

# A Study on Prediction of Probability of Hit for an Anti-Aircraft Artillery

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## Abstract

Background/Objectives: The objective of this paper is to predict probability of hit for an anti-aircraft artillery based on the errors of the muzzle velocity and the cant error related to the fire power. **Methods/Statistical Analysis:** The muzzle velocity is obtained by using Le Duc equation and the errors of the projectile and the propellant. The cant angle error is obtained by the rotation of coordinate system and the cant sensor error. Then the root of the sum of squares of errors gives us the total error and then by using the equation of probability of hit, we may obtain probability of hit for an anti-aircraft artillery. **Findings:** The muzzle velocity error due to the projectile weight error, the propellant weight error and the chamber volume error are obtained. The cant angle error due to cant angle sensor error may be obtained. Assume that all the errors are given, then we obtain the total errors in the horizontal error and the vertical error. Then using the total errors and the equation of probability of hit, we obtain probability of hit for an anti-aircraft artillery. **Improvements/Applications:** When we design a combat vehicle, this approach will be good resources for performance analysis of fire power for an anti-aircraft artillery.

**Keywords:** Antiaircraft Artillery, Cant Variation, Hit probability, Muzzle Velocity Variation, Prediction of Probability of Hit, Vertical and Horizontal Error

## 1. Introduction

When we consider prediction of probability of hit for an anti-aircraft artillery, four steps are repeated: detection, tracking, arms control and shooting. During repetition of four steps, many error budgets occur. Because of these error budgets, the bullets are dispersed at the target. Major error budgets<sup>1</sup> include muzzle velocity error, cant related error, wind error, density error and pressure error. In this paper we consider major two errors: muzzle velocity error and cant sensor error. Using these errors and other errors, we may predict the probability of hit for an anti-aircraft artillery. These approaches will give a valuable tool for design a cant sensor and muzzle velocity detector.

## 2. Error Estimates and Hit Probability Prediction

There are two error estimates methods: one is the engineering based method and the other is the experimental based method. These have been described in the paper of<sup>2</sup>. We use the experimental approach as the error estimates. The experimental approach uses the standard deviations of the cant sensor error and the muzzle velocity error. The total errors of cant sensor error and muzzle velocity error are transferred to the total errors at the target. In addition, there are two methods of hit probability prediction: one is an analytic method and the other is a Monte Carlo method. These have been described in the

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paper of<sup>2</sup>. In this paper, hit probability prediction may be obtained using the analytic method. The analytic method assumes that hit probability is expressible as the product of probability of hit in the horizontal direction and vertical direction.

### 3. Trajectory Differential Equations

The trajectory equations by<sup>3</sup> are expressed as follows:

$$\begin{aligned} \dot{v}_x &= -C_d^* \tilde{V}(v_x - w_x) \\ \dot{v}_y &= -C_d^* \tilde{V}(v_y - w_y) - g \\ \dot{v}_z &= -C_d^* \tilde{V}(v_z - w_z) \end{aligned} \tag{1}$$

where,

$$\tilde{V} = \sqrt{\sum_i^{x,y,z} (v_i - w_i)^2}$$

The x-axis is a range direction and the y-axis is a upward direction against the gravitational force and z-axis is a deflection direction. The trajectory of a bullet is shown in Figure 1. The approximate trajectory of bullet was studied in the paper of<sup>4</sup>.

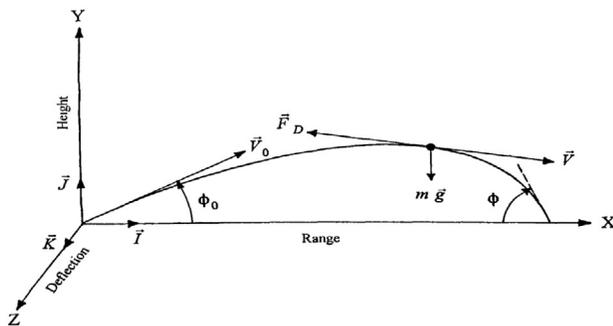


Figure 1. Bullet trajectory.

### 4. Muzzle Velocity Variation

The Le Duc equation is expressible as

$$v = \frac{ax}{(b+x)} \tag{2}$$

where, a is a muzzle velocity when the barrel is assumed to be infinite and b is a twice of the length at

which the pressure inside the chamber volume is maximum. Specifically we may describe a and b as follows:

$$a = J(W_c / W_p)^{1/2} \Delta^n \tag{3}$$

$$b = \beta(1 - \frac{\Delta}{\delta})(\frac{W_c}{W_p})^{2/3} \Delta^{-2/3} \tag{4}$$

where, J,  $W_c$ , and  $W_p$  are constant related to propellant, propellant weight, projectile weight, respectively. The notations,  $\hat{a}$  and  $\hat{a}$  are a constant related to distance and specific gravity of propellant, respectively and

$$\Delta = \frac{W_c / V_{ch}}{W_{H_2O} / V_{ch}} \tag{5}$$

where,  $V_{ch}$ , and  $W_{H_2O}$  are a chamber volume and weight of water  $H_2O$  inside the chamber volume. The muzzle velocity is given by

$$V_m = \frac{aL}{(b+L)} \tag{6}$$

where, L, and  $W_{H_2O}$  is a length of the barrel. Taking differentiation of the natural logarithm of equation (6), we obtain

$$\frac{dV_m}{V_m} = \frac{da}{a} + \left(1 - \left(\frac{b}{b+L}\right)\right) \frac{dL}{L} - \frac{db}{b+L} \tag{7}$$

Inserting modification of equations (3) and (4) into (7), we obtain

$$\begin{aligned} \frac{dV_m}{V_m} &= \left[ \frac{1}{2} + n + \left(\frac{b}{b+L}\right) \left(\frac{\Delta/\delta}{1-\Delta/\delta}\right) \right] \frac{dW_c}{W_c} - \left[ \frac{1}{2} - \frac{2}{3} \left(\frac{b}{b+L}\right) \right] \frac{dW_p}{W_p} \\ &\quad \left[ n + \left(\frac{b}{b+L}\right) \left(\frac{2}{3} + \frac{\Delta/\delta}{1-\Delta/\delta}\right) \right] \frac{dV_{ch}}{V_{ch}} \end{aligned} \tag{8}$$

which is described in the book<sup>5</sup>.

Table 1. Parameters and their values

Parameters	Values
a	251.1811m/sec
b	0.0218 m
L	0.52 m
$V_{ch}$	0.0001 m <sup>3</sup>
	0.612
$\delta$	1.618

J	2083 m/s
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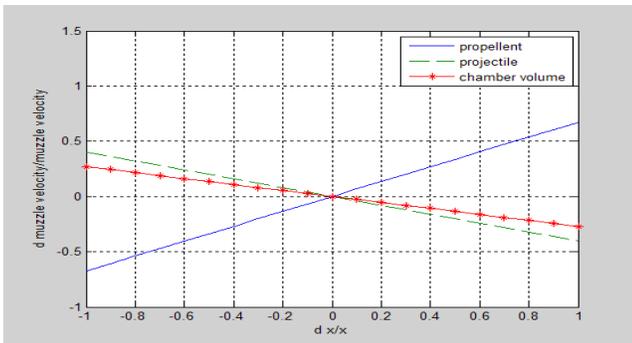
The parameter n is obtained by

$$n = Ln\left(\frac{a}{J}\left(\frac{m_c}{m_p}\right)^{-0.5} Ln(\Delta)\right) \quad (9)$$

Table 1 and the equation (8), the percentage error  $V_m$  is given by

$$\frac{dV_m}{V_m} = 3.2353 \frac{dm_c}{m_c} - 0.4732 \frac{dm_p}{m_p} - 2.7621 \frac{dV_{ch}}{V_{ch}} \quad (10)$$

The percentage error  $V_m$  is depicted by the percentages of  $m_c$ ,  $m_p$  and  $V_{ch}$ , shown in Figure 2.



**Figure 2.** Percentage error of muzzle velocity versus percentage of propellant, projectile and chamber volume.

If we assume that

$$\frac{dm_c}{m_c} = \frac{dm_p}{m_p} = \frac{dV_{ch}}{V_{ch}} = 0.01 \quad (11)$$

Then the maximum and minimum perturbations of the percentage error  $V_m$  is given by

$$\frac{dV_m}{V_m} = \pm 0.0647 \quad (12)$$

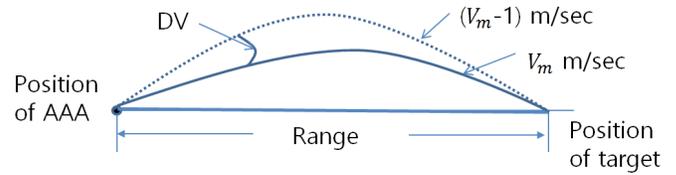
If the variation of  $V_m$  is assumed to be Gaussian, by the equation (12) and  $V_{m=241 \text{ m/s}}$ , then the standard deviation of muzzle velocity  $V_m$  is given by

$$\sigma_{V_m} = 0.0647 \times V_m \times 0.1975 = 3.08 \quad (13)$$

When the range is 2000m, the vertical error at the target by the muzzle velocity error is

$$\sigma_y = DV \times \sigma_{V_m} = .0205 \times 3.08 = 0.0631 \quad (14)$$

where, DV is an elevation angle variation when the muzzle velocity is changed by the 1m/s and the range is fixed, as shown in Figure 3.

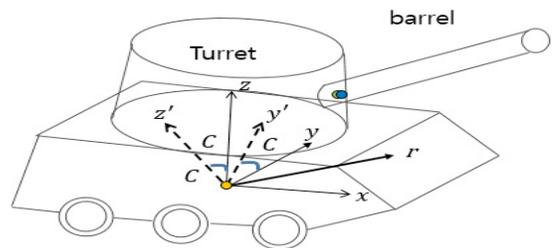


**Figure 3.** Diagram of DV.

If DV is assumed to be 0.0205, the standard deviation of the vertical error is given by 0.0631.

## 5. Elevation and Azimuth Variations due to a Cant Angle

When the anti-aircraft artillery stops, the direction of gun is assigned to the range direction. When an anti-aircraft artillery (AAA) stands on the tiled ground, the direction of gun is changed. The  $(x, y, z)$  coordinate system is a reference system. The x axis represents the front and back line along the AAA. The y axis directs from the left center of the track or wheel to the right center of the track or wheel. The z axis starts from the center of the AAA to the upward direction which means the opposite direction of the gravity. When the AAA is tilted to side, then a new coordinate system is the  $(x', y', z')$  as shown in Figure 4. The plane  $(y', z')$  is obtained by counterclockwise rotation (cant angle C) along the positive x axis. In the reference coordinate system, the direction of barrel is described by the elevation angle  $e$  and the azimuth angle  $\varphi$ :



**Figure 4.** Relationship between the reference coordinate system and the new coordinate system when a gun tube is tilted.

$$e = \cos^{-1}\left(\frac{\sqrt{x_0^2 + y_0^2}}{\sqrt{x_0^2 + y_0^2 + z_0^2}}\right) \quad (15)$$

$$\varphi = \tan^{-1}\left(\frac{y_0}{x_0}\right) \quad (16)$$

Using the rotation of the coordinate system,

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos C & \sin C \\ 0 & -\sin C & \cos C \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad (17)$$

The elevation angle  $e'$  in the new coordinate system is expressible as

$$\sin(e') = \sin(e) \cos C - \cos(e) \sin \varphi \sin C \quad (18)$$

where,  $C$  denotes the cant angle. Using the small value of  $e'$ ,  $e$ ,  $\varphi$  the new elevation angle is

$$e' \approx e \cos C - \varphi \sin C \approx e \cos C \quad (19)$$

The difference of angles is

$$e - e' \approx e(1 - \cos C) \quad (20)$$

Figure 5 shows, the error of elevation angle versus cant angle is shown. Using the cant sensor error  $CE$ , the cant error<sup>6</sup> is given by

$$e(1 - \cos(C + CE)) - e(1 - \cos C) = e(\cos C - \cos(C + CE)) \quad (21)$$

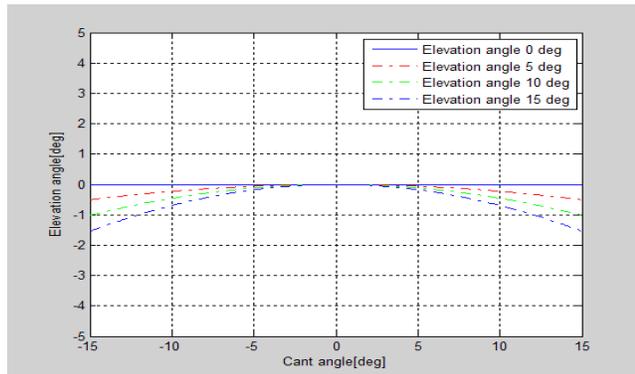


Figure 5. The elevation error versus the cant angle.

The new azimuth angle  $\varphi'$  is given by

$$\tan(\varphi') = \frac{y'_0}{x'_0} = \frac{y_0 \cos C + z_0 \sin C}{x_0} = \frac{r \sin(\theta) \sin \varphi \cos C + r \cos(\theta) \sin C}{r \sin(\theta) \cos \varphi} \quad (22)$$

Using the small value of  $e'$ ,  $e$ ,  $\varphi$  the new azimuth angle is

$$\varphi' = \varphi \cos C + e \sin C \quad (23)$$

By the small value of  $c$ , then the difference is

$$\varphi' - \varphi \approx eC \quad (25)$$

Figure 6 shows, the error of azimuth angle versus cant angle<sup>6</sup> is shown. Using the cant sensor error  $CE$ , the cant error is given by

$$\varphi' - \varphi \approx eCE \quad (25)$$

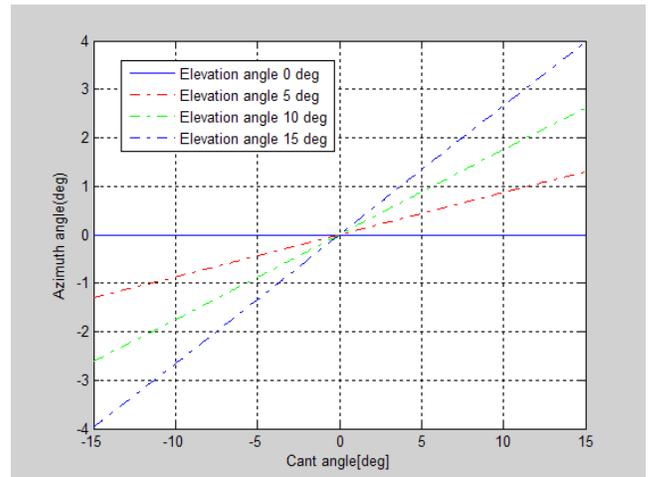


Figure 6. The elevation error versus the cant angle.

## 6. Hit Probability

If the variances of vertical and horizontal direction at the target are given by

$$\sigma_x^2 = \sigma_{V_m}^2 + \sigma_{Range}^2 \approx \sigma_{V_m}^2 \quad (26)$$

and

$$\sigma_y^2 = \sigma_{cwind}^2 + \sigma_{cant}^2 \quad (27)$$

If the fixed bias errors at the target are zeros in the horizontal and vertical directions and the width and height of the target are assumed to be 4 m, then hit probabilities<sup>6</sup> in both directions are given by

$$P_{hx} = \frac{1}{\sqrt{2\pi}\sigma_x} \int_{-\frac{w}{2R} \times 1000}^{-\frac{w}{2R} \times 1000} \exp\left[-\frac{(x-h)^2}{2\sigma_x^2}\right] dx \quad (28)$$

and

$$P_{hy} = \frac{1}{\sqrt{2\pi}\sigma_y} \int_{-\frac{ht}{2R} \times 1000}^{-\frac{ht}{2R} \times 1000} \exp\left[-\frac{(x-k)^2}{2\sigma_y^2}\right] dx \quad (29)$$

Hit probability is expressible using the error function as

$$P_h = P_{hx} P_{hy} \quad (30)$$

By using the data in Tables 2 and 3, we obtain

$$P_{hx}(2000)=.5470 \text{ and } P_{hy}(2000)=.5470 \quad (31)$$

and finally obtain

$$P_h(2000)= P_{hx}(2000) \times P_{hy}(2000)=.3088 \quad (32)$$

**Table 2.** Azimuth angle errors at a distance of 2000m

Item of Error	Standard deviation
crosswind	.01987
cant sensor error	0.0505
other errors	0.634
Root sum square	0.6663

**Table 3.** Elevation angle errors at a distance of 2000m

Item of Error	Standard deviation
Muzzle velocity error	.0631
other error	0.638
Root sum square	0.6411

## 7. Conclusion

In this paper, when an anti-aircraft artillery shoots an enemy target, the probability of hit has been predicted. Using the cant sensor error and muzzle velocity error, we have predicted probability of hit at the target. It seems

that these approaches will give a valuable tool for design a cant sensor and muzzle velocity detector.

## 8. Acknowledgement

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