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Development of Hybrid Algorithm for Integrated Aircraft Routing Problem and Crew Pairing Problem

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Abstract

Traditionally, aircraft routing and crew pairing problems are solved sequentially with the aircraft routing problem solved first followed by the crew pairing problem. But in some cases, the results are suboptimal. In order to overcome this problem, both problems will be composed in one model. Although the integration model is challenging to solve but it is practically useful in airlines operations for getting the optimal solutions. In this study, we proposed the constructive heuristic method and the genetic algorithm (GA) in producing the feasible paths. After that, we will solve those two types of feasible paths in the integrated model by using three approaches which are the integer linear programming (ILP), Dantzig Wolfe decomposition method and Benders decomposition method. Computational results show that the obtained feasible path from the constructive heuristic method and solved by the Dantzig Wolfe decomposition method is more effective while the paths from the GA and solved by the Dantzig Wolfe decomposition method is good in finding the minimum computational time. From the results obtained, all the flight legs and crew pairing are used only once. There are four type of aircrafts are used in testing the performance of the approaches which based on local flights in Malaysia for seven days. The solutions of the feasible paths from GA is more advantageous in term of the computational times compare to the solutions by using the feasible paths from constructive heuristic method.

Keywords: Aircraft Routing Problem, Crew Pairing Problem

1. Introduction

The aircraft routing problem is to seek the minimal solution for yielded routing of each aircraft while covered each flight legs in the schedule. Then, the crew pairing problem is solved by using the solution from the aircraft routing problem in order to find the optimal solution for each crew pair while satisfy some rules and covered each flight leg. To avoid the suboptimality of the results, the integrated model of aircraft routing and crew pairing problem are suggested in this work.

Many solution approaches for aircraft routing problem and crew pairing problem had been proposed since those problems have been received attention from industry all over the world. Reviews of individual aircraft routing and crew pairing problem had been done widely since decades ago. Those works can be found in [1–6].

Benders decomposition method is used widely in solving the integrated aircraft routing and crew pairing problem in these past years as the method allows solving the large linear programming since it has a special block structure in order to divide the big problem into a master problem and a few sub problems. The first work that used Benders decomposition method in solving the integrated aircraft routing and crew pairing problem is by[7], which are the aircraft routing problem as the master problem

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and the crew scheduling as the sub problem. By using the column generation, both of the problems are solved, followed by branch and bound is used in order get the integer solutions of the problems.

[2]produces a new formulation of the integrated aircraft routing and crew pairing problem and solved it by using Benders decomposition method. In this work, the crew pairing is used as the master problem while aircraft routing is used as the sub problem. Then, the methodology followed with obtained the optimal results by using column generation. Pareto optimal cut is included in this approach and it helpful in develop a good speed of the convergence.

An integrated aircraft routing, crew scheduling and flight retiming problem had been formulated in[3]. By including the time windows in the integrated model, the minimal costs can be produced in this work. The integrated model then solved by using the Benders decomposition method collaborated with the dynamic constraint generation.

Work in[5] integrates the aircraft routing, crew pairing and schedule design problem in one model but approach is contra compared to the approach in[7] as the crew pairing problem is solved first followed by the aircraft routing problem. Schedule design problem allow varies of time windows for the flight legs as to increase the robustness of performance for the airlines.

The fleet assignment, aircraft routing, crew pairing and crew rostering problems had been integrated in[8] and heuristic algorithm is used as the approach in order to solve the integrated model. While the other works are based on many days of flight legs, but in this work, only daily flight legs are used in the model.

While[10] used the stochastic model in solving the integrated aircraft routing, crew pairing problem. The flight legs can be re-timed in this work as to decrease the delays which consequently reduce the costs in airlines operations.

A few integrated airline scheduling problems are proposed in[12] and the integrated Benders decomposition and an accelerated column generation is introduced in this work. The experimental results are based on the real data from the European and North America airlines.

While in[13], the iterative method is the approach of the integrated aircraft routing and crew scheduling problem. The crews are divided into two groups which are technical crews and flight attendants. From the tests, they found that if a crew pairing used two aircrafts, then the solutions are more optimal and the robustness is increased.

The integrated of the fleet assignment, aircraft routing and crew pairing in the linear programming model is invented in[15]. The approach in obtaining the results is column generation and the results are good in aspect of decreasing the number of aircrafts and crews used.

[16]take the delays of the flight legs as their problem to be solved as the changing of the aircraft in a crew pairing can increase the percentage of the delays. In each airline, flight legs delay is one of the causes that increase the costs. So, they combine the iterative method from the previous work[13] and the column generation to get the results of the integrated stochastic aircraft routing and crew scheduling problem.

The contribution of our study include (i) an approach based on the constructive heuristic method to produce the paths, (ii) a formulation based on the GA to produce the paths, (iii) a hybrid approach that combine the constructive heuristic method and ILP, (iv)a formulation that combine the constructive heuristic method and Dantzig Wolfe decomposition method, (v) a hybrid approach that combine the constructive heuristic method and Benders decomposition method, (vi) a hybrid approach that combine the GA and ILP, (vii) a formulation of the GA and Dantzig Wolfe decomposition in solving the problem, (viii) a hybrid approach that combine GA and Benders decomposition method and (ix) the experimental results of the proposed approaches.

This paper is organized as follows: in Section 2 we describe, including the approaches for generating the feasible paths, the problem formulations of the integrated aircraft routing and crew pairing problem, the different approaches followed by the experimental results in Section 3 and conclusion of the works in Section 4.

2. Problem Formulation

A brief on both aircraft routing and crew pairing problems are given in this section. The integrated model of aircraft routing and crew pairing problems is presented in this work. In order to solve the integrated model, the feasible paths of aircraft routing and crew pairing are needed. Because of that, we also proposed two approaches that generate the feasible paths which known as the constructive heuristic method and the GA. After that, three methods that mentioned before are used for solving the integrated model by using the feasible paths obtained.

The maximum day of aircraft routing and crew pairing in this study is assumed as seven days which is equal to one week and only the local flight legs are considered in this study. Since the local flight legs are not scheduled after the midnight, the maintenance operations can be done during the nights.

2.1 Aircraft Routing Problem

The aircraft routing problem is solved in order to find the minimal solution for each type of aircraft by determines the sequence of the flight legs to be flown. Besides that, the rule that each flight leg is covered exactly once need to be satisfied. Let $N^A = (D^A, R^A)$ is a network of aircraft routes, as D^A is a set of nodes while R^A is a set of arc. A set of flight leg, F is represented for each type of aircraft. Each node $i \in N^A$ is describes as the departure time and arrival time of the flight leg $f_i \in F$. A path in the aircraft routing problem represents the cyclic route of the flight legs for each type of aircraft that starts and end at the maintenance stations.

Let M be the maintenance station for paths which is $m \in M$ as the home base of the flight legs. The start and the end of a route are representing with p_m^A and q_m^A as the source and sink nodes. The source nodes are the starting station of the routes at the *m* station while the sink nodes are the ending station of the routes at the *m* station.

2.2 Crew Pairing Problem

The crew pairing problem is solved to assign the set of pairings to the flight legs while covered legs exactly once with minimum costs. A crew pairing is a sequence of crew pair that begins and end at the same station which is known as the crew base[2]. The limit of the duty period in one crew pair is does not exceed five. The meaning of the duty period is a sequence of the crew works for flight legs but separated by sit time which is the rest periods. A path for crew pairing represents a complete pairing of a crew that begins and end at the same station which is known as the crew base.

Let $N^C = (D^C, R^C)$ is a network of aircraft routes, as D^C is a set of nodes and R^C is a set of arc. Each node $i \in N^C$ is describes as the departure time and arrival time of the flight leg $f_i \in F$. The crew base is represented as B .The p_b^C and q_b^C are the source and sink nodes for each crew pairing for each crew base $b \in B$. The source nodes are the starting node of a crew pairing at the crew base *b* while the sink nodes are the ending node of a crew pairing at the crew base *b*.

2.3 Feasible Paths of Aircraft Routing and Crew Pairing Problems

In order to solve the integrated model of aircraft routing and crew pairing problems that proposed in this work, the possible path of aircraft routes and crew pair need to be generated. There are two methods are proposed for producing the feasible paths. The first method called the constructive heuristic method that generates all the possible paths for aircraft routes and crew pair. The second one is called the GA that produces some of the possible paths. There are some data are needed for producing the possible paths which are the departure time, the arrival time, the departure station, the arrival station and the type of aircraft for each flight leg.

2.3.1 The Constructive Heuristic Method

The constructive heuristic method is the first method that being proposed in this work for producing the possible paths. The brief description of this method is provided and summarized in Figure 1.

Figure 1. The algorithm of constructive heuristic method

Phase 1
Step 1: Define the parameters of the algorithm: the number of path *h*, the number of flight legs *i*, the flight leg number defined in the flight schedule $k = 1,2,3,\ldots,n$, the departure node of flight leg *k*, $p = 1,2,3,\ldots,m$, the arrival node of flight leg *k*, $q = m+1, m+2, m+3,\ldots,m+u$, the initial home base *j*, the departure time of flight leg *k* as *v*, the arrival time of flight leg *k* as *w*, the departure station of flight leg *k* as *r*, the arrival station of flight leg *k* as *s*, the flight time *ft*.
Step 2: $h = 1, i = 0, ft = 0$.
Step 3: Choose an initial home base *j*.
Phase 2
Step 1: Choose a flight leg (*k*) that has a departure station at the home base *j*.
Step 2: Add it to the current pairing, $i = i + 1$. Delete *k* from the flight schedule.
Step 3: Choose a flight leg *(k)* that has these criteria: i. The *r* of *i* is the same with *s* of $i - 1$. ii. The *v* of *i* minus the *w* of $i - 1$ is bigger or equal to 20 minutes. iii. The arrival station of *k* equals to *j*, add the pairing into the current pairing. Retrieve all of the flight legs *(k)* except for flight leg *(k)* in Step 1 into the list. $h = h + 1$.
Phase 3
Step 1: Use *k* in the Step 1 in Phase 2, go to Step 2 in Phase 2.
Step 2: Choose a flight leg *(k)* that has these criteria: i. The *r* of *i* is the same with *s* of $i - 1$. ii. The *v* of *i* minus the *w* of $i - 1$ is bigger or equal to 20 minutes. iii. Add it to the current pairing, $i = i + 1$. Delete *k* from the flight schedule.
Step 3: If $i < 4$, choose a flight leg *(k)* that has these criteria: i. The *r* of *i* is the same with *s* of $i - 1$. ii. The *v* of *i* minus the *w* of $i - 1$ is bigger or equal to 20 minutes. iii. If the arrival station of *k* equals to *j*, stop. Add the pairing into the current pairing. Retrieve all of the flight legs *(k)* into the list. $h = h + 1$.
Phase 4
Step 1: Choose other flight leg (*k*) that has a departure station at the home base *j* but exclude the *k* that had been used. Go to Phase 2 and 3. Stop when all the *k* that has a departure station at the home base *j* is used.

An illustrative example of the constructive heuristic method is as in Table 1.

Table 1. Small example of flights schedule A

Flight Number (k)	Departure station (#departure node)	Arrival station (#arrival node)	Departure time (v)	Arrival time (w)
1	A (1)	B (5)	08.00	09.00
2	A (2)	B (6)	12.00	12.55
3	B (3)	A (7)	09.30	10.25
4	B (4)	A (8)	15.00	14.00

Phase 1

- The $p = 1,2,3,4$ and $q = 5,6,7,8$. The departure time, the arrival time, the departure station and the arrival station are as in Table 3. The home base, $j = A$.

Phase 2

- Choose the flight leg that the same as home base which is *A*. The paths obtained from this phase are (in *k* terms):
  1. $1 \rightarrow 3$
  2. $2 \rightarrow 4$

Phase 3

- There is no feasible path from this phase since the example above is a small one.

Phase 4

- The paths obtained from this phase are (in *k* terms):
  1. $1 \rightarrow 3 \rightarrow 2 \rightarrow 4$

As the summary, the constructive heuristic method is able to produce all the feasible paths.

2.3.2 The Genetic Algorithm

A genetic algorithm (GA) is a great heuristic method in finding the solutions of the complex problems. The beginning of the GA is a set of solutions called initial populations which is represented by chromosomes are needed to be obtained. The initial population is usually generated randomly and the quantity of the initial population can be in any range, from a few to a thousand. The GA is an iterative process that maintains the best solution from the populations until a stop criterion is reached. For each iteration, the two populations that represent the solution are being evaluated and two new populations are produced using some genetic process such as reproduction, crossover and mutation with the desire that the two new populations will be improve than the old one. In order to identify the best new populations (offspring), their fitness function are being evaluated and in our case, the costs of the chromosomes are expressed as the fitness function.

2.3.2.1 Initialization

In GA, the representation is important as it affect the process. The structures of the GA are different for each problem. In this study, each gene represents a feasible path obtained from the constructive heuristic method above while the chromosome represents the combination of the feasible paths that contained all the respected flight legs from the schedules. The representation of the chromosomes for the aircraft routing problem and the crew pairing problem are same as the form of both problems are same. The initial population is generated by the initial population algorithm (IPA) as below:

Step 1: Define the population size *ps=0*, the flight leg number in the flight schedule $k = 1,2,3,\ldots,n$, the flight time *ft*, and the sit time *st*.

Step 2: Choose randomly a feasible path obtained from the constructive heuristic method in the *k* term.

Step 3: Add it to the current chromosome. Record the total *ft*, *st* and the flight leg number *k*. Delete all the feasible paths that involved of *k* which are generated from the exact method.

Step 4: Determine the candidates of *k* from the flight schedule.

Step 5: Repeat the Step 2 and Step 3 until all the *k* covered. Add it to the current population, *ps= ps+1*. The genes in each chromosome must not duplicate the other genes within the chromosomes.

Step 6: Calculate the costs of the developed path by using Constraint 1.

Step 7: Recover all the flight leg in the flight schedule $k = 1,2,3,\ldots,n$. Repeat the Step 2, Step 3, Step 4, Step 5 and Step 6 until the maximum number of *ps* is reached, and then stop.

2.3.2.2 Evaluation and Selection

In this process, each member of the population is being evaluated based on the fitness function. The cost of each population is taken as the fitness function. Besides that, the flight time and the sit time are also being calculated for each member of the population. If the costs are same, then the chromosome that has the minimum *ft* and *st* is selected. The cost is stated in Constraint 1

The cost = the cost of aircraft routing problem (CAR) + the cost of crew pairing problem (CCP) + the penalty cost (CP) (1)

where the CAR is the cost that involved in the aircraft routing problem and given in Constraint 2

CAR = the cost of fuel per gallon x gallon of fuel used per hour x flight time of the flight leg (hour) (2)

While the CCP is the costs of the crew pairing problem and stated in Constraint 3

CCP = total cost PPA of pilot + total cost PPA of co-pilot + total cost PPA for cabin crew (3)

2.3.2.3 Crossover

The crossover plays a big part in the GA because the crossover is responsible for the changes done between the genes within the chromosomes in the selected parents. Since the feasible path is the genes, the feasible paths in the parents are exchanging but all the flight legs must be included in each member of the chromosome.

Table 2. Small example of flight schedule B

Flight Number (k)	Departure station (#departure node)	Arrival station (#arrival node)	Departure time (v)	Arrival time (w)
1	L (1)	M (13)	08.00	09.00
2	L (2)	M (14)	12.00	12.55
3	L (3)	N (15)	10.00	10.45
4	L (4)	N (16)	13.15	14.00
5	L(5)	Q(17)	06.30	08.55
6	L(6)	Q(18)	14.30	17.00
7	M (7)	L (19)	09.30	10.25
8	M (8)	L (20)	15.00	14.00
9	N (9)	L (21)	12.00	12.45
10	N (10)	L (22)	16.00	16.45
11	Q(11)	L(23)	09.35	12.00
12	Q(12)	L(24)	18.35	21.00

The overall process of crossover is summarized as follows:

Step 1: From the IPA, the initial population is generated. Two parents P_1 and P_2 are selected by using the selection procedure.

Step 2: In order to generate the P_1' and P_2' offspring, repeat Step 2, Step 3, Step 4 and Step 5 of IPA.

An example of the crossover is as Table 2.

The parents as in Figures 2 and 3.

Figure 2. Parents P_1

1 → 7	2 → 8	3 → 9 → 4 → 10	5→11→6→12

Figure 3. Parents P_2

1→7→2→8	3→9	4→10	5→11→6→12

From the parents, the two children that will be produced are as Figures 4 and 5.

Figure 4. Child P_1'

1→7→2→8	3→9→4→10	5→11→6→12

Figure 5. Child P_2'

1→7	2→8	3→9	4→10	5→11→6→12

As we can see above, there are some same genes used in each parents and child but it must be at least one gene is different within other chromosomes.

2.3.2.4 Mutation

The mutation is the process that changes the genes within the chromosomes which is to regenerate a new solution space. This process is very important in finding the solution because without it, the algorithm cannot find the optimal solution. In our work, the mutation process produces the solutions that include all the flight legs but used different paths. The most important point that needs to be highlight is to make sure that each gene within the chromosomes are not identically with the other chromosomes since the flight legs are required to cover once.

An example of the mutation that based on the Figures 4 and 5 are as Figures 6 and 7.

The overall procedure is as construct below:

Step 1: Define the initial population size, *ps=0*.

Step 2: Produce the initial population by using IPA and evaluate the fitness for each of them by using the Constraint 1 and the total time of the *ft* and *st*.

Step 3: Choose two parents, P_1 and P_2 by selection procedure.

Step 4: Produce the children which are P_1' and P_2' by the crossover process. Evaluate each of them by using the Constraint 1 and the total time of the *ft* and *st*.

Step 5: Choose the best solution for the current population size, L_B based on the Constraint 1. If the fitness values for those are same, then take the minimal total time of the *ft* and *st*.

Step 6: Generate the mutation of the P_1' and P_2'. Evaluate each of them by using the Constraint 1 and the total time of the *ft* and *st*. Choose

Step 7: Choose the L_B and the best solution for all the cumulative solution, C_B based on the Constraint 1. If the fitness values for those are same, then take the minimal total time of the *ft* and *st*. *ps= ps+1*

Step 8: By using the mutation of the P_1' and P_2', repeat the Step 4 until Step 7. Stop the process when the C_B are repeated three times. C_B is the optimal solution.

2.4 Model Formulation

The set of feasible paths of aircraft routes that start at source nodes p_m^A and end at sink nodes q_m^A is represented as α^m for each maintenance station $m \in M$. The set of feasible paths of crew pairing that start at source nodes p_b^C and end at sink nodes q_b^C is represented as α^b for each crew base $b \in B$.

The brief of the notation is presented in Table 3.

2.4.1 The Integrated Model

The integrated model of aircraft routing and crew pairing problemis refers as (M1) can be expressed as follows:

Minimize

$$\sum_{b\in B}\sum_{\mu\in\alpha^b} c_\mu \eta_\mu + \sum_{m\in M}\sum_{\mu\in\alpha^m} c_\mu \xi_\mu + \sum_{(f_i,f_j)\in R} z_{ij} R_{ij} \qquad (4)$$

subject to

$$\sum_{m\in M}\sum_{\mu\in\alpha^m} w_\mu^f \xi_\mu = 1 \qquad (f \in F) \qquad (5)$$

$$\sum_{b\in B}\sum_{\mu\in\alpha^b} w_\mu^f \eta_\mu = 1 \qquad (f \in F) \qquad (6)$$

$$\sum_{m\in M}\sum_{\mu\in\alpha^m} l_\mu \xi_\mu \le \omega^A \qquad (7)$$

$$\sum_{m\in M}\sum_{\mu\in\alpha^m} s_\mu \xi_\mu \le \omega^C \qquad (8)$$

$$\sum_{b\in B}\sum_{\mu\in\alpha^b} v_\mu \eta_\mu \le \omega^B \qquad (9)$$

$$\sum_{b\in B}\sum_{\mu\in\alpha^b} n_\mu^{ij} \eta_\mu - \sum_{m\in M}\sum_{\mu\in\alpha^m} n_\mu^j \xi_\mu \le 0 \qquad ((f_i,f_j)\in S) \qquad (10)$$

$$\sum_{b\in B}\sum_{\mu\in\alpha^b} n_\mu^{ij} \eta_\mu - \sum_{m\in M}\sum_{\mu\in\alpha^m} n_\mu^j \xi_\mu - R_j \le 0 \qquad ((f_i,f_j)\in R) \qquad (11)$$

$$R_{ij} \ge 0 \qquad ((f_i,f_j)\in R) \qquad (12)$$

$$\xi_\mu \in \{0,1\} \qquad (m \in M; \mu \in \alpha^m) \qquad (13)$$

$$\eta_\mu \in \{0,1\} \qquad (b \in B; \mu \in \alpha^b) \qquad (14)$$

The objective function (4) aims to minimize the costs of integrated aircraft routing problem, crew pairing problem and penalty costs. The equations (5) and (6) are to ensure that each flight leg use one aircraft and one crew only. Equation (7) is to make sure that each flight leg operated does not exceed the available aircraft. Equation (8) ensures the short connection used in each path does not exceed ω^C. While equation (9) ensures that each crew pairing does not exceed the limit of the duty periods in one crew pairing. Equation (10) ensures that the crews do not change the aircraft if the connection time is too short which is in between 20 minutes to 59 minutes. Equation (11) imposes the penalty costs when the second flight leg uses the same crew only but not the aircraft. While equation (12) is to make sure the penalty costs are non-negativity. Lastly, equation (13) and (14) are the binary decision variables.

2.4.2 The Dantzig Wolfe Decomposition Method

Dantzig Wolfe decomposition method which we denote to as (M2) is stated as below by reformulate the integrated model (4)-(14). The new formulation is including one master problem with two sub-problems.

(a) The master problem

Minimize

$$\sum_{b\in B}\sum_{\mu\in\alpha^b} c_\mu V_\mu^P \lambda + \sum_{m\in M}\sum_{\mu\in\alpha^m} c_\mu V_\mu^A \pi + \sum_{(f_i,f_j)\in R} z_j R_j \quad (15)$$

subject to

$$1^T \lambda = 1 \qquad \rightarrow \delta^P \quad (16)$$

$$1^T \pi = 1 \qquad \rightarrow \delta^A \quad (17)$$

$$\sum_{b\in B}\sum_{\mu\in\alpha^b} n_\mu^{ij} V_\mu^P \lambda - \sum_{m\in M}\sum_{\mu\in\alpha^m} n_\mu^j V_\mu^A \pi - R_j \le 0$$

$$(f_i, f_j) \in R) \qquad \rightarrow \delta \quad (18)$$

where, $\lambda \in \{0,1\}^{|V_\mu^P|}$ and $\pi \in \{0,1\}^{|V_\mu^A|}$ and $R_{ij} \in \{0,1\}$. The matrix $V_\mu^P = [v_1^P, v_2^P, v_3^P, \ldots, v_k^P]$ spans the polyhedra $P^P = \{\eta^P \in \mathrm{R}_+^n \mid w_\mu^f \eta_\mu = 1\}$ and $P^P = \mathrm{conv}(\{v_1^P, v_2^P, v_3^P, \ldots, v_k^P\})$. While the matrix $V_\mu^A = [v_1^A, v_2^A, v_3^A, \ldots, v_k^A]$ spans the polyhedra $P^A = \{\xi^A \in \mathrm{R}_+^n \mid w_\mu^f \xi_\mu = 1\}$ and $P^A = \mathrm{conv}(\{v_1^A, v_2^A, v_3^A, \ldots, v_k^A\})$.

Equations (16) and (17) ensure that all the flight legs covered exactly one aircraft and one crew pair. Equation (18) is corresponding to the restricted connection variables.

(b)The crew pairing problem (sub-problem 1)

The crew pairing problem contains the crew pairing constraints only.

Minimize

$$\sum_{b\in B}\sum_{\mu\in\alpha^b} (c_\mu - \delta^P n_\mu^j)\eta_\mu \quad (19)$$

subject to

$$\sum_{b\in B}\sum_{\mu\in\alpha^b} w_\mu^f \eta_\mu = 1 \qquad (f \in F) \quad (20)$$

$$\sum_{b\in B}\sum_{\mu\in\alpha^b} v_\mu \eta_\mu \le \omega^B \quad (21)$$

$$\eta_\mu \in \{0,1\} \qquad (b \in B; \mu \in \alpha^b) \quad (22)$$

(c) The aircraft routing problem (sub-problem 2)

The aircraft routing problem also contains aircraft routing constraints only.

Minimize

$$\sum_{m\in M}\sum_{\mu\in\alpha^m} (c_\mu + \delta^A n_\mu^j)\xi_\mu \quad (23)$$

subject to

$$\sum_{m\in M}\sum_{\mu\in\alpha^m} w_\mu^f \xi_\mu = 1 \qquad (f \in F) \quad (24)$$

$$\sum_{m\in M}\sum_{\mu\in\alpha^m} l_\mu \xi_\mu \le \omega^A \quad (25)$$

$$\sum_{m\in M}\sum_{\mu\in\alpha^m} s_\mu \xi_\mu \le \omega^C \quad (26)$$

$$\xi_\mu \in \{0,1\} \; (m \in M; \mu \in \alpha^m) \quad (27)$$

The approach of M2 starts by solving the master problem. Then, the solutions from the master problem are used as the input in order to solve the subproblems. If the reduced costs are negative, the generated columns from the solutions of the subproblems are passed to the master problem. The reduced cost in this approach is decrease if the objective function had been improved since the problem is the minimization problem. The process iterates until there is no more reduce costs are produced.

The process of M2 works is stated in Figure 8.

Figure 6. Mutation of P_1'

1→7	2→8	3→9→4→10	5→11	6→12

Figure 7. Mutation of P_2'

1→7→2→8	3→9→4→10	5→11	6→12

Figure 8. Dantzig Wolfe decomposition approach

Step 1- Solve the master problem (eqs 15-18) and go step 2.
Step 2- Solve the 2 sub-problems
2a: Solve the crew pairing problem (eqs 19-22)
2b: Solve the aircraft routing problem (eqs 23-27)
Step 3- If the reduced costs are negative in the generated columns from steps 2a and 2b then augment the set V_μ^P and V_μ^A of the master problem by using the generated columns and solve the master problem.
Otherwise (i.e., there is no column with negative reduced cost) stop.

2.4.3 The Benders Decomposition Method

The integrated model (3)-(13) are reformulated as a Benders decomposition method which denotes as (M3). The Benders decomposition method consists of the master problem, the primal sub-problem and the dual sub-problem that stated as below.

(s) The primal sub-problem

Minimize

$$\sum_{m\in M}\sum_{\mu\in\alpha^m} c_\mu \xi_\mu + \sum_{(f_i,f_j)\in R} b_{ij} R_j \quad (28)$$

subject to

$$\sum_{m\in M}\sum_{\mu\in\alpha^m} w_\mu^f \xi_\mu = 1 \qquad (f \in F) \quad (29)$$

$$\sum_{m\in M}\sum_{\mu\in\alpha^m} l_\mu \xi_\mu \le \omega^A \quad (30)$$

$$\sum_{m\in M}\sum_{\mu\in\alpha^m} s_\mu \xi_\mu \le \omega^C \tag{31}$$

$$\sum_{m\in M}\sum_{\mu\in\alpha^m} n_\mu^{ij}\xi_\mu \ge \sum_{b\in B}\sum_{\mu\in\alpha^b} n_\mu^{ij}\overline{\eta}_\mu \qquad ((f_i, f_j)\in S) \tag{32}$$

$$\sum_{m\in M}\sum_{\mu\in\alpha^m} n_\mu^{ij}\xi_\mu + R_{ij} \ge \sum_{b\in B}\sum_{\mu\in\alpha^b} n_\mu^{ij}\overline{\eta}_\mu \qquad ((f_i, f_j)\in R) \tag{33}$$

$$R_{ij} \ge 0 \quad ((f_i, f_j)\in R) \tag{34}$$

$$\xi_\mu \ge 0 \quad (m\in M; \mu\in\alpha^m) \tag{35}$$

(b) The dual sub-problem

The dual variables associated with constraints (29)-(33) are

$\beta = (\beta_f | f\in F)\ \gamma \le 0, \delta \le 0, \vartheta = (\vartheta_j \ge 0 | (f_i, f_j)\in S)\ \chi = (\chi_j \ge 0 | (f_i, f_j)\in R)$.

The dual subproblem is as follows:

Maximize

$$\sum_{f\in F}\beta_f + \omega^A\gamma + \omega^C\delta + \sum_{(f_i,f_j)\in S}\sum_{b\in B}\sum_{\mu\in\alpha^b} n_\mu^j \eta_\mu \vartheta_j + \sum_{(f_i,f_j)\in R}\sum_{b\in B}\sum_{\mu\in\alpha^b} n_\mu^j \eta_\mu \chi_j \tag{36}$$

subject to

$$\sum_{f\in F} w_\mu^f \beta_f + l_\mu\gamma + s_\mu\delta + \sum_{(f_i,f_j)\in S} n_\mu^j \vartheta_j + \sum_{(f_i,f_j)\in R} n_\mu^j \chi_j \le c_\mu$$

$$(m\in M; \mu\in\alpha^m) \tag{37}$$

$$\chi_{ij} \le z_{ij} \qquad ((f_i, f_j)\in R) \tag{38}$$

$$\gamma, \delta \le 0 \tag{39}$$

$$\vartheta_{ij} \ge 0 \qquad ((f_i, f_j)\in S) \tag{40}$$

$$\chi_{ij} \ge 0 \qquad ((f_i, f_j)\in R) \tag{41}$$

(c) The master problem

$$\text{Minimize} \qquad \sum_{b\in B}\sum_{\mu\in\alpha^b} c_\mu\eta_\mu + y_0 \tag{42}$$

subject to

$$y_0 + \sum_{(f_i,f_j)\in S}\sum_{b\in B}\sum_{\mu\in\alpha^b} n_\mu^j \eta_\mu \vartheta_j + \sum_{(f_i,f_j)\in R}\sum_{b\in B}\sum_{\mu\in\alpha^b} n_\mu^j \eta_\mu \chi_j \ge \sum_{f\in F}\beta_f + \omega^A\gamma + \omega^C\delta$$

$$((\beta, \gamma, \delta, \vartheta, \chi)\in P_\Delta) \tag{43}$$

$$\sum_{b\in B}\sum_{\mu\in\alpha^b} w_\mu^f \eta_\mu = 1 \qquad (f\in F) \tag{44}$$

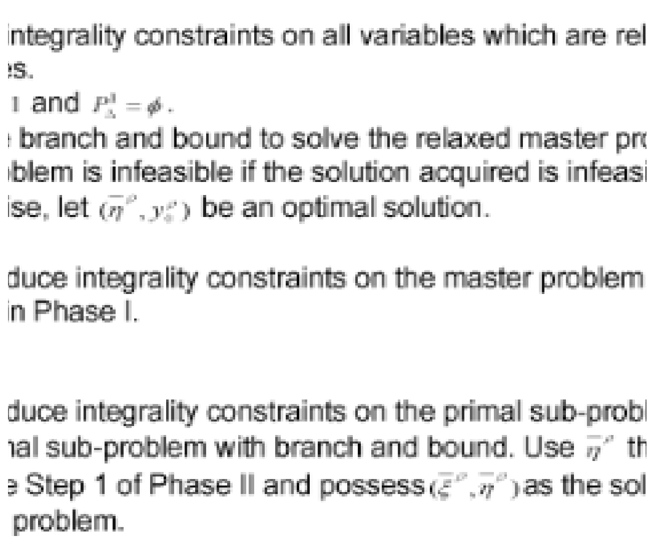
$$\sum_{b\in B}\sum_{\mu\in\alpha^b} v_\mu \eta_\mu \le \omega^B \tag{45}$$

$$\eta_\mu \in \{0,1\} \qquad (b\in B; \mu\in\alpha^b) \tag{46}$$

Let y_0 be the additional free variable of the master problem while P_Δ denote the sets of extreme points of Δ and ρ be the iteration number of the solution for this approach. Let P_Δ^ρ is used as the replacement of the P_Δ .

The process of the approach is summarized in Figure 9.

Figure 9. The Benders decomposition approach


ntegrality constraints on all variables which are rel
s.
1 and $P_\Delta^1 = \phi$.
branch and bound to solve the relaxed master pr
blem is infeasible if the solution acquired is infeas
se, let $(\overline{\eta}^\rho, y_0^\rho)$ be an optimal solution.

duce integrality constraints on the master problem
n Phase I.

duce integrality constraints on the primal sub-probl
al sub-problem with branch and bound. Use $\overline{\eta}^\rho$ th
Step 1 of Phase II and possess $(\overline{\xi}^\rho, \overline{\eta}^\rho)$ as the sol
problem.

3. Experimental Results

We consider four types of aircraft involving in several destinations based on Malaysia. All the approaches are figured out on an Intel Core Duo processor running at 2.10 GHz using high level modelling languages, Microsoft Visual Studio C++, interface with libraries for mathematical programming, ILOG CPLEX Callable Library.

3.1 Data Sets

The aircraft types involved are based on the local flights in Malaysia which are EQV, 738, AT7 and 734 for seven days.

3.1.1 Aircraft Type EQV

The aircraft type EQV operates 126 flight legs and the possible paths produced for EQV type by the constructive heuristic method are 147 paths. The possible paths obtained from the GA for this type of aircraft are 56 paths. Since the network path of the aircraft routing and crew pairing are the same, the possible paths for both problems are the same.

Table 3. The notation

Notation	Description
M	Set of maintenance stations
B	Set of crew bases
R	Set of two flight legs that has a restricted connection
S	Set of two flight legs that has a short connection
p_m^A	The source nodes for aircraft paths
q_m^A	The sink nodes for aircraft paths
p_b^C	The source nodes for crew paths
q_b^C	The sink nodes for crew paths
α^m	Set of possible paths from the source node p_m^A to a sink node q_m^A in N_m^A
α^b	Set of possible paths from the source node p_b^C to the sink node q_b^C in N_b^A
w_μ^f	Equal to 1 if leg f belongs to path μ, and 0 otherwise
η_μ	Binary constant that represents the flow on the crew path μ
ξ_μ	Binary constants that represents the flow on the aircraft path μ
R_{ij}	Binary constant that will be 1 if connection $(f_i, f_j) \in R$ is operated by the same crew but not the same aircraft, and 0 otherwise
c_μ	The cost of using the path μ
n_μ^{ij}	Equal to 1 if leg i and j are operated sequentially in path μ, and 0 otherwise
l_μ	The number of required aircrafts in the path μ
ω^A	The number of available aircrafts
ω^B	The number of duty periods allowed in a crew pairing
ω^C	The number of short connections allowed in one path
v_μ	The number of duties in path μ
s_μ	The number of short connections in path μ
z_{ij}	Penalty cost associated with $(f_i, f_j) \in R$

3.1.2 Aircraft Type 738

There are 70 flight legs for this type of aircraft. The aircraft type 738 produced 112 paths for the aircraft routing and crew pairing by using the constructive heuristic method while the 70 feasible paths yielded by using the GA.

3.1.3 Aircraft Type AT7

This aircraft type consists of 364 flight legs and the possible paths obtained from the constructive heuristic method are 672 paths while there are 420 paths produced by using the GA.

3.1.4 Aircraft Type 734

The involved flight legs for this type of aircraft are 588. The possible paths produced by the constructive heuristic method for this type of aircraft are 854 paths while there are 518 feasible paths obtained by using the GA.

3.2 Solution Approaches

The feasible paths are needed in order to solve the integrated aircraft routing and crew pairing model. In this study, the feasible paths are yielded by two approaches which are the constructive heuristic method and the GA. By using the feasible paths from both methods, the integrated model then solved by three methods. Firstly, the ILP is used as the first approach to solve the integrated problems, followed by the Dantzig Wolfe decomposition method and Benders decomposition method. All those problems are coded in C++ Language interface with ILOG 12.4 CPLEX.

The solution results are given in the Table 4 that include the costs and the respective CPU time used for all four instances involved. The costs used are in Ringgit

Table 4. Summary Results for the three proposed models (M1, M2 & M3)

Aircraft type (#legs, #nodes, #paths)	Feasible paths from constructive heuristic method						Feasible paths from genetic algorithm (GA)					
	Model 1 (M1)		Model 2 (M2)		Model 3 (M3)		Model 1 (M1)		Model 2 (M2)		Model 3 (M3)	
	Cost (RM)	Time (secs)	Cost (RM)	Time (secs)	Cost (RM)	Time (secs)	Cost (RM)	Time (secs)	Cost (RM)	Time (secs)	Cost (RM)	Time (secs)
EQV (126, 252, 147)	1034222	0.07	1034215	0.15	1034348	0.05	1034230	0.08	1034224	0.12	1034360	0.03
738 (70, 140, 112)	1048033	7.84	1048033	8.54	1048075	7.68	1048037	7.12	1048038	7.29	1048101	6.78
AT7 (364, 728, 672)	2296210	544.2	2296210	655.9	2297918	588.3	2491506	434.2	2491507	569.7	2493428	511.3
734 (546, 1092, 854)	5723228	1562.6	5723228	1657.6	5725587	1598.4	5730214	1226.7	5730214	1324.5	5731345	1167.8

Malaysia (RM). From the solution, all the flight legs and crew pairing are used only once. In the aspect of computational costs, the feasible paths from the constructive heuristic method and then solved by using the M1 is the best. While for the aspect of the computational time, the hybrid algorithms of the GA and M3 is found more advantageous compared to the others.

To analyse the effects of the size of the feasible paths used in the problem, the comparison of the results are done. From the Table 4, the computational time of the results grows as the feasible paths are increase. The smaller feasible paths used in the problems only used a small amount of time in solving the problems while the AT7 and 734 type of aircrafts need more efforts in solving the problems.

4. Conclusion

In this paper, we proposed two approaches to produce the feasible paths. The first approach is called the constructive heuristic method that generates all the feasible paths from the flight schedule while the second approach is the GA that only produces some of the feasible paths. The feasible paths obtained from those two approaches are used in the integrated model and lastly solved them in the three methods which are ILP, Dantzig Wolfe decomposition and Benders decomposition. There are four instances used to test the performances of the constructive heuristic method and the GA that based on the airline in Malaysia for local flights. The results show that the number of feasible paths involved affect the run time. The hypothesis that we can concluded is the bigger number of feasible paths, the more time needed in the computational.

In the future, we can improve the computational time of the problems by solving the integrated model in heuristic methods. Since the feasible paths were produced by two approaches which are the constructive heuristic method and the GA, we can develop a heuristic method in solving the integrated model so that the run time involved can be improved. Thus, a comparison between the results of the hybrid method and the heuristic method can be made. We believe that the results of the heuristic method are more encouraging for CPU time.

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