ISSN (Print) : 0974-6846
Indian Journal of Science and Technology, Vol 9(46), DOI: 10.17485/ijst/2016/v9i46/98606, December 2016 ISSN (Online) : 0974-5645

# Upper Bound for the Radio Number of Some Families of Sunlet Graph

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## Abstract

**Objectives:** In this study, Radio Coloring is used to color the graphs. The objective of this article is to analyze the bounds of Line, Middle, Total and Central graphs of Sunlet graph. **Methods/Analysis:** Combinatorics is an important part of discrete mathematics that solves counting problems without actually enumerating all possible cases. Combinatorics has wide applications in Computer Science, especially in coding theory, analysis of algorithms and others. An equation that expresses $a_n$, the general term of the sequence $\{a_n\}$ is called a recurrence relation. Using the generating function of a sequence and few coloring techniques we prove the results. **Findings:** The Problem of finding radio coloring with small or optimal k arises in the concept of radio frequency assignment. The radio chromatic score rs(G)[17] of a radio coloring is the number of used colors. The number of colors used in a radio coloring with the minimum score is the radio chromatic rn(G) of G.

The radio chromatic number of Sunlet graph $S_n$ is [10]

4 if n=3i, i=1,2,...

5 if n=3i+1, i=1,2,...

6 if n=3i+2, i=1,2,...

and is improved to the radio chromatic number of Sunlet graph $S_n$ is

5 if n is congruent to 1mod 3

4 if otherwise

In this paper we improve the radio chromatic number of Sunlet graph $S_n$ and obtain the radio number of Line, Middle, Total and Central graphs of Sunlet graph. Radio Coloring has wide range of significance because radio coloring has its applications in communication theory. The paper contributes the researches in the field of computer science and combinatorics.

**Applications:**

Radio coloring is of great significance because the frequency assignment problem is modeled as a graph coloring problem assuming transmitters as vertices and interference as adjacencies between two vertices.

**Keywords:** Sunlet Graph, Line Graph, Middle Graph, Radio Number, Total Graph and Central Graph

## 1. Introduction

Radio frequency assignment is a broad area of research. The task is to assign radio frequencies to transmitters at different locations without causing interference. The problem is closely related to graph coloring where the vertices of a graph represent the transmitters and adjacencies indicate possible interferences. In[1] Griggs and Yeh introduced a problem proposed by Roberts which they call the Radio Coloring problem. It is the problem of assigning radio frequencies (integers) to transmitters such that transmitters that are close (distance 2 apart) to each other receive different frequencies and transmitters that are very close together (distance 1 apart) receive frequencies that are at least two apart. To keep the frequency bandwidth small, frequencies that have been assigned to

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the radio network. The minimum frequencies assigned is radio number (rn).

Subsequently, different bounds of rn were obtained for various graphs. A common parameter used is $\Delta$, the maximum degree of a graph. The obvious lower bound for rn is $\Delta+1$, achieved for the tree $K_{1,n}$. In [1] it was shown that for every graph G, $rn \leq \Delta^2 + 2\Delta$. This upper bound was later improved to $rn \leq \Delta^2 + \Delta$ in [2].

For some classes of graphs, tight bounds are known and can be computed efficiently. These include paths, cycles, wheels and complete k-partite graphs,trees[2], cographs[2], interval graphs[2], unicycles and bicycles[3], outerplanar planar graphs[3], hexagons-meshes[4], hexagons and unit interval graphs[5], hypercubes[6,7], bipartite graphs[8], k-almost tress[9], cacti, and grids, and Sunlet graphs[10].

In this paper, we extend the upper bounds of rn to sunlet families of graphs. Other types of graphs have also been studied, and their bounds are given in this paper. All graphs considered in this paper are finite, nontrivial, simple, undirected and connected.

## 2. Preliminaries

**Definition. 2.1**

A k-vertex coloring of G is an assignment of k-colors 1,2,....k to the vertices of G . The coloring is proper if no two distinct adjacent vertices have the same color. G is k-vertex colorable if G has a proper k-vertex coloring. The chromatic number $\chi(G)$ of G is the minimum k for which G is k-colorable [11].

**Definition 2.2.**

A k-radio coloring of a graph is a function **f** from the vertex set V(G) to the set of all nonnegative integers 1,2,…,k such that

(i) $|f(x)-f(y)| \geq 1$ if d(x,y)=2 and

(ii) $|f(x)-f(y)| \geq 2$ if d(x,y)=1

The radio number of G is the smallest k for which G has radio coloring and is denoted by rn(G)[12].

**Definition 2.3.**

The n-sunlet graph is a graph on 2n vertices obtained by attaching a n-pendant edges to the vertices of cycle $C_n$ and is denoted by $S_n$ [13].

**Definition 2.4.**

The Line graph of a graph G, denoted by L(G), is a graph whose vertices are the edges of G, and if $u, v \in E(G)$ then $u, v \in E(L(G))$ if u and v share a vertex in G [13].

**Definition 2.5.**

Let G be a graph with vertex set V(G) and edge set E(G). The Middle graph of G, denoted by M(G), is defined as follows. The vertex set of M(G) is $V(G) \cup E(G)$. Two vertices x,y in the vertex set of M(G) are adjacent in M(G) in case one of the following holds:

(i) x,y are in E(G) and x,y are adjacent in G.
(ii)x is in V(G), y is in E(G), and x,y are incident in G [13].

**Definition 2.6.**

Let G be a graph with vertex set V(G) and edge set E(G). The Total graph of G, denoted by T(G), is defined as follows. The vertex set of T(G) is $V(G) \cup E(G)$. Two vertices x,y in the vertex set of T(G) are adjacent in T(G) in case one of the following holds:

(i) x,y are in E(G) and x,y are adjacent in G.
(ii) x,y is in V(G), and x,y are adjacent in G.
(iii) x is in V(G), y is in E(G), and x,y are incident in G [13].

**Definition 2.7.**

The central graph of a graph G, C(G) is obtained by subdividing each edge of G exactly once and joining all the non adjacent vertices of G[14].

***Analysis:***

By the analysis of various graph families to obtain radio number, it is found that on radio coloring of a graph using positive integers, the radio number is obtained by using either only odd integers or even integers. The usage of both odd and even integers alternatively for coloring the graphs gives maximum rn compared to the above method.

## 3. Results

***Result: 1***

The radio chromatic number of path $P_n$ is $\Delta+1$ for $n \geq 3$[10].

***Result: 2***

The radio chromatic number of comb graph G is $\Delta+1$ for $n \geq 3$[10].

***Theorem: 3***

The radio chromatic number of star graph $K_{1,n}$ is $\Delta+1$ [10].

***Theorem: 4***

The radio chromatic number of cycle[10]

$$rn(C_n) = \begin{cases} \Delta+1 & n \equiv 0 \bmod 3 \\ \Delta+2 & n \equiv 1 \bmod 3 \\ \Delta+3 & n \equiv 2 \bmod 3. \end{cases}$$

***Theorem: 5***

The radio chromatic number of Sunlet graph $S_n$

$$rn(S_n) = \begin{cases} \Delta+2 & n \equiv 1 \bmod 3 \\ \Delta+1 & \text{Otherwise} \end{cases}$$

### 3.1 Structural properties of Sunlet graph

- Number of Vertices in $S_n$ is $P = 2n$
- Number of Edges in $S_n$ is $q = 2n$
- Maximum degree in $S_n$ is $\Delta = 3$
- Minimum degree in $S_n$ is $\delta = 1$

### 3.2 Structural properties of Line graph of Sunlet graph

- Number of Vertices in $L(S_n)$ is $P = 2n$
- Number of Edges in $L(S_n)$ is $q = 3n$
- Maximum degree in $L(S_n)$ is $\Delta = 4$
- Minimum degree in $L(S_n)$ is $\delta = 2$

### 3.3 Structural properties of Middle graph of Sunlet graph

- Number of Vertices in $M(S_n)$ is $P = 4n$
- Number of Edges in $M(S_n)$ is $q = 7n$
- Maximum degree in $M(S_n)$ is $\Delta = 6$
- Minimum degree in $M(S_n)$ is $\delta = 1$

### 3.4 Structural properties of Total graph of Sunlet graph

- Number of Vertices in $T(S_n)$ is $P = 4n$
- Number of Edges in $T(S_n)$ is $q = 9n$
- Maximum degree in $T(S_n)$ is $\Delta = 6$
- Minimum degree in $T(S_n)$ is $\delta = 2$

### 3.5 Structural properties of Central graph of Sunlet graph

- Number of Vertices in $C(S_n)$ is $P = 4n$
- Number of Edges in $C(S_n)$ is $q = 2n + \binom{2n}{2}$
- Maximum degree in $C(S_n)$ is $\Delta = 6$
- Minimum degree in $C(S_n)$ is $\delta = 2$

## 4. Radio Number of Line, Middle, Total and Central Graphs of Sunlet Graph.

***Lemma: 6***

The radio chromatic number of Middle graph of Sunlet graph $S_n$ is $\Delta+3$ for n=3.

**Proof:**

Let us define the vertex set V and the edge set E of $S_n$ as $V(S_n) = \{v_1,\dots,v_n\} \cup \{u_1,\dots,u_n\}$ where $v_i$ are the vertices of cycles taken in cyclic order and $u_i$ are the pendent vertices such that $v_iu_i$ is a pendent edge and $E(S_n) = \{e'_i : 1 \le i \le n\} \cup \{e_i : 1 \le i \le n-1\} \cup \{e_n\}$, where $e_i$ is the edge $v_iv_{i+1}$ $(1 \le i \le n-1)$, $e_n$ is the edge $v_nv_1$ and $e'_i$ is the edge $v_iu_i$ $(1 \le i \le n)$.

By the definition of middle graph

$V(M(S_n)) = V(S_n) \cup E(S_n) = \{v_i : 1 \le i \le n\} \cup \{u_i : 1 \le i \le n\} \cup \{v'_i : 1 \le i \le n\} \cup \{u'_i : 1 \le i \le n\}$, where $v'_i$ and $u'_i$ represents the edge $e_i$ and $e'_i$ $(1 \le i \le n)$ respectively.

Consider the following 9-coloring of (1,3,5,7,9,11,13,15,17) of $M(S_n)$.

Assign the color 1 to $v_1$, 3 to $v'_1$, 5 to $v_2$, 7 to $v'_2$, 9 to $v_3$, 11 to $v'_3$, 5 to $u_1$, 13 to $u'_1$, 1 to $u_2$, 15 to $u'_2$, 3 to $u_3$, 17 to $u'_3$.

**Note:**

Every vertex in the cycle (ie) $v_1,\dots v_n$, $v'_1,\dots, v'_n$ can be reached from the remaining vertices in the cycle by distance two .So, they are colored using different colors.

$$\left| c(v_1,\dots v_n, v'_1,\dots v'_n) \right| = 2n$$

Then the remaining 2n vertices are colored in such a way that pendant vertices are colored using repeated colors and vertices with degree 3 is colored with n different colors.

***Lemma: 7***

The radio chromatic number of Total graph of Sunlet graph $S_n$ is $\Delta+3$ for n=3.

**Proof:**

The vertex set and edge set of $S_n$ are as described in lemma 6.

By the definition of total graph

$V(T(S_n)) = V(S_n) \cup E(S_n) = \{v_i : 1 \le i \le n\} \cup \{v'_i : 1 \le i \le n\} \cup \{u_i : 1 \le i \le n\} \cup \{u'_i : 1 \le i \le n\}$, where $v'_i$ and $u'_i$ represents the edge $e_i$ and $e'_i$ $(1 \le i \le n)$ respectively.

Consider the following 9-coloring of (1,3,5,7,9,11,13,15,17) of $T(S_n)$.

Assign the color 1 to $v_1$, 3 to $v'_1$, 5 to $v_2$, 7 to $v'_2$, 9 to $v_3$, 11 to $v'_3$, 7 to $u_1$, 13 to $u'_1$, 11 to $u_2$, 15 to $u'_2$, 3 to $u_3$, and 17 to $u'_3$.

The above mentioned coloring is radio coloring.

***Theorem: 8***

For n≥3, the radio chromatic number of line graph of sunlet graph $L(S_n)$ is

$$rn(L(S_n)) = \begin{cases} \Delta+1 & n \equiv 2 \bmod 3 \\ \Delta+1 & n \equiv 0 (\bmod 6),\ n \equiv 0 (\bmod 10) \\ \Delta+2 & n \equiv 0 \bmod (25+10i),\ i=0,1,2,\ldots \\ \Delta+2 & n \equiv 0 \bmod 2, \text{ except } n=5i\&6i,\ i=1,2,\ldots \\ \Delta+2 & n \equiv 0 \bmod 3, n \equiv 1 \bmod 3 \ \text{ except } n=6i,\ i=1,2,\ldots \end{cases}$$

**Proof:**

The vertex set and edge set of $S_n$ are as described in lemma 6.

By the definition of line graph $V(L(S_n)) = E(S_n) = \{u'_i : 1 \le i \le n\} \cup \{v'_i : 1 \le i \le n-1\} \cup \{v'_n\}$ where $v'_i$ and $u'_i$ represent the edge $e_i$ and $e'_i$ $(1 \le i \le n)$ respectively.

**Case(i): $rn(L(S_n))=\Delta+1$ $n\equiv 2 \bmod 3$**

Consider the following 5-coloring of (1, 3, 5, 7, 9) of $L(S_n)$.

Assign the color

1 to $v'_1, v'_4, v'_7, v'_{10},\ldots,v'_{n-4}$ and $u'_{n-1}$
3 to $v'_2, v'_5, v'_8,\ldots,v'_{n-3}$ and $u'_n$
5 to $v'_3, v'_6, v'_9,\ldots, v'_{n-2}$ and $u'_1$
7 to $u'_2, u'_4, u'_6,\ldots,u'_{n-3}$ and $v'_{n-1}$
9 to $u'_3, u'_5, u'_7,\ldots, u'_{n-2}$ and $v'_n$

**Case(ii): $rn(L(S_n))=\Delta+1$ $n\equiv 0 \bmod 6$, $n\equiv 0 \bmod 10$**

Consider the following 5-coloring of (1,3,5,7,9) of $L(S_n)$.

**Sub case(i): $rn(L(S_n))=\Delta+1$ $n\equiv 0 \bmod 6$**

Assign the color

1 to $v'_1, v'_4, v'_7, v'_{10},\ldots, v'_{n-2}$
3 to $v'_2, v'_5, v'_8,\ldots,v'_{n-1}$
5 to $v'_3, v'_6, v'_9,\ldots,v'_n$
7 to $u'_2, u'_4, u'_6,\ldots, u'_n$
9 to $u'_1, u'_3, u'_5,\ldots,u'_{n-1}$

**Sub case(ii): $rn(L(S_n))=\Delta+1$ $n\equiv 0 \bmod 10$**

Assign the color

1 to $v'_1, u'_4, v'_6,\ldots$
3 to $u'_2, v'_4, u'_7,\ldots$

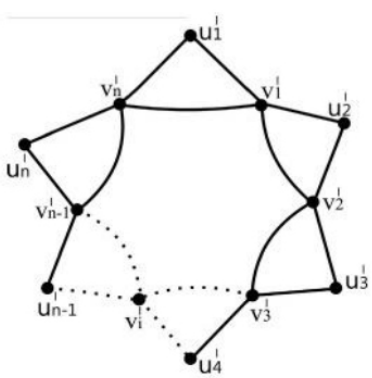


**Figure 1.** Line graph of Sunlet graph $L(S_n)$.

5 to $v'_2, u'_5, v'_7,\ldots$
7 to $u'_3, v'_5, u'_8,\ldots$
9 to $v'_3, u'_6, v'_8,\ldots$

(Note: 1,3,5,7,9 are consecutively assigned to vertices $v'_i$ and $u'_i$ (ie) if $c_1$ is assigned to $v'_1$ and 3 to $u'_2$ and so on)

**Case(iii):**

**$rn(L(S_n))=\Delta+2$ $n\equiv 0 \bmod(25+10i)$, $i=0,1,\ldots$**

Consider the following 6-coloring of (1,3,5,7,9,11) of $L(S_n)$.

**Sub case(i): $n=3k+1$, $k=8+10i$, $i=0,1,\ldots$**

Assign the color

1 to $v'_1, v'_4, v'_7, \ldots,v'_{n-3}$ and $u'_{n-1}$
3 to $v'_2, v'_5, v'_8,\ldots,v'_{n-2}$
5 to $v'_3, v'_6, v'_9,\ldots,v'_{n-1}$
7 to $u'_3, u'_5, u'_7,\ldots,u'_{n-3}$ and $v'_n$
9 to $u'_2, u'_4, u'_6,\ldots,u'_{n-2}$
11 to $u'_1$

**Sub case(ii): $n=3k+2$, $k=11+10i$, $i=0,1,\ldots$**

**Sub case(iii): $n=3k$, $k=15+10i$, $i=0,1,\ldots$**

For Sub cases (ii) and (iii) the colors are assigned in a suitable way as in sub case (i)

**Case(iv):**

**$rn(L(S_n))=\Delta+2$ $n\equiv 0 \bmod 2$ expect $n=5i$ and$6i$**

Consider the following 6-coloring of (1,3,5,7,9,11) of $L(S_n)$.

**Sub case(i): $n\equiv 2 \bmod 3$**

Assign the color

1 to $v'_1, v'_4, v'_7, \ldots,v'_{n-4}$ and $u'_{n-1}$
3 to $v'_2, v'_5, v'_8,\ldots, v'_{n-3}$ and $u'_n$
5 to $v'_3, v'_6, v'_9,\ldots, v'_{n-2}$ and $u'_1$
7 to $u'_2, u'_4, u'_6,\ldots, u'_{n-4}$ and$v'_{n-1}$
9 to $u'_3, u'_5, u'_7,\ldots, u'_{n-3}$ and $v'_n$
11 to $u'_{n-2}$

**Sub case(ii): $n\equiv 1 \bmod 3$**

Assign the color

1 to $v'_1, v'_4, v'_7, \ldots,v'_{n-3}$
3 to $v'_2, v'_5, v'_8,\ldots, v'_{n-2}$
5 to $v'_3, v'_6, v'_9,\ldots,v'_{n-1}$
7 to $u'_3, u'_5, u'_7,\ldots, u'_{n-3}$ and $v'_n$
9 to $u'_2, u'_4, u'_6,\ldots,u'_n$
11 to $u'_1, u'_{n-1}$

**Case(v):**

**$rn(L(S_n))=\Delta+2$ $n\equiv 0 \bmod 3$, $n\equiv 1 \bmod 3$**

Consider the following 6-coloring of (1,3,5,7,9,11) of $L(S_n)$.

**Sub case(i): n≡0 mod3 except n=6i**

Assign the color

1 to $v'_1, v'_4, v'_7, v'_{10}, \ldots, v'_{n-2}$

3 to $v'_2, v'_5, v'_8, \ldots, v'_{n-1}$

5 to $v'_3, v'_6, v'_9, \ldots, v'_n$

7 to $u'_2, u'_4, u'_6, \ldots, u'_{n-1}$

9 to $u'_3, u'_5, u'_7, \ldots, u'_n$

11 to $u'_1$

**Sub case(ii): n≡1 mod3 when n is prime**

The colors are assigned in a suitable way as in subcase (i)

***Theorem: 9***

Let $n \geq 4$, then the radio chromatic number of Middle graph of sunlet graph $S_n$ is $\Delta+2$.

**Proof:**

The vertex set and edge set of $S_n$ are as described in lemma 6.

By the definition of middle graph

$V(M(G))=V(S_n)\cup E(S_n) =\{v_i:1\leq i\leq n\}\cup\{u_i:1\leq i\leq n\}\cup\{v'_i :1 \leq i \leq n\}\cup\{u'_i :1 \leq i \leq n\}$, where $v'_i$ and $u'_i$ represents the edge $e_i$ and $e'_i$ $(1\leq i \leq n)$ respectively.

Consider the following 8-coloring of (1,3,5,7,9,11,13,15) of $M(S_n)$.

Assign the color in the following manner

(a) The vertices $\{v_i:1\leq i\leq n\}$ and $\{v'_i :1 \leq i \leq n\}$ are colored using the following patterns

1,3,5,7

1,9,3,5

1,7,3,9

1,5,3,7

Here $v'_n$ is assigned 11, $v'_1$ as 3 and $v_1$ as 1

(b) The vertices $\{u'_i :1 \leq i \leq n\}$ are colored by 11, 13, and 15

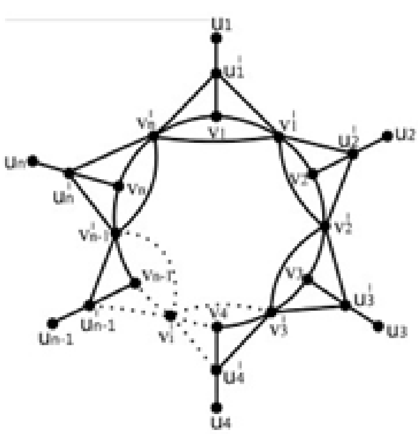

**Figure 2.** Middle graph of Sunlet graph $M(S_n)$.

(c) The vertices $\{u_i :1 \leq i \leq n\}$ are colored by 1, 3, and 5

Here $u_1$ is assigned 1, and $u_n$ as 5.

An easy check shows that this coloring is radio coloring.

***Theorem: 10***

Let $n \geq 5$, then the radio chromatic number of Total graph of sunlet graph $S_n$ is

$$rn(T(S_n)) = \begin{cases} \Delta+1 & n \equiv 0 \bmod 10 \\ \Delta+2 & n \equiv 0 \bmod 5 \\ \Delta+2 & n \equiv 0 \bmod (k+5i),\ i=0,1,2,\ldots \ \&\ k=8,9 \\ \Delta+3 & n \equiv 0 \bmod (k+5i),\ i=0,1,2,\ldots \ \&\ k=6,7 \end{cases}$$

**Proof:**

The vertex set and edge set of $S_n$ are as described in lemma 6.

By the definition of total graph

$V(T(S_n))=V(S_n)\cup E(S_n)=\{v_i :1\leq i \leq n\}\cup\{v'_i : 1 \leq i \leq n\}\cup\{u_i : 1 \leq i \leq n\}\cup\{u'_i : 1 \leq i \leq n\}$, where $v'_i$ and $u'_i$ represents the edge $e_i$ and $e'_i$ $(1 \leq i \leq n)$ respectively.

**Case(i)**

**rn(T(S$_n$))=7, for n≡0mod10**

Consider the 9-coloring (1,3,5,7,9,11,13,15,17) of $T(S_n)$.

Assign the color

(a) 1 to $v_k$, k=1+5i, i=0,1,2,…

1 to $v'_k$, k=3+5i, i=0,1,2,…

(b) 3 to $v_k$, k=4+5i, i=0,1,2,…

3 to $v'_k$, k=1+5i, i=0,1,2,…

(c)5 to $v_k$, k=2+5i, i=0,1,2,…

5 to $v'_k$, k=4+5i, i=0,1,2,…

(d) 7 to $v_k$, k=5+5i, i=0,1,2,…

7 to $v'_k$, k=2+5i, i=0,1,2,…

(e) 9 to $v_k$, k=3+5i, i=0,1,2,…

9 to $v'_k$, k=5+5i, i=0,1,2,…

11 to $u_i$, when i is even

11 to $u'_i$, when i is odd

13 to $u_i$, when i is odd

13 to $u'_i$, when i is even

**Case(ii)**

**rn(T(S$_n$))=8, for n≡0 mod5**

The coloring (a) to (e) of above case follows here. Then assign

11 to $u_i$, when i is even

11 to $u'_i$, when i is odd

13 to $u_i$, when i is odd

13 to $u'_i$, when i is even
15 to $u'_n$

**Case(iii)**
**rn(T($S_n$))=8, for n≡0 mod(k+5i) i=0,1,2,and K=8,9**
The coloring (a) to (e) of above case follows here.

**Subcase(i) For k=8**
**when n is even** assign
11 to $v'_n$
13 to $u'_i$, when i is odd
15 to $u'_i$, when i is even
The remaining vertices $v'_i$ with degree 2 are assigned with some colors in an easy manner.
**when n is odd** assign
11 to $v'_n$ and $u'_i$, i is odd except $v_1$
13 to $u'_i$, when i is odd except $u'_1$
15 to $v_2$, $v_n$
The remaining vertices with degree 2 are assigned with some colors in an easy manner.

**Subcase(ii) For k=9**
**when n is odd** assign
11 to $v'_{n-1}$, 13 to $v_n$, 15 to $v'_n$
9 to $u'_1$, 1 to $u'_{n-1}$, 5 to $u'_n$
11 to $u'_i$, when i is even
13 to $u'_i$, when i is odd
The remaining vertices with degree 2 are assigned with some colors in an easy manner.
**when n is even** assign
11 to $v'_{n-1}$, 13 to $v_n$, 15 to $v'_n$
9 to $u'_1$, 1 to $u'_{n-1}$, 5 to $u'_n$
11 to $u'_i$, when i is even
13 to $u'_i$, when i is odd
15 to $u'_{n-2}$

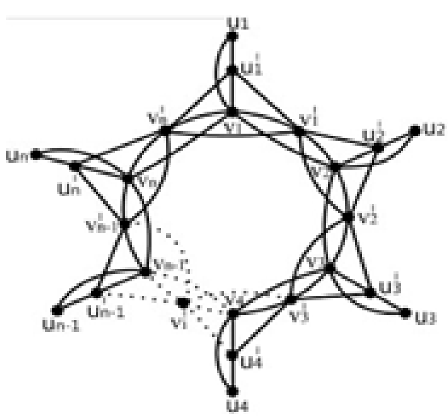


**Figure 3.** Total graph of Sunlet graph T($S_n$).

The remaining vertices with degree 2 are assigned with some colors in an easy manner.

**Case(iv)**
**rn(T($S_n$))=9, for n≡0mod(k+5i) i = 0,1,2, and K = 6,7**
The coloring (a) to (e) of above case follows here.

**Subcase(i) For k=6**
**when n is even** assign
11 to $v_{n-1}$, 13 to $v'_n$
15 to $u'_1$, $u'_{n-1}$
11 to $u'_i$, when i is even
13 to $u'_i$, when i is odd
11 to $u'_n$
The remaining vertices with degree 2 are assigned with some colors in an easy manner.
**when n is odd** assign
11 to $v_{n-1}$, $c_{13}$ to $v'_n$
15 to $u_1$, $u'_{n-1}$
11 to $u'_i$, when i is odd except $u'_{n-3}$
13 to $u'_i$, when i is even except $u'_{n-2}$
17 to $u'_n$
The remaining vertices with degree 2 are assigned with some colors in an easy manner.

**Subcase(ii) For k=7**
**when n is odd** assign
9 to $u'_1$, 1 to $u'_{n-1}$, 1 to $u'_n$
11 to $v_{n-1}$, 13 to $v'_{n-1}$
15 to $v_n$, 17 to $v'_n$
11 to $u'_i$, when i is odd except $u'_{n-4}$
13 to $u'_i$, when i is odd except $u'_{n-3}$
15 to $u'_{n-2}$
The remaining vertices with degree 2 are assigned with some colors in an easy manner.
(Note: for n=7, $u'_{n-3}$=11)
**when n is even** assign
11 to $v_{n-1}$, 13 to $v'_{n-1}$
15 to $v_n$, 17 to $v'_n$
9 to $u'_1$, 1 to $u'_{n-1}$, 5 to $u'_n$
11 to $u'_i$, when i is even except $u'_{n-3}$
13 to $u'_i$, when i is odd except $u'_{n-2}$
15 to $u'_{n-2}$
The remaining vertices $v'$ with degree 2 are assigned with some colors in an easy manner.

***Theorem 11***. For n≥ 4 the radio chromatic number of Central graph of Sunlet graph [15]

$$rn(C(S_n)) = \begin{cases} \Delta+4 & n \text{ is even} \\ \Delta+5 & n \text{ is odd} \end{cases}$$

***Proof.***

The vertex set and edge set of $S_n$ are as described in lemma 6.

By the definition of Central graph

$V(C(S_n)) = V(S_n) \cup v'_i \cup u'_i = \{v_i: 1 \le i \le n\} \cup \{v'_i: 1 \le i \le n \cup \{u_i: 1 \le i \le n\} \cup \{u'_i: 1 \le i \le n\}$ where $v'_i$ represent the vertices subdiving $v_i, v_{i+1} (1 \le i \le n-1)$, and $u'_{i,}$ represent the vertices subdiving $u_i, v_i (1 \le i \le n)$ respectively.

**Case 1**: **rn(C(Sn)) = Δ + 4 for n is even**

The following Δ+4 coloring for C(Sn) admits radio coloring. For $(1 \le i \le n)$ assign the color i to $v_i$, and for$(1 \le i \le n)$ assign the color n+i to $u_i$.

Assign the color 2n+1 to $u'_i$ $(1 \le i \le n)$, for $(1 \le i \le n-1)$ 2n+2 to $v'_i$ if i isodd, and 2n + 3 to $v'_i$ if i is even.

**Case 2:rn(C(Sn)) = Δ + 5 for n is odd**

The following Δ+5 coloring for C(Sn) admits radio coloring. For $(1 \le i \le n)$ assign the color i to $v_i$, and for $(1 \le i \le n)$ assign the color n+i to $u_i$.

Assign the color 2n+1 to $u'_i$ $(1 \le i \le n)$, for $(1 \le i \le n-1)$ 2n+2 to $v'_i$ if i is odd , 2n + 3 to $v'_i$ if i is even, and 2n + 4 to $v'_n$.

## 5. Conclusion

In this paper we obtain the radio chromatic number for Line, Middle, Total and Central graphs of Sunlet graph and improved bounds of radio chromatic number of Sunlet graph. Despite of the few results in radio coloring we still experiment on radio number of various families of graphs and thuis radio coloring is useful in channel assignment problem which has wide range of applications[16].

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