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Three Machines Flowshop Scheduling Model with Bicriterion Objective Function

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Abstract

Objectives: To find the optimum solution for minimization of bicriterion (makespan, weighted mean flowtime) objective function of three machines flowshop scheduling problem with transportation times and weight of the jobs. **Methods/ Statistical Analysis:** In this paper, we used two types of methodologies first one is based on a Branch and Bound (B&B) technique of exact algorithms and second one is based on Palmer approach of heuristic algorithms. First of all, we originated a new algorithm using B&B technique later on; we developed a new heuristic algorithm using Palmer approach for obtaining the optimal or near optimal sequence to minimize the bicriterion objective function of three machines scheduling problem in flowshop environments with transportation times and weights of the jobs. Comparative study between both the proposed algorithms is also considered to select the best methodology of our bicriterion objective function with the help of numerical illustration. Directed graphs, Gantt chart and Branch Tree are also generated to understand the process of lower bound and effectiveness of proposed algorithms. **Findings**: We solved the same numerical by constructed Branch & Bound (B&B) algorithm and Palmer based heuristic algorithm. Hence, comparatative result show that our originated B&B algorithm gives the optimal solution or better result as compare to Palmer based heuristic algorithm for minimization of bicriterion (makespan and weighted mean flowtime) objective function. We also calculated the percentage improvement of our constructive B&B algorithm over palmer based new heuristic algorithm and it is examined that constructive B&B algorithm gives the 8.33% improvement in make span and 6.52% improvement in weighted mean flowtime. The directed graph of each computational level is also originated to understand the computational process of the lower bounds easily. The Gantt chart between both the proposed algorithms is also generated to verify the effectiveness of new originated B&B algorithm. Directed graph is also generated of the optimal sequence. Finally, Branch Tree is generated to empathize the process of Lower Bound. **Application/Improvements:** Our constructed B&B algorithm provide an important tool for decision maker to minimize the makespan and weighted mean flowtime together as bicriterion objective function of three machine flowshop scheduling problems.

Keywords: Algorithm, Branch Tree, Branch & Bound, Directed Graph, Gantt Chart, Makespan, Percentage Improvements, Three Machines Scheduling, Transportation Time, Weighted Mean Flowtime

1. Introduction

Scheduling is an important tool in production and project management and is very useful in increasing the productivity, improving quality of products, fulfilling the demands of market in time and to minimize the flow time, idle time of machines, cost etc. In day to day life, the decision makers are very curious to find the best way to successfully manage the resources in order to produce product of the most efficient way for manufacturing and service industries. During last 4 decades many researchers work on scheduling and sequencing problems. Order of the jobs on different machines is called sequencing and the process on which jobs are sequenced is called sched-

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uling. There are different types of shops (environments) using in scheduling problems like job shop, flow shop, mixed shop, open shop etc... Here we deal with flowshop scheduling environment for three machines n jobs. In flowshop scheduling environment, sequenced jobs have to be processed on all the machines with identical flow patterns. The idea of flowshop sequencing is given by[1] in 1954. The classical scheduling problems, numerous researchers are primarily considered the single criterion as makespan but in modernistic time, production companies and manufacturing industries are considered the bicriterion scheduling problems because of customer's satisfactions. Nowadays, always changing customer's demands and arising competitions in the global market are motivating the manufacturers to reassess their manufacturing industries and production systems. Hence, customers satisfaction and production cost both are important concerns in the combat with other rivals. Therefore, bicriterion objective function is considered for flowshop scheduling problem of three machines. The two criteria which we used in this chapter are makespan and weighted mean flowtime. The objective of this study is to minimize both the criteria as makespan and weighted mean flowtime of the three machines flowshop scheduling problem. Johnson's provided an exact solution for three machines flowshop scheduling problems if these problems satisfied the Johnson's condition of three machines otherwise we used some other techniques like exact algorithms, heuristic algorithms, metaheuristic algorithms etc. for finding the optimal or near optimal solutions of three machines flowshop scheduling problems. The exact algorithm has the guarantee of optimum solution although heuristic algorithm provides the near optimal solution. Hence, in this paper we developed a new algorithm based on B&B technique to optimize the bicriterion objective function. Furthermore, we also developed a new heuristic algorithm using Palmer approach for the problem which we considered. Afterwards, we compared our new developed algorithm of B&B based with new developed heuristic algorithm using Palmer approach with the help of numerical illustrations.

In[1] developed the algorithms for two and three machines flowshop scheduling problems to obtain the optimal solution of for minimization of makespan in production managements.In[2] studied the concept of optimization of maximum tardiness and mean flowtime. The parameter as transportation time was considered by[3] for two machines flowshop scheduling problems. In[4] formulated the B&B technique of scheduling problems in flowshop environment. The bicriteria for single machine scheduling problem was studied by[5]. In[6] discussed the bicriterion objective function for flowshop scheduling problem of two machines. In[7,8] analyzed the bicriteria objective function under assigned rental policy for three machines flowshop scheduling problems. In[9] studied Bicriteria scheduling problems on parallel machines. Branch and Bound (B&B) methodology was established by[10]. In[11] proposed an algorithm based on branch and bound algorithm to optimize the makespan of two machines flowshop scheduling problem. Palmer heuristic algorithm is based on slope index value and[12] was the first who developed the concept of slope index value for prioritizing the jobs. In[13] proposed a new heuristic algorithm for "n""n" job "m" machines scheduling problems, which is generalization of Johnson's algorithm and called CDS heuristic algorithm. A new heuristic algorithm was developed by[14] for three machines flowshop scheduling problem with transportation time as a parameters. Many other researchers who developed the heuristic algorithm for "n" job "m" machines scheduling problems such as[15–17] etc.

2. Practical Significance of this Model

In real life situation, flowshop scheduling problems occurs in so many fields such as, educational intuitions, hospitals, factories banking, aircraft, and garments manufacturing industries, Automobile dent car repairing[18] etc., where various types of products are prepared along their relative importance i.e., weights in jobs. Transportation times and weight of the jobs have remarkable role in the production management. In real life situations transportations times are considering apart from processing time if machines are placed distantly. In addition, weights of the jobs are also having the remarkable role in the real life practical situations because of urgency or demand of its relative importance. Here we provide a real life practical situation, where three machines flowshop scheduling problems occurs. Hence, we provide an example of garments manufacturing industry where three machines are used, first one is used for cutting, second one is for sewing or assembling and third one is for pressing and these are indicated "M_1", "M_2" and "M_3" respectively.

If these three machines are located at different places, consequently, sometime is taken for transferring the jobs from one machine to another machine by conveyers in the form of loading times, moving times and unloading times. It is called transportation times (TR) which is used for transferring the jobs from machine "M_1" to "M_2" and from "M_2" to "M_3". The processing order of all the jobs will be the same as "$M_1M_2M_3$" that means firstly jobs will be process on machine "M_1" then operated on Machine "M_2" and lastly on machine "M_3". In garments manufacturing industry which has mainly three machines centers as cutting, sewing and pressing and also considered that every center has only one machine. In garments manufacturing industry, first fabrics have to be cut according to manufacturer of garments after that transporter transferring the fabrics on the sewing center. In this center, row materials of cloths are stitched and prepare the complete apparels. Now, transporter is again transferring the stitched garments to pressing center. In this center complete garments are ironed or treated as steam and needful finishing are completed in this center. Many times different quality of garments is produced because of their relative importance. So weights of jobs also become significant. The garments manufacturing process of three machines in flowshop environments is shown in Figure 1.

3. Assumptions and Notations Used

3.1 Assumptions

- All the jobs and machine are available at times Zero.
- In our problem fixed set of jobs are studied. Number of jobs doesn't change. (Static Scheduling problem).
- Transportations times should be considering apart from processing time.
- Jobs release times are considered zero, $r_j = 0$.
- More than one operation is not allowed to process on the machine at a time.
- Order of the jobs should be remain the same throughout complete the process of jobs on all the different machines as "$M_1M_2M_3$".
- No preemption is allowed. Once the jobs are started to operate on the machine, it must be finished before some other functions can start on that machine.
- The machine is assumed to be continuously available and Machine breakdowns or maintenance tasks are not considered.
- Setup times are included within the processing time.

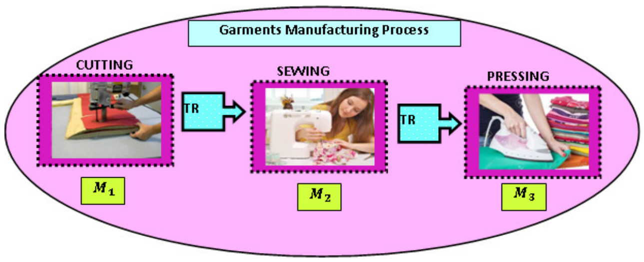

Figure 1. Garments manufacturing process.

3.2 Notations

- M_l = Machine where $l = 1, 2, 3$.
- m_{il} = Processing time of i^{th} job on l^{th} machine. (Where $i = 1\ to\ n$ & $l = 1, 2, 3$).
- r_i = Release time of i^{th} job.
- C_{max} =Total completion time of jobs or makespan.
- $O_{(\alpha_n)out}{}^{M_3}$ = Out going time of the last job on the machine M_3.
- w_f =Total weighted flow time.
- w_i = Weighted flowtime of i^{th} job (Where $i = 1\ to\ n$).
- f_i = Flowtime of i^{th}job.
- C_i = Completion time of i^{th} job.
- $\overline{w_f}$ = Weighted mean flow time.
- j_i = i^{th}job (Where $i = 1\ to\ n$).
- $t_{i,1\rightarrow 2}$ =Transportation time of i^{th} job from machine M_1 to machine M_2.
- $q_{i,2\rightarrow 3}$ = Transportation time of i^{th} job from machine M_2 to machine M_3.

- w_i = Weight of i^{th} job.
- $D(0)$ =Sequence where no job assigned.
- $D(i)$ =Sequence where i^{th} job assigned.
- $S_k = k^{th}$ Partial Sequence.
- $\mathrm{LB}(k)$ = Lower Bound of k^{th} Partial Sequence.
- U_s = Set of unscheduled jobs.
- $C_l(g)$ =Completion time of jobs on l^{th} machine.
- Y_i =Slope of i^{th} job.
- S_1 =Optimal sequence obtained by Branch & Bound algorithm.
- S_3 = Near optimal sequence obtained by Palmer approach.
- $(C_{max})^P$ = Makespan obtained by Palmer approach.
- $(C_{max})^{B\&B}$ = = Makespan obtained by Branch & Bound algorithm.
- % IMP = Percentage improvement.
- $(\overline{w_f})_p$ = Weighted mean flow time obtained by Palmer approach.
- $(\overline{w_f})_{B\&B}$ =Weighted mean flow time obtained by Branch & Bound algorithm.

4. Performance Measures and Objective Function

4.1 Performance Measures

In this paper, we dealt with these performance measures as follows:

4.1.1 Completion Time Measures

- Total completion time, C_{max} = out going time of the last job on the machine M_3.

$$C_{max} = O_{(\alpha_n)out}{}^{M_3}$$

- Total weighted flow time, $w_f = \sum_{i=1}^{n} w_i f_i$ (where flow time, $f_i = C_i - r_i$).
- Weighted mean flow time, $\overline{w_f} = \frac{\sum_{i=1}^{n} w_i f_i}{\sum_{i=1}^{n} w_i}$

4.2 Objective Function

In this paper we dealt with bi-criterion objective function. First we minimize the makespan and so weighted mean flowtime.

Objective Function

1. $\min\{C_{max} = T_{(\alpha_n)out}{}^{M_3}\}$
2. $\min\{\overline{w_f} = \frac{\sum_{i=1}^{n} w_i f_i}{\sum_{i=1}^{n} w_i}\}$

5. Problem Description and Mathematical Model

5.1 Problem Description and Mathematical Model in Matrix Form

Three machines scheduling problem in flowshop environment is consider in this paper. where a set of n independent jobs $j_i(j_1, j_2, j_3, j_4, \ldots\ldots j_n)$ are processed on these three machines "M_1", "M_2" and "M_3" with processing times m_{i1}, m_{i2} and m_{i3} $(i = 1 \text{ to } n)$ respectively. We also considered that machines are distantly located. Hence, some transportation times exists apart from processing times to transfer the jobs from one machine to other machines. Let $t_{i,1\to 2}$ be the transportation times to transfer the jobs from machine "M_1" to "M_2" and $g_{i,2\to 3}$ be the transportation times to transfer the jobs from machine "M_2" to "M_3". Let weights $w_i(w_1, w_2, w_3 \ldots.. w_n)$ be also attached with their respective jobs $j_i(j_1, j_2, j_3 \ldots.. j_n)$ because of their relative importance. The main aim is to find the optimal sequence of jobs to minimize the bicriterion objective function as makespan and weighted mean flowtime. The

Table1. Three machines flowshop scheduling model in matrix form

Jobs (i)	Machine M_1 (m_{i1})	Transportation Time T_i $(t_{i,1\to 2})$	Machine M_2 (m_{i2})	Transportation Time G_i $(q_{i,2\to 3})$	Machine M_3 (m_{i3})	Weight of jobs (w_i)
j_1	m_{11}	$t_{1,1\to 2}$	m_{12}	$q_{1,2\to 3}$	m_{13}	w_1
j_2	m_{21}	$t_{2,1\to 2}$	m_{22}	$q_{2,2\to 3}$	m_{23}	w_2
j_3	m_{31}	$t_{3,1\to 2}$	m_{32}	$q_{3,2\to 3}$	m_{33}	w_3
j_4	m_{41}	$t_{4,1\to 2}$	m_{42}	$q_{4,2\to 3}$	m_{43}	w_4
j_n	m_{n1}	$t_{n,1\to 2}$	m_{n2}	$q_{n,2\to 3}$	m_{n3}	w_n

mathematical model of the given problem is showed in Table 1 in the matrix form.

5.2 Johnson's Condition of Three Machines Flowshop Scheduling Problem

In this problem we also assumed that Johnson's condition of three machines flowshop scheduling problem is not satisfied. Structural Conditions; or Structural Relationship of Johnson's Algorithm for Three Machines Flowshop Scheduling Problems

- $\min\ (m_{i1}+t_{i,1\to 2}) \geq \max\ (t_{i,1\to 2}+m_{i2})$.
- $\min\ (g_{i,2\to 3}+m_{i3}) \geq \max\ (g_{i,2\to 3}+m_{i2})$.

Suppose that either one or both of the above structural conditions involving the processing time and transportation time of jobs does not holds.

6. Constructed Branch and Bound Algorithm for Three Machines Flowshop Scheduling Problem

The Branch and Bound Algorithm is introduced by[4,10]. It is based on permutations scheduled and Branch Tree.

Branch and Bound Algorithm involves the following steps:

- The zero level is considered as first level. At level zero, root node will be placed with all "n" empty sequenced jobs. In this level we start with no job sequenced and root node along with the nodes equal to the number of jobs involved in the production.
- At level one, there will be "n" number of nodes. Each node will contain a partial sequence of jobs. At this level we assign or schedule the first job for first position and lower bound on makespan is calculated for one scheduled job and the node having least lower bound is continued further. If some nodes having equal least lower bound value then we branch from all these nodes and the remaining nodes are discontinued as fathomed nodes.
- At level two, we scheduled the second job for second position and computations again calculate the lower bound of two scheduled job and decide the continuation of further nodes. We proceed this way until all the jobs are scheduled and obtained the optimal sequence.

6.1 New Developed B&B Algorithm for three machine Bicriterion Objective Function

Step 1: First we check out the Johnson's conditions and assure that whether it is satisfied or not. if Johnson's condition is not satisfied so we cannot covert three machine flowshop scheduling problem into two machine flowshop scheduling problem. Hence we applied Branch and Bound technique for obtaining optimal solution of objective function.

Step 2: Now we calculated new processing time as m'_{i1}, m'_{i2} and m'_{i3} of machines M_1, M_2 and M_3 respectively.

i. If $\min(m_{i1}, m_{i2}, m_{i3}) = m_{i1}$

Then $m'_{i1} = \frac{(m_{i1}+W_i)}{W_i}, m'_{i2} = \frac{m_{i2}}{W_i}$ and $m'_{i3} = \frac{m_{i3}}{W_i}$

ii. If $\min(m_{i1}, m_{i2}, m_{i3}) = m_{i2}$

Then $m'_{i1} = \frac{m_{i1}}{W_i}, m'_{i2} = \frac{(m_{i2}+W_i)}{W_i}$ and $m'_{i3} = \frac{m_{i3}}{W_i}$

iii. If $\min(m_{i1}, m_{i2}, m_{i3}) = m_{i3}$

Then $m'_{i1} = \frac{m_{i1}}{W_i}, m'_{i2} = \frac{m_{i2}}{W_i}$ and $m'_{i3} = \frac{(m_{i3}+W_i)}{W_i}$

Step 3: Now we will apply the branch and bound algorithm in new reduced problem obtained in Step 2. We start from the root node called "0" level or initial node. If there are "n" potential jobs then, we have "n" branches as each branch concern to assign single job for the first position in the sequence. Hence if there are "n" number of nodes then, we have $(n-1)$ number of branches corresponding to $(n-1)$ potential or possible jobs. At this level, we start with no job sequenced. It is designated by $(*,*,*,* \ldots\ldots\ldots\ldots *)$ where" $*$ "indicates that no job from "n" jobs has yet been assigned to that position and it will be placed with all "$\boldsymbol{n}$"empty sequenced jobs.

$D(0) = (*,*,*,* \ldots\ldots\ldots\ldots *)$

Step 4: We move from level 0 to level 1. At this level, we have "$\boldsymbol{n}$" number of nodes and each node will contain a partial sequence of jobs. We assign or schedule the first job for first position in every partial sequence S_k. The problem $D(0)$ divide $\boldsymbol{n}$ sub problems, {$D(1), D(2), D(3)$ ……….., $D(n)$} by assigning the last position in sequence. We move from $D(0) = (*,*,*,* \ldots\ldots\ldots\ldots *)$ to $\{D(1) = (1,*,*,* \ldots\ldots *), D(2) = (2\ *,*,*,* \ldots\ldots *),$ ….., $D(n) = (n\ *,*,*,* \ldots\ldots\ldots\ldots *)\}$.

Now calculate the Lower Bound (LB) for all the partial sequences like

$\{LB(1)\ for\ D(1), L\ B(2)\ for\ D(2), LB(3) for\ D(3)$ …, $LB(n) for\ D(n)\}$

respectively. We select that node which has minimum lower bound value and further proceed to branch from this node. If some nodes have equal minimum lower bound value then we branch from all these nodes and eliminate all the other nodes.

Step 5: This is level 2, at this level we assign the two jobs of all the partial sequences and calculate the Lower Bound (LB) for all the partial sequences like

$\{LB(12) for\ D(12), LB(13) for\ D(13) \ldots\ldots\ldots . \}$

Select that node which has minimum lower bound value and further proceed to branch from this node. Now we scheduled or assign the three jobs at level 3, of all the partial sequences and calculate the Lower Bound Value for all the partial sequences like

$\{LB(123) for\ D(123), LB(134) for\ D(134) \ldots\ldots\ldots . \}$

Step 6: We repeat the process until all the jobs are scheduled or assign. At last, we find the optimal sequence by scheduling all the jobs.

Step 7: For this optimal sequence which we obtained in step 6, we construct the In – Out table for calculating the value of objective function.

6.1.1 Calculating the Lower Bound

The Lower Bound (LB) for any partial sequence S_k

$$\mathrm{LB(k)} = \max(\mathrm{H_1}, \mathrm{H_2}, \mathrm{H_3})$$

Where, $\mathrm{H_1}$, $\mathrm{H_2}$ and $\mathrm{H_3}$ are calculated as follows:

i. $\mathrm{H_1} = \mathrm{C_1(g)} + \sum_{\mathrm{j \in u_s}} \mathrm{T_{j1}} + \min \sum_{\mathrm{j \in u_s}} (\mathrm{T_{j2}} + \mathrm{T_{j3}})$

ii. $\mathrm{H_2} = \mathrm{C_2(g)} + \sum_{\mathrm{j \in u_s}} \mathrm{T_{j2}} + \min\ (\mathrm{T_{j3}})$

iii. $\mathrm{H_3} = \mathrm{C_3(g)} + \sum_{\mathrm{j \in u_s}} \mathrm{T_{j3}}$

Where, $\mathrm{C_1(g)}, \mathrm{C_2(g)}$ and $\mathrm{C_3(g)}$ are completion times on machines $\mathbf{M_1}$, $\mathbf{M_2}$and $\mathbf{M_3}$ respectively for scheduled jobs or partial sequence S_k.

$\mathbf{U_s}$ = Set of unscheduled jobs orJobs are not in the partial sequence S_g.

$\mathrm{T_{j1}}, \mathrm{T_{j2}}$ and $\mathrm{T_{j3}}$ are processing times of j^{th} job on machines $\mathbf{M_1}$, $\mathbf{M_2}$and $\mathbf{M_3}$ respectively.

7. Palmer Heuristic Algorithm

Palmer heuristic algorithm is based on slope index value. First, we calculate the slope index Y_i for i^{th} jobs after that all the jobs are sequenced according to non-increasing or decreasing order of slope index value Y_i of their respective jobs.

7.1 Proposed New Heuristic Algorithm using Palmer Approach

Step 1 and **Step 2** is same as the above heuristic algorithm.

Step 3: Now compute the slope Y_i for i^{th} jobs ($i = 1$ to n) and Three machines ($L = 3$) as follows.

$$Y_i = -\sum_{x=1}^{L}\{L-(2x-1)\}m_{ix} + \sum_{i=1}^{n}(T_i + G_i)$$

Step 4: Now sequenced the jobs based on non - increasing order or descending order of Y_i such that, $Y_i = Y_1 \geq Y_2 \geq Y_3 \geq Y_4 \ldots\ldots\ldots\ldots\ldots\ldots\ldots \geq Y_n$

Step 5: For the sequence, which we obtained in step 5, we construct the In – Out table for calculating the value of objective function.

8. Numerical Illustrations

A numerical illustration is provided to verify the effectiveness of our constructed heuristic algorithms. Let 4 jobs are processed on 3 different machines and processing times are attached with their respective weights. Transportation times are also given to transfer the jobs from one machine to other machines. The problem is expressed in matrix form as in Table 2.

8.1 Numerical Solved By Constructive Heuristic Algorithm using B&B Methodology

As per Step 1: check the Jonhnson's Condition for three machines

$\min(m_{i1}+t_{i,1\to2}) = 5, \max(t_{i,1\to2}+m_{i2}) = 17$

$\min(m_{i1}+t_{i,1\to2}) \ngeq \max(t_{i,1\to2}+m_{i2})$ condition one is not satisfied,

$\min(g_{i,2\to3}+m_{i3}) = 8, \max(g_{i,2\to3}+m_{i2}) = 22$

$\min(g_{i,2\to3}+m_{i3}) \ngeq \max(g_{i,2\to3}+m_{i2})$ second condition is also not satisfied, hence we go to step 2.

As per Step 2: Calculate the new processing time m'_{i1}, m'_{i2} and m'_{i3} of machines $\mathbf{M_1}$, $\mathbf{M_2}$ and $\mathbf{M_3}$ respectively. The modified problem is as follows in Table 3.

Table 3. Modified problem

Job (i)	M_1 (m'_{i1})	T_i ($t_{i,1\to2}$)	M_2 (m'_{i2})	G_i ($g_{i,2\to3}$)	M_3 (m'_{i3})
1	3	2	4	3	5
2	2	6	5	7	4
3	2	4	2	5	3
4	1	5	3	6	4

As per Step 3 and 4: we start from root node or level 0. We have four jobs in our problem hence, there will be four branches from the root node to level 1. Now we calculate the Lower Bound, LB(1) for partial sequence D(1) ,L B(2) for for partial sequence D(2),LB(3)for

Table 2. Five jobs three machines flowshop scheduling problem of three machines

Job (i)	M_1 (m_{i1})	T_i ($t_{i,1\to2}$)	M_2 (m_{i2})	G_i ($g_{i,2\to3}$)	M_3 (m_{i3})	W_i
1	3	2	8	3	10	2
2	2	6	5	7	4	1
3	3	4	3	5	9	3
4	4	5	12	6	16	4

partial sequence D(3) and LB(4)for partial sequence D(4)

8.1.1 Computation for the Level 1

Calculate the Lower bound, LB(1) for partial sequence D(1)=(1, *, *, *). First, we calculate C_1 (g),C_2 (g) and C_3 (g) with the help of directed graph. Directed Graph for assigning Job 1 is designated in Figure 2.

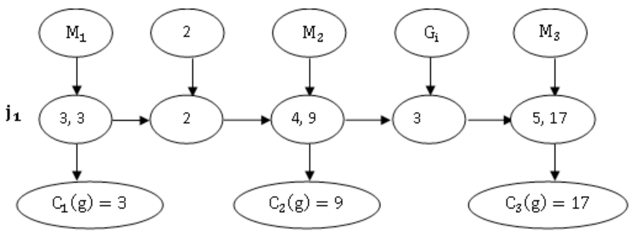

Figure 2. Directed graph for the partial sequence D(1)= (1, *, *, *).

Now, the set $U_s=\{j_2,j_3,j_4\}$ and calculations are shown as follows in Table 4.

Where, $Y=\sum_{j\in u_s}\{(T_{ji}+m_{j2})+(m_{j3}+G_{ji})\}$ and $X=\sum_{j\in u_s}(G_{ji}+m_{j3})$

$$H_1=C_1(g)+\sum_{j\in u_s} m_{j1}+\min\sum_{j\in u_s}\{(T_{ji}+m_{j2})+(m_{j3}+G_{ji})\}=3+5+14=22$$

$$H_2=C_2(g)+\sum_{j\in u_s} m_{j2}+\min\sum_{j\in u_s}(G_{ji}+m_{j3})=9+10+8=27$$

$$H_3=C_3(g)+\sum_{j\in u_s} m_{j3}=17+11=28$$

$$LB(1)=\max(H_1,H_2,H_3)=\max(22,27,28)=28$$

Similarly, calculate the $L\ B(2), L\ B(3)$ and $L\ B(4)$.

$L\ B(2)=36, L\ B(3)=29$ and $L\ B(4)=31$

The Lower Bound values, for the first level are entered to the respective partial sequence and find the least lower bound value node that is LB(1). Hence, we further branching from this node 1. The branching tree of the first level is shown in Figure 3.

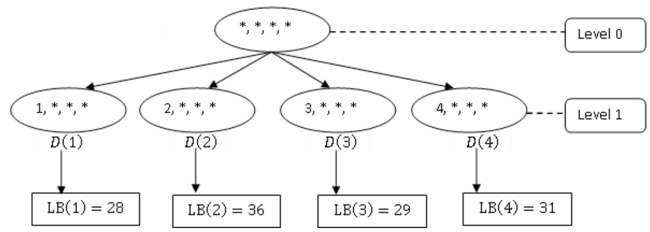

Figure 3. Branching tree of first level.

8.1.2 Computations for the Level 2

We proceed from the partial sequence *D*(1) for further branching because it has the least lower bound value. Now we scheduled the two jobs and calculate the lower bound LB(12), LB(13) and LB(14) for partial sequence *D*(12), *D*(13) and *D*(14) respectively.Calculate the Lower bound, LB(12) for partial sequence *D*(12)=(1, 2, *, *)with the help of directed graph.Directed Graph for assigning Job 1 & Job 2 is designated in Figure 4.

Now, the set $U_s=\{j_3,j_4\}$ because job $\{j_1,j_2\}$ are scheduled. The Lower Bound calculation for the partial sequence $D(12)=$ (1, 2, *, *) is shown in Table 5:

$$H_1=C_1(g)+\sum_{j\in u_s} m_{j1}+\min\sum_{j\in u_s}\{(T_{ji}+m_{j2})+(m_{j3}+G_{ji})\}=5+3+14=22$$

$$H_2=C_2(g)+\sum_{j\in u_s} m_{j2}+\min\sum_{j\in u_s}(G_{ji}+m_{j3})=16+5+8=29$$

$$H_3=C_3(g)+\sum_{j\in u_s} m_{j3}=27+7=34$$

$$LB(12)=\max(H_1,H_2,H_3)=\max(22,29,34)=34$$

Table 4. Lower bound calculation for the partial sequence *D*(1)= (1, *, *, *)

Job (i)	M_1 (m'_{i1})	T_i $(t_{i,1\to 2})$	M_2 (m'_{i2})	G_i $(g_{i,2\to 3})$	M_3 (m'_{i3})	Y	X
1	3	2	4	3	5		
2	2	6	5	7	4	22	11
3	2	4	2	5	3	14	8
4	1	5	3	6	4	18	10
$\sum_{j\in u_s} m_{j1}=5$		$\sum_{j\in u_s} m_{j2}=10$		$\sum_{j\in u_s} m_{j3}=11$		Min=14	Min=8

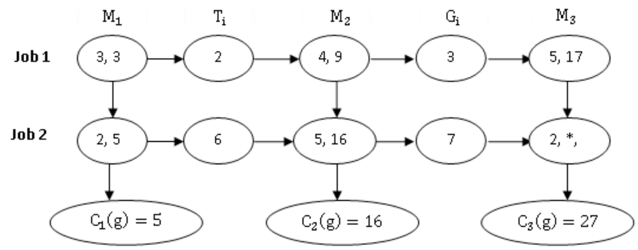

Figure 4. Directed graph with transportation times for the partial sequence $D(12) = (1, 2, *, *)$.

Similarly, calculate the $L\,B(13)$ and $L\,B(14)$ for partial sequence $D(13)$ *and* $D(14)$ respectively.

L B(13)=29
L B(14)=29

Now select the least lower bound value. Since the L B(13) and L B(14) having the same least value hence, we further branching from these both the nodes for the partial sequence *D*(13) and *D*(14).

8.1.3 Computations for the Level 3

We proceed from the partial sequence $D(13)$ and $D(14)$for further branching because both of these partial sequences have the same east lower bound value. Now we scheduled the three jobs and calculate the lower bound LB(132), LB(134), LB(142) and LB(143) for partial sequence $D(132)$, $D(134)$, $D(142)$ and $D(143)$ respectively. Calculate the Lower bound, LB(132) for partial sequence $D(132) = (1, 3, 2, *)$ with the help of directed graph. Directed Graph for assigning the Job 1, Job 3 & Job 2 is shown in Figure 5.

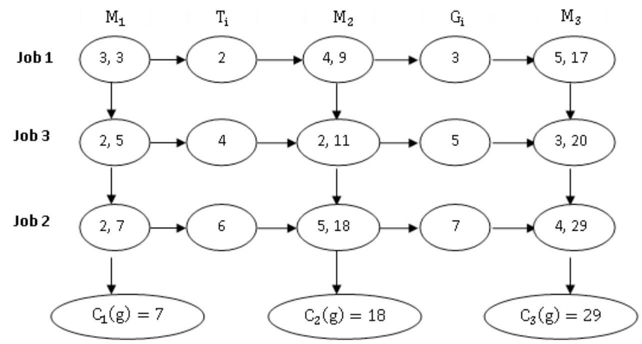

Figure 5. Directed graph with transportation times for the partial sequence $B(132) = (1, 3, 2, *)$.

Now, the set $U_s = \{j_1, j_3, j_2\}$ because job $\{j_4\}$ are scheduled, the lower Bound calculation for the partial sequence $B(132) = (1, 3, 2, *)$ is shown in Table 6.

$H_1 = 7 + 1 + 18 = 26$, $H_2 = 18 + 3 + 10 = 31$, $H_3 = 29 + 4 = 33$

$LB(132) = \max(H_1, H_2, H_3) = \max(26, 31, 33) = 33$

Similarly, calculate the LB(134), LB(142) and LB(143) for partial sequence $D(132)$, $D(134)$, $D(142)$ and $D(143)$ respectively at level 2.

$LB(134) = 30, LB(142) = 31$ and $LB(143) = 30$

Now select the least lower bound value. Since the $L\,B(134)$ and $L\,B(143)$ having the same least value

Table 5. Lower bound calculation for the partial sequence $B(12) = (1, 2, *, *)$

Job (i)	M_1 (m'_{i1})	T_i $(t_{i,1\to2})$	M_2 (m'_{i2})	G_i $(g_{i,2\to3})$	M_3 (m'_{i3})	Y	X
1	3	2	4	3	5		
2	2	6	5	7	4		
3	2	4	2	5	3	14	8
4	1	5	3	6	4	18	10
$\sum_{j\in u_s} m_{j1} = 3$		$\sum_{j\in u_s} m_{j2} = 5$		$\sum_{j\in u_s} m_{j3} = 7$		Min=14	Min=8

hence, we further branching from these both the nodes for the partial sequence $D(134)$ *and* $D(143)$. Finally we find the two sequence $S_1 = (1, 3, 4, 2)$ and $S_2 = (1, 4, 3, 2)$ and no job is left for scheduling hence, we go to step 7.

As per Step 7: Construct the In – Out table for both the sequence S_1 & S_2 in Table 7 and obtained the optimal sequence which has least makespan

Remarks: The optimal sequence is $S_1 = (1, 3, 4, 2)$ because it gives more effective result for minimization of

Table 6. Lower bound calculation for the partial sequence $B(132) = (1, 3, 2, *)$

Job (i)	M_1 (m'_{i1})	T_i $(t_{i,1\to2})$	M_2 (m'_{i2})	G_i $(g_{i,2\to3})$	M_3 (m'_{i3})	Y	X
1 3 2 4	3 2 2 1	2 4 6 5	4 2 2 3	3 5 7 6	5 3 4 4	 14 18	 8 10
$\sum_{j\in u_s} m_{j1} = 1$		$\sum_{j\in u_s} m_{j2} = 3$		$\sum_{j\in u_s} m_{j3} = 4$		Min=18	Min=10

Table 7. In – out table for the sequences $S_1 \& S_2$

In- Out table for sequence, $S_1 = (1, 3, 4, 2)$

Job (i)	M_1 In	M_1 Out	T_i	M_2 In	M_2 Out	G_i	M_3 In	M_3 Out	w_i	$w_i f_i$
1	0	3	2	5	13	3	16	26	2	2×(26-0)=52
3	3	6	4	13	16	5	26	35	3	3×(35-3)=96
4	6	10	5	16	28	6	35	51	4	4×(51-6)=180
2	10	12	6	28	33	7	51	55	1	1×(55-10)=45
$\overline{w_f} = \frac{\sum_{i=1}^n w_i f_i}{\sum_{i=1}^n w_i} = \frac{373}{10} = 37.3$						$C_{max} = 55$			$\sum_{i=1}^{n} w_i f_i = 373$	

In- Out table for sequence, $S_2 = (1, 4, 3, 2)$

Job (i)	M_1 In	M_1 Out	T_i	M_2 In	M_2 Out	G_i	M_3 In	M_3 Out	w_i	$w_i f_i$
1	0	3	2	5	13	3	16	26	2	2×(26-0)=52
4	3	7	5	13	25	6	31	47	4	4×(47-3)=176
3	7	10	4	25	28	5	47	56	3	3×(56-7)=147
2	10	12	6	28	33	7	56	60	1	1×(60-10)=50
$\overline{w_f} = \frac{\sum_{i=1}^n w_i f_i}{\sum_{i=1}^n w_i} = \frac{427}{10} = 42.7$						$C_{max} = 60$			$\sum_{i=1}^{n} w_i f_i$ =425	

Table 8. Compute the slope Y_i

Job (i)	$M_1\ (l=1)$ $\{3-[(2\times1)-1]\}=2$ (m_{11})	T_i	$M_2\ (l=2)$ $\{3-[(2\times2)-1]\}=0$ (m_{22})	G_i	$M_3\ (l=3)$ $\{3-[(2\times3)-1]\}=-2$ (m_{33})	Y_i
1	3×2= 6	2	4×0=0	3	5×(-2)= -10	1
2	2×2=4	6	5×0=0	7	4×(-2)= -8	9
3	2×2=4	4	2×0=0	5	3×(-2)= -6	7
4	1×2=2	5	3×0=0	6	4×(-2)= -8	5

Table 9. In- out table for the sequence S_3

In- Out table for sequence, $S_3=(2,3,4,1)$										
Job (i)	M_1		T_i	M_2		G_i	M_3		w_i	$w_i f_i$
	In	Out		In	Out		In	Out		
2	0	2	6	8	13	7	20	24	1	1×(24-0)=24
3	2	5	4	13	16	5	24	33	3	3×(33-2)=93
4	5	9	5	16	28	6	34	50	4	4×(50-5)=180
1	9	12	2	28	36	3	50	60	2	2×(60-9)=102
$\overline{w_f}=\frac{\sum_{i=1}^{n} w_i f_i}{\sum_{i=1}^{n} w_i}=\frac{399}{10}=39.9$						$C_{max}=60$			$\sum_{i=1}^{n} w_i f_i=399$	

makespan and weighted mean flowtime as compare to $S_2=(1,4,3,2)$.

8.2 Numerical solved by Palmer Approach

Step 1 and **Step 2** are the same as above algorithm.

As per Step 3: Compute the slope Y_i and it is calculated in Table 8.

As per Step 4: Sequenced the jobs based on non - increasing order or descending order of Y_i

$$S_3=(2,3,4,1)$$

As per Step 5: Construct the in – Out table for the sequence $S_3=(2,3,4,1)$ in Table 9.

9. Percentage Improvement

The percentage improvement of proposed heuristic algorithm using Branch and Bound approach over Palmer approach is calculated. The comparison of makespan and weighted mean flow time using the percentage improvement are calculated as follows:

Percentage improvement of makespan

$$=\frac{\left[(C_{max})_P-(C_{max})_{B\&B}\right]}{(C_{max})_P}\times100$$

The Percentage improvement of B&B approach over Palmer approach for the sequence $S_1 \& S_3$

$$\%\ \text{IMP}=\frac{(60-55)}{60}\times100=8.33\ \%$$

Percentage improvement of weighted mean flow time

$$=\frac{\left[(\overline{w_f})_p-(\overline{w_f})_{B\&B}\right]}{(\overline{w_f})_p}\times100$$

The Percentage improvement of B&B approach over Palmer approach for the sequence

$$\%\ \text{IMP}=\frac{(39.9-37.3)}{39.9}\times100=6.52\ \%$$

Table 10. Comparatative study between B&B and palmer approach

	Sequence	Makespan C_{max}	Weighted Flowtime $\sum_{i=1}^{n} w_i f_i$	Weighted Mean Flowtime $\overline{w_f} = \frac{\sum_{i=1}^{n} w_i f_i}{\sum_{i=1}^{n} w_i}$
Constructive (B&B) Heuristic Algorithm	S_1	55 units	373 units	37.3 units
Palmer Based Heuristic Algorithm	S_3	60 units	399 units	39.9 units
% Improvement				
% improvement of Constructive (B&B) Heuristic Algorithm over Palmer Approach	% IMP of the makespan C_{max}		8.33 %	
	% IMP of the Weighted Mean Flowtime $\overline{w_f}$		6.52 %	

10. Comparatative Study between Constructive Branch and Bound Algorithm and Palmer based Heuristics Algorithm

Comparatative study between Branch and Bound Algorithm and Palmer Based Heuristics Algorithm is showed in Table 10.

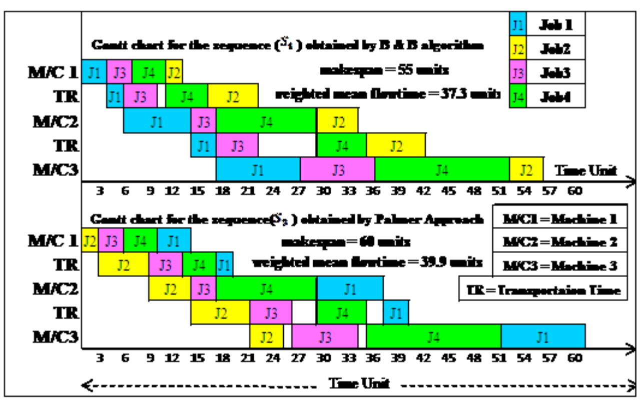

Figure 6. Gantt chart of between S_1 obtained by B&B and S_3 obtained by palmer approach.

11. Gantt Char, Directed Graph and Branch Tree

The Gantt chart between optimal sequences $S_1 = (1, 3, 4, 2)$ obtained by constructed B&B algorithm and a near optimal sequence $S_3 = (2, 3, 4,1)$ obtained by Palmer based new heuristic algorithm is generated in Figure 6. Directed graph of the optimal sequence $S_1 = (1, 3, 4, 2)$ is designated in Figure 7. Furthermore, branching tree is also developed to show the computational analysis of lower bound at each computation level in Figure 8.

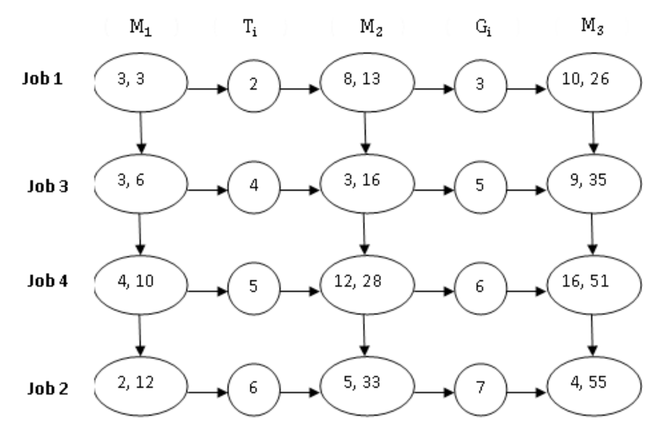

Figure 7. Directed graph with transportation times of the optimal sequence S_1 obtained by B&B algorithm.

12. Conclusion and Future Research

Branch and Bound algorithm is an exact algorithm thus it has the guarantee to provide the optimal solution while

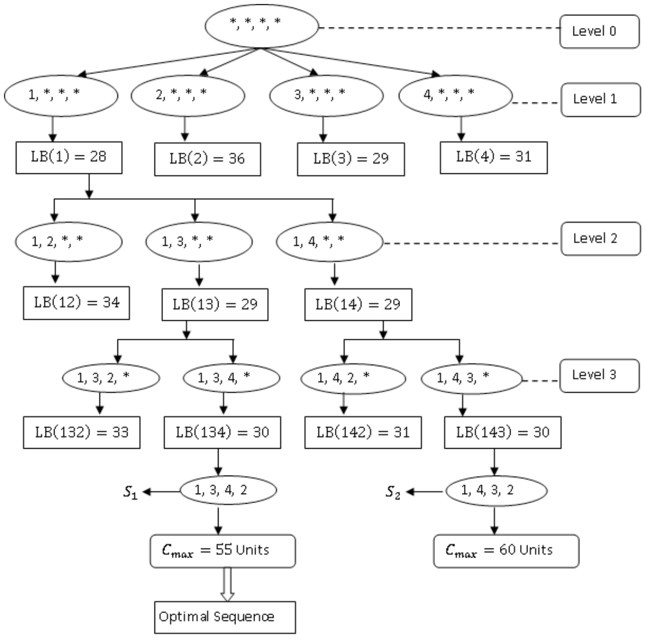

Figure 8. New constructed B&B algorithm's branching tree.

heuristic algorithm as palmer approach does not has the guarantee to provide an optimal solution. Hence, we have developed the new algorithm based on Branch and Bound technique for finding the optimal solution of bi criterion objective function of three machines flowshop scheduling problem with transportation time and weight of the jobs. Most of the researchers have worked in a single criterion objective function and so we used bicriterion objective function. Our first criterion is a makespan and second one is the weighted mean flowtime. Moreover, a new heuristic algorithm based on Palmer approach is also developed for the same considered problem. We solved the same numerical by both the algorithm and Comparatative result show that our constructed B&B algorithm is outperform as compare to Palmer based heuristic algorithm for minimization of bicriterion (makespan, weighted mean flowtime) objective function. Furthermore, with the help of percentage improvements we observed that our constructive algorithm using B&B methodology provides 8.33 % improvement for makespan and 6.52 % improvement for weighted mean flow time over the Palmer heuristic approach. We also generated the Gantt chart for verifying the effectiveness of our Constructed B&B algorithm with respect to Palmer based heuristic algorithm. In addition, directed graphs for optimal sequence and for each computational level of Lower Bounds are also designated to understand the lower bound computation. Lastly, a branch tree is originated to show the process of lower bounds in Branch and Bound algorithm.

12.1 Future Research

For future research, we can extend our bicriterion objective function to multi criterion objective function. We also use some other parameters like Breakdown of machine, earliness, tardiness, setup time etc. in different scheduling environments as job shop; Openshop, flexible environment etc in scheduling problems. Some other heuristic algorithms like NEH, CDA, Gupta and Meta heuristic algorithms like Genetic Algorithm, Ant Colony Optimization, Beam Search, Simulated Annealing, Tabu Search are also be used for future research with multi objective functions.

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