |1\rangle_2) \quad (6)$$

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}(|0\rangle_1 \otimes |0\rangle_2 - |1\rangle_1 \otimes |1\rangle_2) \quad (7)$$

The concept of the Bloch sphere to understand the reconstruction of quantum state.

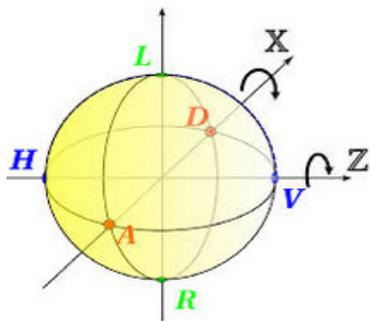
<sup>12</sup>A pure Qubit state  $|\psi\rangle$  is a point in Complex plane. The standard convention is to assume that it is a unit vector in  $C^2$  and ignore the global phase. Then without the loss of generality we can write

$$|\psi(\theta, \varphi)\rangle = \begin{pmatrix} \cos \frac{\theta}{2} \\ e^{i\varphi} \sin \frac{\theta}{2} \end{pmatrix} \quad (8)$$

We can represent our pure photonic qubit based on horizontal and vertical polarization given by -

$$|\psi\rangle = \cos \frac{\theta}{2} |H\rangle + e^{i\varphi} \sin \frac{\theta}{2} |V\rangle \quad (9)$$

Where,  $0 \leq \theta < \pi$  and  $0 \leq \varphi < 2\pi$ .



**Figure 3.** Representation of quantum states in Bloch sphere. H & V denoted to horizontal and vertical polarization which are along to Z-axis and D & A as diagonal and anti diagonal polarization which are along to X-axis, L & R as left circular and right circular polarization which are along to Y-axis<sup>12</sup>.

The quantum state  $|\psi(\theta, \varphi)\rangle$  can be represented on Bloch sphere by its vector Figure 3. A mixed state is described by its density matrix.

The density matrix is classical statistical distribution for pure states. The density matrix is written by-

$$\rho = \sum_j P_j |\psi(\theta_j, \varphi_j)\rangle \langle \psi(\theta_j, \varphi_j)| \quad (10)$$

Where  $P_j$  is denoted probability, for mixed state with probability

$$P_j \geq 0 \text{ and } \sum_j P_j = 1$$

We can write density operator in matrix form as -

$$\rho_{ij} = \langle i | \rho | j \rangle \quad (11)$$

## 5. Properties of Density Matrix

1.  $\rho$  is Hermetian:  $\rho^* = \rho$
2. Normalization:  $\text{Tr}(\rho) = 1$
3.  $\text{Tr}(\rho^2) = \begin{cases} = 1 \text{ For pure state} \\ < 1 \text{ For mixed state} \end{cases}$

The eigenvector of our observable based on particular basis direction can be found by -

$$\sigma_{(\theta, \varphi)} = \sin \theta \cos \varphi \sigma_x + \sin \theta \sin \varphi \sigma_y + \cos \theta \sigma_z \quad (12)$$

We define the polarization of the Pauli spin matrices as

$$\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = |H\rangle \langle V| + |V\rangle \langle H| \quad (13)$$

$$\hat{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = i(|V\rangle \langle H| - |H\rangle \langle V|) \quad (14)$$

$$\hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = |H\rangle \langle H| - |V\rangle \langle V| \quad (15)$$

as the projection onto one of the eigen states of the observable and the measurement result is the corresponding eigen value. Therefore, we can calculate the probability of occurrence with the expectation value of the projector. The probability of projection of a given state  $\beta$  into the state  $|\psi\rangle$  is given by

$$P_{(\theta, \varphi)}^+ = \frac{(1 + \sigma_{(\theta, \varphi)})}{2} = \langle \psi_{(\theta, \varphi)} | \psi_{(\theta, \varphi)} \rangle \quad (16)$$

$$P_{(\theta, \varphi)}^- = \frac{(1 - \sigma_{(\theta, \varphi)})}{2} = \langle \psi_{(\theta, \varphi)}^\perp | \psi_{(\theta, \varphi)}^\perp \rangle \quad (17)$$

Any ensemble of single-Qubit state can be represented by an ensemble of only two orthogonal pure states (Two pure states are

$$|\psi_{(\theta, \varphi)}\rangle \text{ and } |\psi_{(\theta, \varphi)}^\perp\rangle \text{ are orthogonal only if}$$

$$|\langle \psi_{(\theta, \varphi)} | \psi_{(\theta, \varphi)}^\perp \rangle| = 0$$

A multiple qubit states are given by the sum of tensor products of single state vectors. An n-qubit system can be written as –

$$|\psi\rangle = \sum_{i_1, i_2, \dots, i_n=0,1} a_{i_1, i_2, \dots, i_n} |i_1\rangle \otimes |i_2\rangle \otimes \dots \otimes |i_n\rangle \quad (18)$$

Where  $a_i$  is complex and  $\sum a_i = 1$ . For two qubit pure state can be written as

$$|\psi\rangle = \alpha|HH\rangle + \beta|HV\rangle + \gamma|VH\rangle + \delta|VV\rangle \quad (19)$$

Where  $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = 1$ , normalization conditions. And  $00\rangle$  shorthand notation for  $|0\rangle_1 \otimes |0\rangle_2$ .

The state of the individual subsystem cannot be written as a simple tensor product. Such types of quantum states are called Entangled.

We can completely describe a quantum state by its density matrix  $\rho$ . The density matrix is written as function of correlations  $T_{\mu_1, \mu_2, \dots, \mu_n}(\rho)$  because the states are entangled. The correlations tensor  $T(\rho)$  is defined as –

$$T_{\mu_1, \dots, \mu_n}(\rho) = \text{Tr}[\rho(\sigma_{\mu_1} \otimes \sigma_{\mu_2} \dots \otimes \sigma_{\mu_n})] \quad (20)$$

We can determine  $T_{\mu_1, \dots, \mu_n}(\rho)$  by the statistical ensemble of detector click N. This ensemble can be got by the polarization based Pauli vector.

We can reconstruct the density matrix by the determining the correlation function T. We can write the density matrix in term of correlation function as –

$$\rho = \frac{1}{2^N} \sum_{\mu_1, \mu_2, \dots, \mu_n} T_{\mu_1, \dots, \mu_n}(\rho) (\sigma_{\mu_1} \otimes \sigma_{\mu_2} \dots \otimes \sigma_{\mu_n}) \quad (21)$$

## 6. Rotation Operators

Here we describe exponential and rotation matrices among the particular basis direction in 3-D

$$e^{i\theta(\vec{n}\cdot\vec{\sigma})} = I \cos\theta + i(\vec{n}\cdot\vec{\sigma})\sin\theta \quad (22)$$

Now we see a Qubit as simply a unit vector on the complex circle (in Hilbert space representation) or as a unit vector on the Bloch sphere. We can consider rotations of the Qubit's state that keep its length invariant.

When we rotate the X, Y and Z axis of Bloch sphere by  $\theta$  then

$$R_x(\theta) = e^{-i\theta\frac{X}{2}} = I \cos\frac{\theta}{2} - i \sin\frac{\theta}{2} X$$

$$= \begin{bmatrix} \frac{\cos\theta}{2} & -i\sin\frac{\theta}{2} \\ -i\sin\frac{\theta}{2} & \frac{\cos\theta}{2} \end{bmatrix} \quad (23)$$

$$R_y(\theta) = \begin{bmatrix} \frac{\cos\theta}{2} & -\frac{\sin\theta}{2} \\ \frac{\sin\theta}{2} & \frac{\cos\theta}{2} \end{bmatrix} \quad (24)$$

$$R_z(\theta) = \begin{bmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{bmatrix} \quad (25)$$

## 7. Preparation of Qubit State

To get the Qubit state by rotating excited state around X and Y-axis as

$$X_{\pi} \text{ Pulse} \rightarrow \omega_x t = \pi \rightarrow |0\rangle \mapsto X_{\pi} \mapsto |1\rangle \quad (26)$$

$$Y_{\pi} \text{ Pulse} \rightarrow \omega_y t = \pi \rightarrow |0\rangle \mapsto Y_{\pi} \mapsto i|1\rangle \quad (27)$$

We thus prepare the superposition state as

$$X_{\frac{\pi}{2}} \text{ Pulse} \rightarrow \omega_x t = \frac{\pi}{2} \rightarrow |0\rangle \mapsto X_{\frac{\pi}{2}} \mapsto \frac{|0\rangle + |1\rangle}{\sqrt{2}} \quad (28)$$

$$Y_{\frac{\pi}{2}} \text{ Pulse} \rightarrow \omega_y t = \frac{\pi}{2} \rightarrow |0\rangle \mapsto Y_{\frac{\pi}{2}} \mapsto \frac{|0\rangle + i|1\rangle}{\sqrt{2}} \quad (29)$$

## 8. Hadamard Gate

Hadamard gate is the most common gate in quantum computing for creating a uniformly distribution superposition state<sup>13</sup>. It is defined for all zero inputs as  $|\Omega\rangle = |0000 \dots 00\rangle$

$$|\psi\rangle = \mathcal{H}^{\otimes n} |\Omega\rangle = \frac{1}{\sqrt{2^n}} \sum_{i=0}^{2^n-1} |i\rangle \quad (30)$$

Where  $\mathcal{H}^{\otimes n}$  denotes the joint n-qubit Hadamard Gate. If we use the Hadamard gate computation as basis vector  $|x\rangle$  as the initial state then the output in superposition form is given as-

$$|\psi\rangle = \mathcal{H}^{\otimes n} |\Omega\rangle = \frac{1}{\sqrt{2^n}} \sum_{i=0}^{2^n-1} (-1)^{ix} |i\rangle \quad (31)$$

Hadamard matrix is written for one qubit as -

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad (32)$$

$$|x\rangle \rightarrow \mathcal{H} \rightarrow (-1)^x |x\rangle + |1-x\rangle \quad (33)$$

Hadamard matrix is the  $(|0\rangle \& |1\rangle)$  computational basis .

$$|0\rangle \rightarrow \mathcal{H} \rightarrow \frac{|0\rangle + |1\rangle}{\sqrt{2}} \quad (34)$$

$$|1\rangle \rightarrow \mathcal{H} \rightarrow \frac{|0\rangle - |1\rangle}{\sqrt{2}} \quad (35)$$

### 8.1 Shor's Algorithm

In the Shor's algorithm there are two parts and in the first where classically reduces the factoring integer problem to a period finding problem. Then we solve the order finding problem using efficient quantum implementation of quantum Fourier transformation<sup>14</sup>.

Here Shor algorithm for  $N = 15$  factoring is explained. We choose number of qubit as  $2n \geq N$  then make the superposition state using Hadamard gate. We initialize the

register to  $|\psi\rangle = \frac{1}{\sqrt{k}} \sum_{x=0}^{k-1} |x\rangle |0\rangle$ , Where  $0 \leq x < k - 1$  This initial state is a superposition state of  $k$  states. Now we compute for the function  $f(x) = |\psi\rangle_1 = \frac{1}{\sqrt{k}} \sum_{x=0}^{k-1} |x\rangle |f(x)\rangle$ .

Now apply the inverse quantum Fourier transform we will get the quantum state which measures as

$|x\rangle = \frac{1}{\sqrt{k}} \sum_{x=0}^{k-1} e^{\frac{2\pi i u x}{k}} |u\rangle$  after that we adjust the probability amplitude to get the value of the  $r$  period with high probability.

### 8.2 Grover's Algorithm

Grover's algorithm is a quantum search algorithm that runs faster than any other classical algorithm. If there are  $N$  entries in the search space, then the time taken to

complete a search is  $O(N)$  but Grover's algorithm takes time of only  $O(\sqrt{N})$ . The key idea of Grover's algorithm is that recognizing a solution is simple instead finding the solution. We concern searching for a specific object in the database, is by using binary oracle queries. The oracle is a device that recognizes solution of search problem. When  $x$  index of a search problem is given, then oracle raises a flag as  $y_x$ . This algorithm uses the minimum number or oracle queries and locates the desired object<sup>15</sup>.

The following are the main step for Grover algorithm as shown in Figure 4-

1. We begin with  $|\psi\rangle_1 = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} |i\rangle$
2. Apply the oracle to  $|\psi\rangle$

$$|\psi\rangle = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} (-1)^{f(x)} |i\rangle$$

3. Apply the quantum Fourier transform to  $|\psi\rangle$
4. Reverse the sign of all terms in  $|\psi\rangle$  except for the term  $|0\rangle$ .
5. Apply inverse QFT.
6. Return to step 2 and repeat.

### 8.3 Homodyne Tomography

Quantum homodyne tomography may be a reliable technique of reconstructing quantum state in optical domain<sup>16</sup>. Here tend to utilize a mode-locked Ti: sapphire laser together with a pulse picker to get transform-limited pulses at 790 nm with a repetition rate of 816 kHz and a pulse width of 1.6 ps. Most of the radiation was frequency doubled in a single pass through a 3-mm LBO crystal yielding one hundred  $\mu$  W at 395 nm and then passed on to a 3-mm BBO crystal for down conversion. It can be seen from figure 5(a), the orthogonally

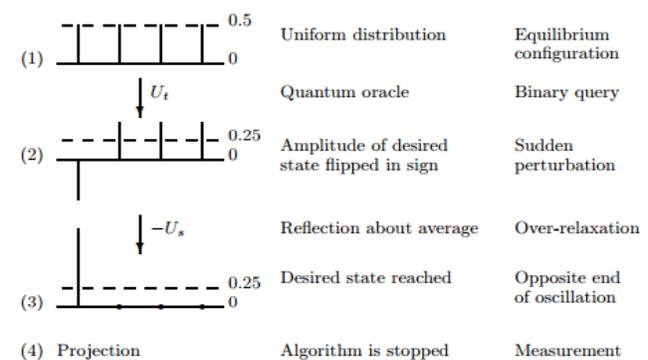


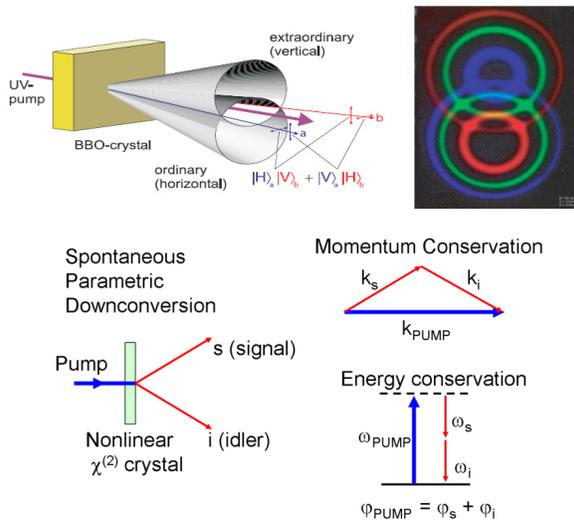
Figure 4. Basic step for Grover algorithm<sup>15</sup>.

Orthogonally polarized twin beam (spontaneous parametric down conversion corresponds to the generation of two fields, called signal and idler, with the frequencies  $\omega_s$  and  $\omega_i$  out of one field with the frequency  $\omega_p$ . These are generated in the same spatial mode. In the Spontaneous Parametric Down Conversion (SPDC) process the energy and momentum conservation must be held:

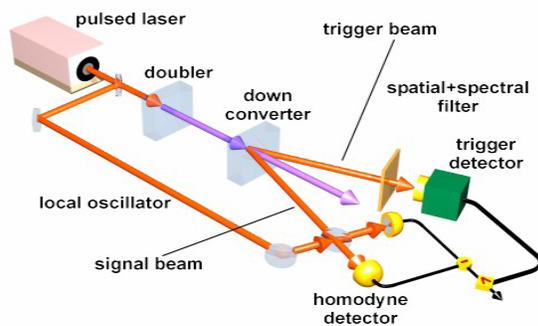
$$\omega_p = \omega_s + \omega_i \tag{36}$$

$$\vec{K}_p = \vec{K}_s + \vec{K}_i \tag{37}$$

Idler and signal are entangled which are generated by type II SPDC crystal (BBO) shown in Figure 5b. One of the photon (idler) is used as a trigger signal which is



**Figure 5a.** (I) Generation of two orthogonally entangled photon by type II nonlinear crystal (BBO) and energy (wavelength) relation of orthogonally entangled photon<sup>23</sup> (II) Energy and moment conservation in entanglement photon generation process<sup>21</sup>.



**Figure 5b.** Experimental set up of quantum homodyne tomography<sup>17</sup>.

used to trigger (start) to read out event of the homodyne tomography detector. Experiment set up for Quantum homodyne tomography is shown in Figure 6. The other photon (signal) interferes with the local oscillator, when they are directed onto a 50-50% beam splitter. After the optical mixing of the signal and local oscillator, each emerging beam is directed to a photo detector (usually a linear response photo diode). The photocurrents  $I_1$  and  $I_2$  are measured, and finally subtracted from each other. The difference  $I = I_2 - I_1$  is the quantity of interest because it contains the interference term of the local oscillator and signal.

## 9. Read out Qubit

The two generated photons are separated into two emission channels according to their propagation direction (Figure 5b) above. A single-photon counter is placed into one of the emission channels (labeled trigger) to detect photon pair creation events and to trigger the readout of a homodyne detector placed in the other (signal) channel<sup>18</sup>.

## 10. Quantum State Tomography

Quantum state tomography is that the procedure of experimentally determining an unknown quantum state by reconstructing its density matrix. This will be done by an explicit measurement of all the components that make up a particular state. The polarization state of a single photonic Qubit can be determined by taking a set of four projective measurements which are represented by the operators

$$\mu_0 = |H\rangle\langle H| \mu_2 = |D\rangle\langle D|$$

$$\mu_1 = |V\rangle\langle V| \mu_3 = |R\rangle\langle R|$$

And similarly the state of two Qubits can be determined by a set of 16 measurements (represented in the following by  $|\psi\rangle_v, v = 1; 2; 3; \dots; 16$ ), which are all possible combinations of the above operator  $\mu_i \otimes \mu_j$ , where  $i, j = 1, 2, 3$ . In general, an n-Qubit system requires  $4^n$  measurements.

The average number of counts in the detector is given by the formula  $n_i = N \text{tr}(\rho \mu_i)$ , where  $\rho$  is the density matrix representing the state of the Qubit and N is a constant that can be determined from the experimental data (includes light intensity, detector efficiency).

We can describe a density matrix in terms of matrix T and it is written as –

$$(38)$$

In case of two qubit system  $\rho$  is dependent on 16 variables of T and matrix T is written as-

$$T = \begin{pmatrix} t_1 & 0 & 0 & 0 \\ t_5 + it_6 & t_2 & 0 & 0 \\ t_{11} + it_{12} & t_5 + it_8 & t_3 & 0 \\ t_{15} + it_{16} & t_{13} + it_{14} & t_9 + it_{10} & t_4 \end{pmatrix} \quad (39)$$

To quantify however good the density matrix  $\rho(t_1, t_2, \dots, t_{16})$  that it is in agreement with the measured experimental information, we have a tendency to introduce the alleged likelihood-function<sup>19</sup>, that represents the deviation of the density matrix  $\rho$  from the measurement. Since the expected number of coincidence counts in a given experimental run is given by

$$n_v = N \langle \psi | \rho | \psi \rangle \quad (40)$$

The chance P that  $\rho$  reproduces the data, is given by the coincidence measurements which has a Gaussian probability distribution, and is given by

$$P = \prod_v^{16} \exp \left[ -\frac{(n_v - N \langle \psi | \rho | \psi \rangle)^2}{2N \langle \psi | \rho | \psi \rangle} \right] \quad (41)$$

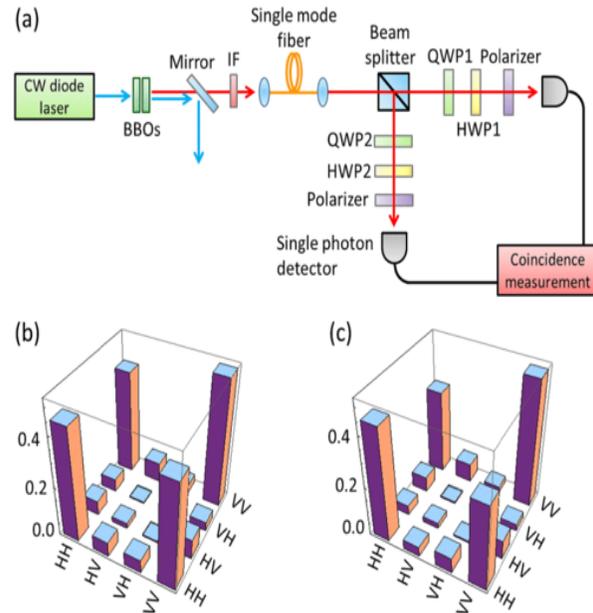
Where the standard deviation  $\rho$  is given approximately given by Poisoning noise, i.e.  $\sqrt{n_v}$ . But it is actually easier to find the minimum of the logarithm of this function P, which leads us to

$$\Delta f = \sum_v^{16} \frac{(n_v - N \langle \psi | \rho | \psi \rangle)^2}{2N \langle \psi | \rho | \psi \rangle} \quad (42)$$

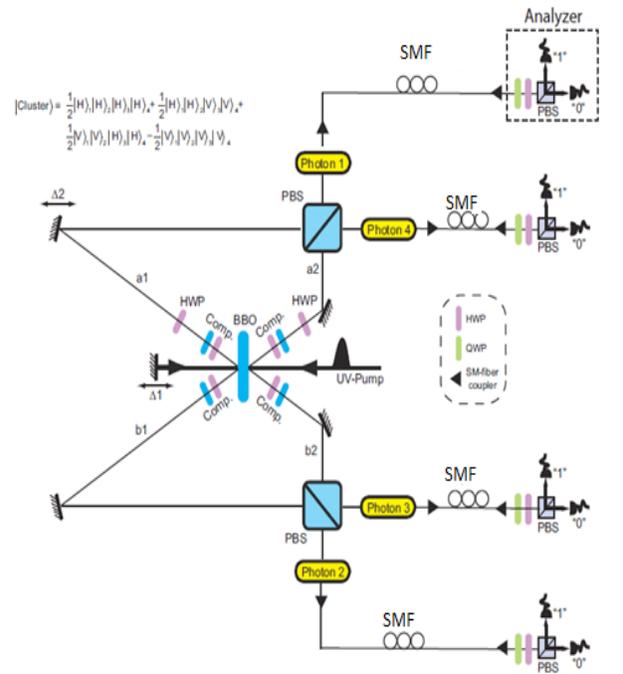
With  $\Delta f$  being the likelihood-function that indicates the deviation from the observed  $V_{th}$  coincidence measurement. An example of such a reconstructed density matrix using experimental data can be seen in Figure 6a, b and c below.

### 11. Four Photon Interference

Schematic setup for generation of four photon linear cluster states<sup>21</sup> is shown in Figure 8. The ultra short (200fs) pulse at a central wavelength 789nm pump laser pulse is passed through a 300  $\mu m$  nonlinear crystal (BBO). When the pulse is passed through the crystal this creates the entangled pair as one pair in forward propagation



**Figure 6.** (a) Schematic of the experimental setup for tomography. An interference filter (IF) with 4-nm bandwidth was used. The tomography was implemented using quarter wave plates (QWP), half wave plates (HWP), and polarizer's. (b) Histogram of the average of 300 density matrices for the normal states. (c) Histogram of the average of 50 density matrices for the erroneous states<sup>20</sup>.



**Figure 7.** Experimental set-up of four photon linear cluster states<sup>21</sup>.

direction  $a_1$  &  $b_1$  and another pair of entangled pair of photon in backward direction because pulse is reflected back and the passage by nonlinear crystal in backward side so it generate an entangled pair in the backward direction  $a_2$  &  $b_2$ . These entangled pairs of photons are coherently combined at the polarization beam splitter. The output of PBS is coupled with single mode fiber and after we use the HWP and QWP and PBS for analysis of polarization in particular basis direction after that we detect these photons by the single photon counting modules (SPDC).

## 12. Conclusion

In this paper an attempt has been made to consolidate the various aspects of Quantum Entanglement and bring forth the various methods adopted by various authors to describe the computational and experimental techniques to find the entangled states using tomography. The whole attempt was to understand the complexities of the Tomography process starting with the Bloch sphere and define the various quantum states. Sufficient understanding has been built as part of this exercise to enable either to create an experimental set-up and computational environment.

## 13. Acknowledgement

I would like to express my sincere gratitude to my advisor Prof. (Dr.) J. P. Raina for his continuous support in understanding this problem and for his motivation, enthusiasm, patience, and immense information. His guidance helped me in writing this paper. I couldn't have imagined having a much better mentor for my study. I must also thank V.I.T. University to give me the opportunity to study at this prestigious University.

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