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Double Diffusive Natural Convective Flow over a Vertical Plate with Chemical Reaction

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Abstract

Chemically reactive fluid flow over a vertical plate placed in a thermally stratified medium is presented here. A parametric study illustrating the influence of Prandtl number, Schmidt number, buoyancy ratio parameter, stratification parameter and chemical reaction parameter on the velocity, temperature, concentration profiles are studied. The chemical reaction parameter increases, the velocity is found to decreases. From the numerical value, we conclude that the variation between the steady state and temporal maximum increases gradually as *S* increases.

Keywords: Chemical Reaction, Nusselt Number, Prandtl Number, Thermal Stratification, Vertical Plate

1. Introduction

Pohlhausen[1] was first considered the convection effects on a vertical plate by using integral method, whereas Ostrach[2] studied the same problem using similarity variables. Siegel[3] used integral method to analysis the convective flow over a vertical plate. Gebhart[4] discussed the same problem using an approximate method. The above problems are related to purely heat transfer.

The difference in species concentration also affects the nature convection together with the convection currents due to the temperature variation. Gebhart and Pera[5] proposed to inspect the free convection effects on vertical plate with joint effects of thermal and mass diffusion. Callahan and Marner[6] found numerical values for the transient convective flow over a vertical plate with uniform surface temperature using an explicit finite difference method. Soundalgekar and Ganesan[7] studied very similar problem using an implicit finite difference method.

Whenever there is a variation in concentrations of chemical species in the binary mixture, mass transfer will exists. A lot of practical operations involve molecular diffusion of a species in the presence of chemical reaction taking place either within the flow or at the boundary. The influence of a homogenous chemical reaction may be mass transfer by diffusion; similarly the influence of an external source of energy may be by heat diffusion. The occurrence of simultaneous heat and mass transfer is encountered in chemical process industries, *i.e.*, food processing and polymer production.

Chambre and Young[8] examined the first order chemical reaction in the region of a flat plate. Apelblat[9] carried out analytical solution for chemical reactive species. First

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order homogenous chemically reactive species on free convection flow past a moving vertical plate was studied by Das *et al*.[10]. Theoretical solution of a moving vertical plate with chemical reactive was considered by Das *et al*.[11]. Anjali and Kandasamy[12] obtained approximate solution for a MHD flow over an accelerating vertical plate in a porous medium. Loganathan *et al*.[13] discussed the problem of chemically reactive species flow on a moving semi-infinite vertical plate. Theoretical solution of a radiative convection flow is chemical reaction over an accelerated vertical plate considered by Muthucumaraswamy and Ravi Shankar[14].

Free convection in a thermally stratified medium is remarkable and useful in the engineering applications. Cheesewright[15] was the first proposed to study the problem of free convective heat transfer from a semi-infinite vertical plate immersed in a thermally stratified medium. Chen and Eichhorn[16] examined natural convection effects from a vertical surface in a thermally stratified fluid. Srinivasan and Angirasa[17] engaged explicit method to solve the problem of convective flow over a vertical surface immersed in a thermally stratified medium with the heat and mass transfer. Double-diffusive free convection flow past a vertical cylinder was analyzed numerically by Loganathan and Ganesan[18].

The reason of the proposed study is to discuss the problem of heat and mass transfer natural convection on a vertical plate with thermally stratified and chemical reaction. A finite difference scheme was engaged to look into the nonlinear system of the particular problem.

2. Basic Governing Equations

The consideration was made in a two-dimensional unsteady flow over a semi-infinite vertical plate with heat and mass transfer in a thermally stratified medium along with its chemical reaction. The axes are taken in such a way that the *x*-axis represents along the plate and *y*-axis represents upward normal to the plate. Initially, the plate and the fluid are equal at the same temperature $T'_{\infty,x}$ and the concentration C'_{∞}. Also when the time t becomes greater than zero the temperature and the species concentration raised to T'_w and C'_w respectively and later they are continued at the same level. Also first-order chemical reaction between the fluid and species is considered.

The boundary layer equations with initial and boundary conditions in non dimensional form are as follows:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{1}$$

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=T+NC+\frac{\partial^2 U}{\partial Y^2} \tag{2}$$

$$\frac{\partial T}{\partial t}+U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y}+SU=\frac{1}{\Pr}\frac{\partial^2 T}{\partial Y^2} \tag{3}$$

$$\frac{\partial C}{\partial t}+U\frac{\partial C}{\partial X}+V\frac{\partial C}{\partial Y}=\frac{1}{Sc}\frac{\partial^2 T}{\partial Y^2}-KC \tag{4}$$

Where the following non-dimensional quantities are used:

$$\begin{gathered} X=\frac{x}{L},\ Y=\frac{y}{L}Gr^{1/4},\ U=\frac{uL}{\nu}Gr^{-1/2},\ V=\frac{vL}{\nu}Gr^{-1/4},\ t=\frac{\nu t'}{L^2}Gr^{1/2}, \\ T=\frac{T'-T'_{\infty,x}}{T'_w-T_{\infty,0}},\quad Gr=\frac{g\beta L^3(T'_w-T'_{\infty,0})}{\nu^2},N=\frac{\beta^*(C'_w-C'_{\infty})}{\beta(T'_w-T'_{\infty,0})} \\ Gc=\frac{g\beta^* L^3(C'_w-C'_{\infty})}{\nu^2},\Pr=\frac{\nu}{\alpha},\ K=\frac{K'L^2}{\nu}Gr^{-1/2},Sc=\frac{\nu}{D},S=\frac{1}{T'_w-T'_{\infty,0}}\frac{dT'_{\infty,0}}{dX} \end{gathered} \tag{5}$$

The corresponding initial and boundary conditions in a dimensionless form are as follows:

$$\begin{array}{llllll} t\le 0: & U=0,\ V=0, & T=0 & C=0 & \text{for all } X \text{ and } Y & \\ t>0: & U=0,\ V=0, & T=1-SX & C=1 & \text{at } Y=0, & \\ & U=0 & T=0 & C=0 & \text{at } X=0 & (6) \\ & U\to 0, & T\to 0 & C\to 0 & \text{as } y\to\infty & \end{array}$$

The local initial non-dimensional skin friction and the local Nusselt number and the local Sherwood number are given by

$$\tau_X=Gr^{3/4}\left(\frac{\partial U}{\partial Y}\right)_{Y=0},\quad Nu_X=\frac{-XGr^{1/4}\left(-\frac{\partial T}{\partial Y}\right)_{y=0}}{T_{Y=0}},\overline{Sh}x=\frac{-XGr^{1/4}\left(-\frac{\partial C}{\partial Y}\right)_{y=0}}{C_{Y=0}} \tag{7}$$

Also, the non-dimensional average skin friction and the average Nusselt number and the average Sherwood number are given by:

$$\overline{\tau}=Gr^{3/4}\int_0^1 X\left(\frac{\partial U}{\partial Y}\right)_{Y=0}dX,\ \overline{Nu}=Gr^{1/4}\int_0^1\left[\frac{\left(\frac{\partial T}{\partial Y}\right)_{Y=0}}{T_{Y=0}}\right]dX,\ \overline{Sh}=Gr^{1/4}\int_0^1\left[\frac{\left(\frac{\partial C}{\partial Y}\right)_{Y=0}}{C_{Y=0}}\right]dX \tag{8}$$

3. Numerical Techniques

An implicit finite difference scheme is deployed to solve the equations (1)-(4) with the corresponding initial and boundary conditions (6). For calculation we consider the region of integration as a rectangles with sides X_{max} (=1) and Y_{max} (=22). An appropriate mesh sizes $\Delta X = 0.05$, $\Delta Y = 0.25$ and $\Delta t = 0.01$ were considered for calculation. The derivatives and integration involved in the equations (7) and (8) are evaluated using a five approximation formula and Newton-closed integration formula.

4. Discussion of Results

For the different values and the effect of variation of *Sc* and *Pr* on transient velocity, temperature and concentration were shown in Figure 1, Figure 2 and Figure 3 respectively. During the transient period, the velocity rising gradually with time *t*, it reaches the temporal maximum and subsequently it reaches the stable solution. Time required reaching the steady state more strongly depends upon the value of the Sc. Time to reaches the temporal maximum is earlier for lighter gases in comparison with the heavier gas. The velocity is observed to decreases with the higher values of Pr.

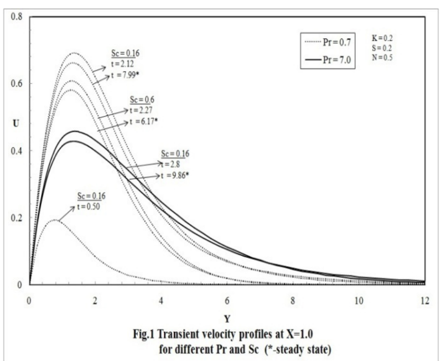

Figure 1. Transient velocity profiles at X=1.0 for different Pr and Sc (*-steady state).

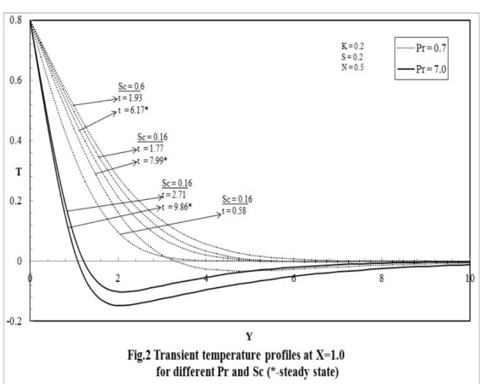

Figure 2. Transient temperature profiles at X=1.0 for different Pr and Sc (*-steady state).

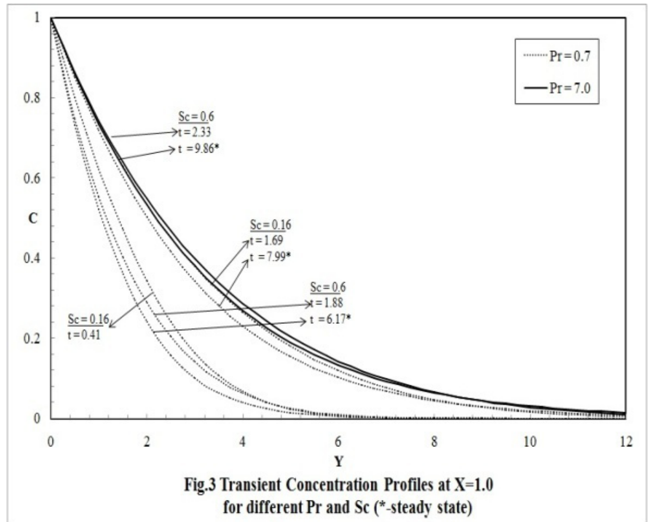

Figure 3. Transient concentration profiles at X=1.0 for different Pr and Sc (*-steady state).

Thermal stratification can cause negative values of temperature, since the fluid near the plate can have a temperature lower than the ambient. The temperature for water is more in the case of air. From the numerical result, we notice that the temperature distribution decreases as *Sc* decreases. A fall in temperature occurs due to increasing value of the Pr. The concentration increase as Pr increases.

Transient velocity, temperature and concentration shapes at the leading edge of the plate for different values of buoyancy ratio parameter and chemical reaction parameter *K* are revealed graphically in Figure 4, Figure 5 and Figure 6 respectively. As increase in N, the combined buoyancy force increases. Therefore, the velocity increases near the plate but velocity decreases near the region Y > 2.5. The enhance in the velocity of thermal buoyancy force in the boundary layer is more pronounced than that of the concentration buoyancy force in the region far away from the plate. The time needed to reach the stable solution increasing with the decreasing value of buoyancy ratio parameter *N*. It is observed that the velocity that the velocity decreases as chemical reaction parameter *K* increases. From the numerical value, we conclude that the increase in the chemical reaction parameter K also increase the temperature. It is very important with the effect of the chemical reaction parameter in the concentration. The decrease in the concentration is observed as the increase in the chemical reaction parameter or the buoyancy ratio parameter.

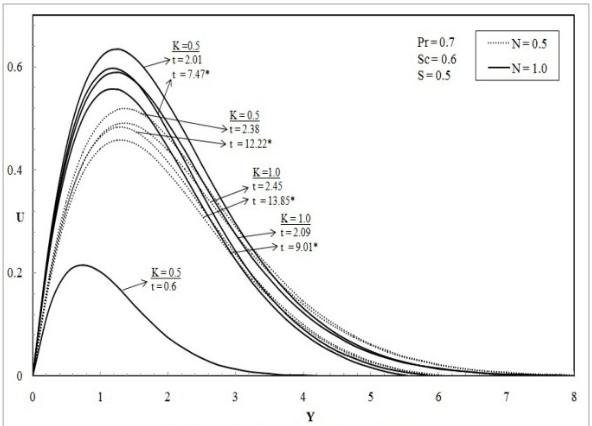

Fig.4 Transient Velocity Profiles at X=1.0 for different N and K (*-steady state)

Figure 4. Transient Velocity profiles at X=1.0 for different N and K (*-steady state).

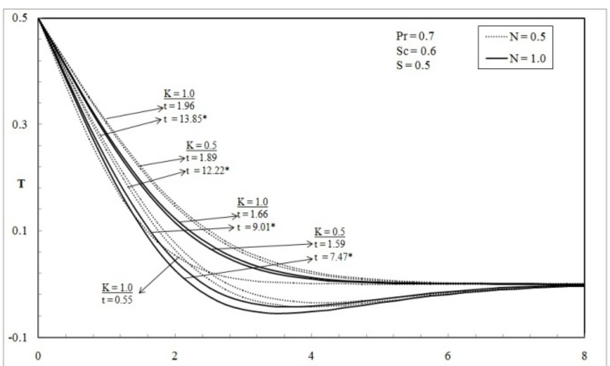

Fig.5 Transient temperature profiles at X=1.0 for different N and K (*-steady state)

Figure 5. Transient temperature profiles at X=1.0 for different N and K (*-steady state).

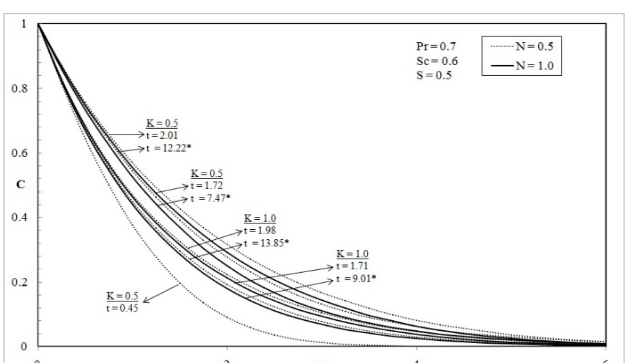

Fig.6 Transient Concentration Profiles at X=1.0 for different N and K (*-steady state)

Figure 6. Transient concentration profiles at X=1.0 for different N and K (*-steady state).

The effects of thermal stratification in the ambient are shown in Figure 7, Figure 8 and Figure 9 for aiding flow. We conclude from the numerical values that an increasing the stratification parameter decreases the velocity. From Figure 8, we see that the temperature decreases as thermal stratification parameter *S* increases. More time is required to reach the steady state for higher value of thermal stratification parameter *S*. The concentration profile decreases as *S* decreases.

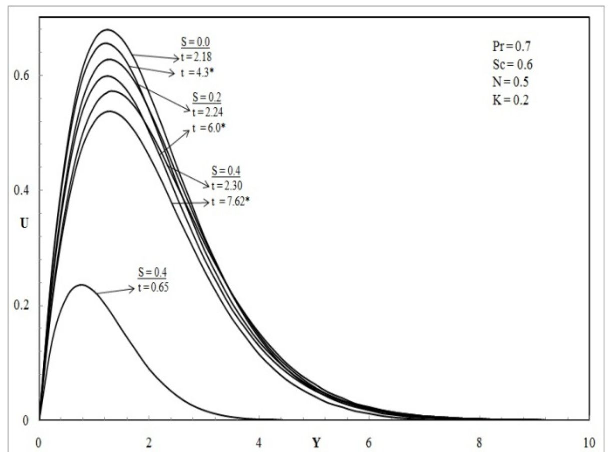

Fig.7 Transient velocity profiles at X=1.0 for different S (*-steady state)

Figure 7. Transient velocity profiles at X=1.0 for different S (*-steady state).

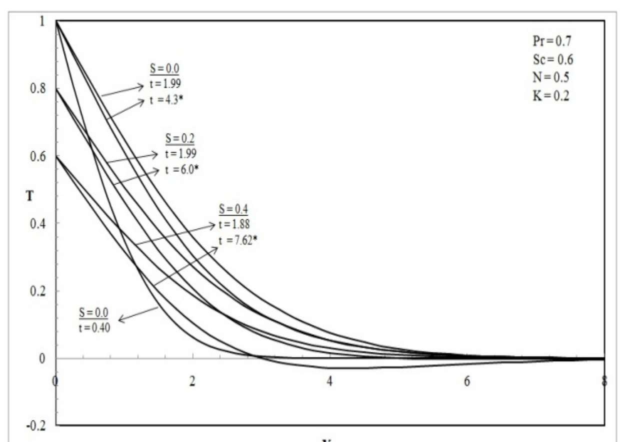

Fig.8 Transient temperature profiles at X=1.0 for different S (*-steady state)

Figure 8. Transient temperature profiles at X=1.0 for different S (*-steady state).

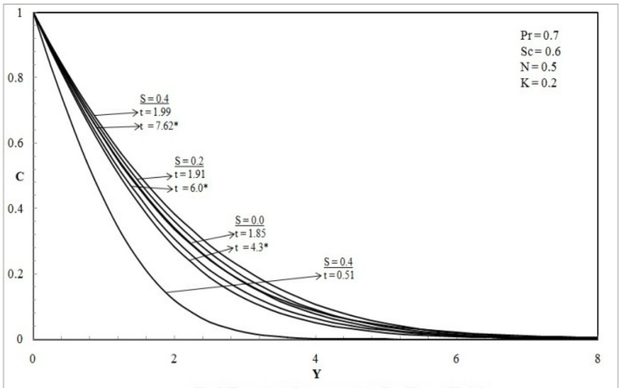

Fig.9 Transient Concentration Profiles at X=1.0 for different S (*-steady state)

Figure 9. Transient concentration profiles at X=1.0 for different S (*-steady state).

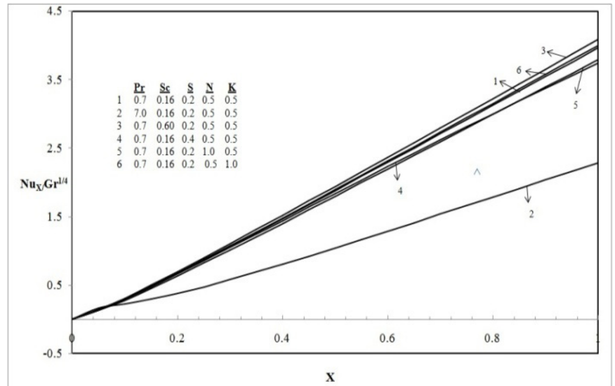

Fig.11 Local Nusselt number

Figure 11. Local nusselt number.

The effects of Sc, Pr, N, K and S are shown in Figure 10. The local skin friction increases as *X* increases. As Sc increase, shear stress in local decreases. The local skin friction increases with increasing N, because the velocity increases as N increases. The skin friction decreases as *K* increases. Local shearing stress decreases with increasing stratification parameter *S*.

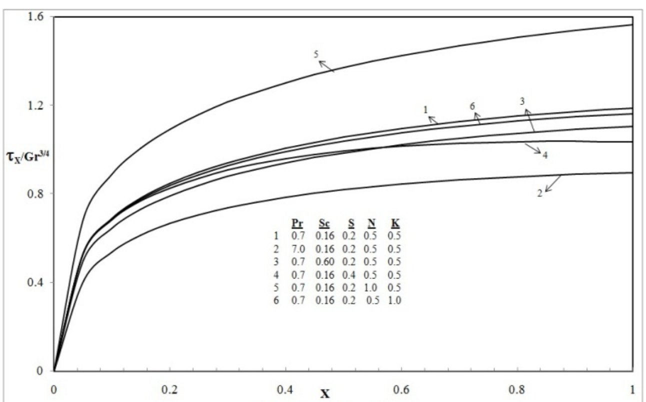

Fig.10 Local skin friction

Figure 10. Local skin friction.

In Figure 11, local Nusselt number is plotted at the steady state. From Figure11, we observe that an increase in Sc show the way to an increase in local Nusselt number but an increase in Pr show the way to a decrease Nusselt number in local. The rate of heat transfer decreases with increasing stratification parameter *S*. Local Nusselt number decreases as N increases. But Local Nusselt number increases *K* or *Sc* increases.

Local Sherwood numbers are shown in Figure 12 for various values of parameters taking place into the problem. Local Sherwood number increases as *Sc* increases. Also we conclude from Figure 12 that the Local rate of mass transfer increases as *N* or *K* increases. The rate of mass transfer in local found to decreases with an increasing value of *S* or *Pr*.

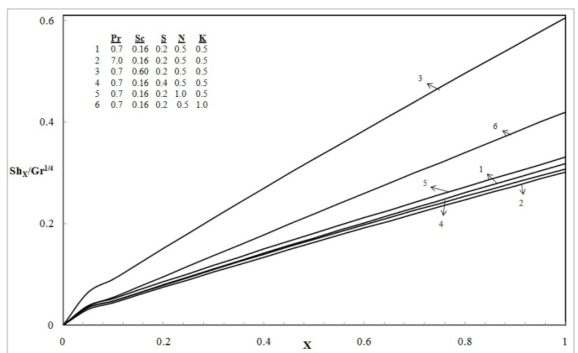

Fig.12 Local Sherwood Number

Figure 12. Local Sherwood number.

Average values of skin friction, Nusselt number and Sherwood number for different values of Parameters occurring into the problem are depicted graphically in Figure 13, Figure 14 and Figure 15 respectively. Skin friction in average found to decreases with the increasing values of Sc. Average skin friction decreases with increasing value of *S*. Also we noticed that the average skin friction decreases as *Pr* increases. The average skin friction increases with *N* throughout the transient period. It is also observed that the average skin friction decreases as *K* increases. The average Nusselt number increases with increasing *Sc* or *K*. But the average Nusselt number decreases with increasing *Pr* or *S* or *N*. Average Sherwood number increases as *Sc* increases, this is quite expected. Also average Sherwood number increases as *N* or *K* increases. Average Sherwood number decreases as *Pr* increases. Sherwood number in average found to decreases as *S* increases.

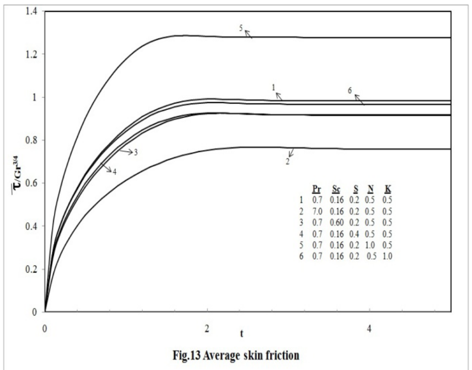

Figure 13. Average skin friction.

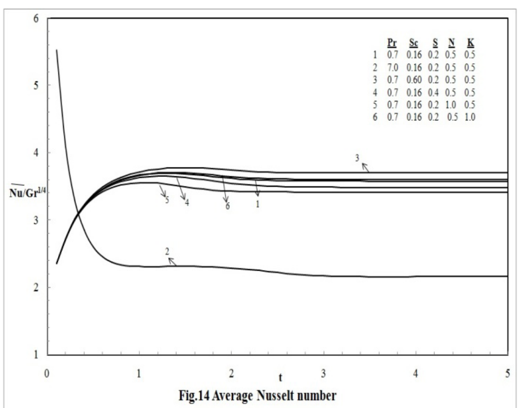

Figure 14. Average nusselt number.

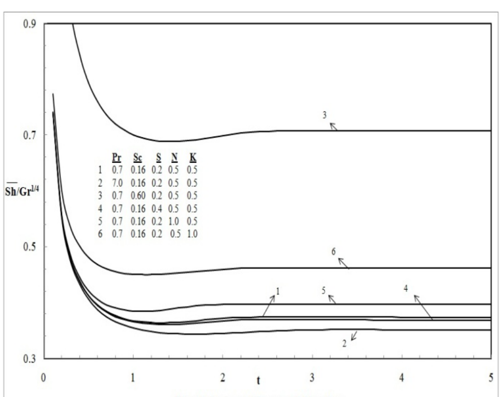

Figure 15. Average Sherwood number.

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