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Experiments with LMS Algorithm

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Abstract

Objectives: This article focuses on improving the convergence rate and reducing the number of operations used to train the Least Mean Square (LMS) algorithm. **Methods/Statistical Analysis:** In this paper, two modifications are suggested to train an adaptive filter using the LMS algorithm; one is based on initialization of weights and another on early termination of the training of a sequence. **Findings:** The optimum weights of an adaptive filter are found by initializing the weights by zeros, providing several random sequences as input and updating the weights according to the error. Moreover, the weights are continuously updated for the entire sequence even if the weights have been converged. In the proposed algorithm, the weights are initialized with zeros only once for the first sequence. The optimum weights obtained for a sequence are used as initial weights for the subsequent sequence to improve the convergence rate. Further, to reduce the number of operations, the weight update process for a sequence is terminated when the error is below some prescribed threshold. **Applications/Improvements:** Results shows that by making these modifications, the rate of convergence increases and number of multiplications decrease.

Keywords: Convergence Rate, Initialization of Weights, LMS Algorithm, Multiplications, Mean Square Error, Threshold

1. Introduction

Adaptive filtering has applications in communications, radar, sonar, seismology, and biomedical engineering. The four basic classes of its applications are system identification, inverse modelling, prediction and interference cancellation[1]. The transversal filter implementation of an adaptive filter consists of a unit delay element, a multiplier and an adder. The input vector with M delay elements is multiplied with a weight vector, called the impulse response, and the output is compared with the desired response. The error so obtained is used to update the weights. There are mainly two approaches to update the weights, namely, stochastic gradient approach and least squares estimation. The respective families of algorithms are LMS family and Recursive Least Squares (RLS) family.

To train an adaptive filter, an ensemble of pilot sequences are given as input to the filter, errors are calculated and weights are adjusted accordingly. Once the optimum weights are obtained, the filter can be used in decision directed mode. This paper discusses the training of an adaptive filter using LMS algorithm. In this case, for each pilot sequence, the weights are initialized with zeros. At every iteration, the weight vector is multiplied with the input vector and the output is subtracted from the desired response. The difference, i.e. the error, is used to update the weight vector. The process continues for the entire sequence. For the next sequence, again the weights are initialized with zeros and the above process repeats. The goal is to find the weight vector which minimizes the mean square error of the ensemble. Multiplication is a costlier operation than addition. So, in the following discussion, for comparison, only multiplication is considered. Mainly, two steps of the algorithm, the output calculation and the weight update, constitute the computational cost. The conventional LMS algorithm requires 2M multiplications per iteration for a filter of order M.

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Several attempts have been made to improve the LMS algorithm. When the input vector is large, the LMS algorithm suffers from a gradient noise amplification problem. This difficulty is overcome by Normalized Least Mean-Square filter which normalizes the weight adjustment vector by the instantaneous power of the input[2,3]. Self-orthogonalizing adaptive filtering algorithm improves the convergence properties at the cost of an increase in computational complexity[4,5]. However, it requires the inversion of the correlation matrix of the input vector which is sometimes not known. The filter is adapted on a block-by-block basis in Block LMS algorithm[6,7]. It uses a more accurate estimate of the gradient vector. The accuracy of the gradient vector increases as the block size increases. But the average time constant also increases in the same proportion. So, this algorithm does not improve convergence properties of the LMS algorithm[1]. It is experimentally shown that fractional LMS algorithm has a higher convergence rate, especially for deterministic signals[8]. High quality Cardiac Signals are required for perfection in diagnosis. Some Adaptive Noise Cancellers (ANC) based on the Proportionate Normalized Least Mean Square algorithm (PNLMS) have been introduced[9] and it is shown that the ANCs based on PNLMS and its variants are useful in eliminating artifacts present in the cardiac signal.. In applications such as acoustic echo cancellation in teleconferencing, a long impulse response is required. To cope with the corresponding increase in computational complexity, frequency-domain and subband adaptive filters are useful[1,10–15]. The filter length is adjusted dynamically to improve the convergence rate[16]. Other variants of LMS algorithms are devised which varies the originally fixed step size of conventional LMS algorithm to achieve faster convergence[17–19]. Comparatively recent trend is to apply evolutionary algorithms such as Genetic Algorithm (GA), Particle Swarm Optimization (PSO), Wind Driven Optimization (WDO), etc. for equalization. An adaptive identification model based on WDO to estimate the parameters of a plant have been proposed[20] . Such a model is useful for tracking the plant dynamics. The results of implementing the model on signals with different noise ratios are compared with LMS algorithm and other evolutionary algorithms and it is concluded that WDO has a promising capability of system identification. For noisy channels, a discrete adaptive equalizer is developed in[21] which is based on Quantum behaved Particle Swarm Optimization (QPSO) technique. Authors have shown that QPSO based equalizer is more efficient compared to equalizers based on other algorithms.

2. The Proposed Method

During the training phase of LMS algorithm, several random sequences are fed as input. For each new sequence, the weights are initialized as zeros. Moreover, even if the sequence gets converged after certain iterations, the remaining samples of the sequence are also given as input to the algorithm.

In this paper, the optimal weights obtained at the end of one sequence are used as the initial weights for the subsequent sequence to improve the convergence rate. Another important modification done is that when a sequence starts converging and the absolute value of the error becomes less than a pre-decided threshold, the remaining sequence is skipped. However, to calculate ensemble average, error values are required for each sample. So, the error for the remaining samples is assumed to be the same as the error obtained at the sample where the sequence is terminated. This helps reduce the number of operations.

3. Experiments and Results

Two experiments are performed to check the validity of our assertions. Application of one step prediction is considered. The filter length is kept fixed at 10. The algorithm is trained using 500 sequences from uniform distribution on the interval $[0, 1]$. Each sequence contains 1001 samples.

The first experiment deals with our assertion that if the optimum weights obtained at the end of one sequence are used as the initial weights for the next sequence, it helps improving the convergence rate. The results are presented in the Figure 1 with step-sizes $\mu = 0.025,$ $0.05.$ In Figure 1, the weights converge after about 30 iterations, whereas if the initial weights are varied, then the convergence is achieved after about 10 iterations only. In LMS algorithm, if the step size is increased, the convergence is faster but at the cost of increased misadjustment. So, the same experiment is repeated for another step size $\mu = 0.05.$ It can be seen from Figure 1 (a-d) that the convergence is achieved in less than 10 iterations. As Figure 1 shows, in this case also, the convergence is even faster when the initial weights are varied.

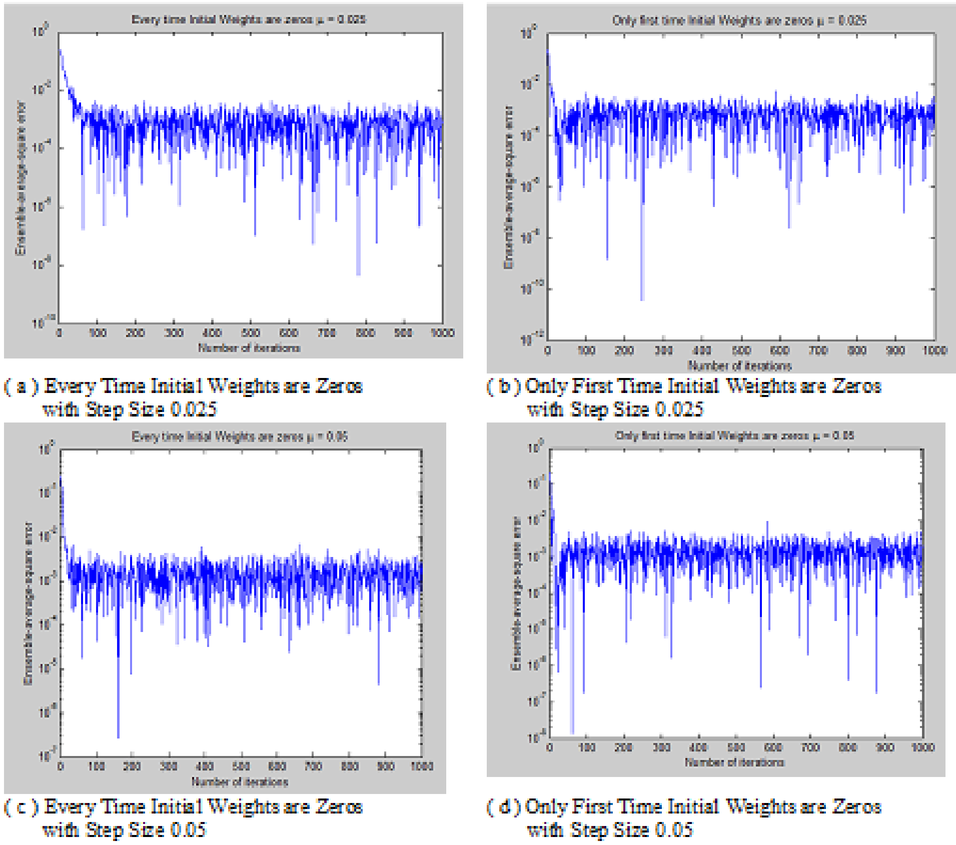


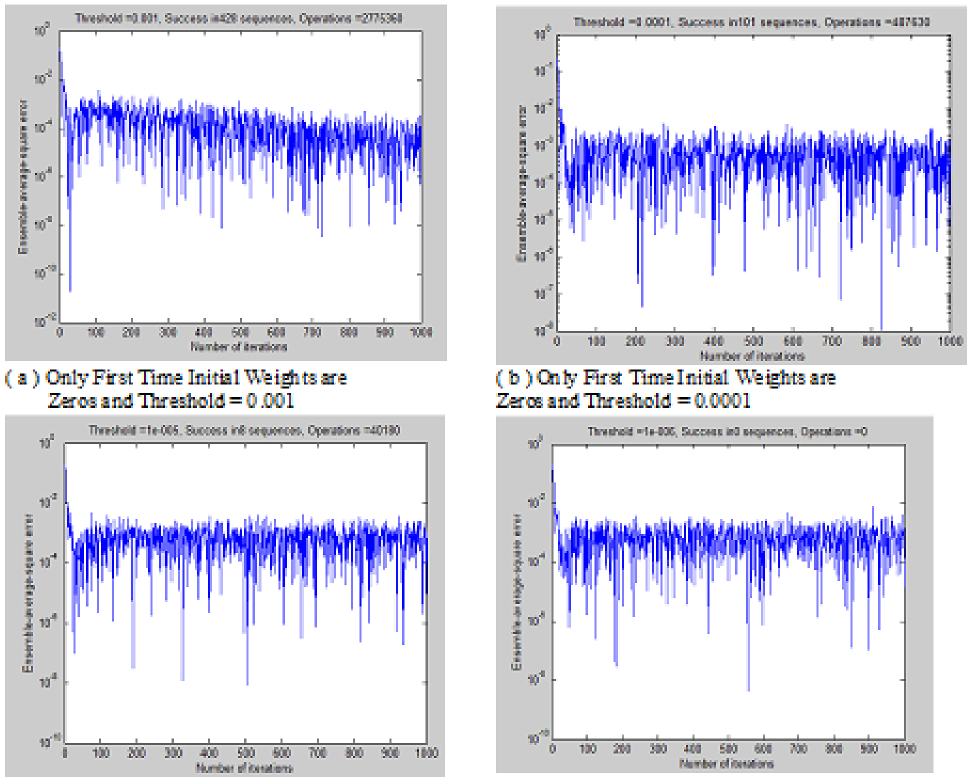
Figure 1. Effect of varying initial weights.

Another assertion regarding the early termination of a sequence if the error is below a particular threshold reduces the number of operations. There are two multiplications in the LMS algorithm, one for calculation of output and the other for weight update equation. Both operations require M multiplications. Thus, for every input sample, $2M$ multiplications are required. Suppose for a particular sequence and for a particular input vector, the magnitude of the error is below some threshold, then it is assumed that further training from that sequence is not useful. Suppose for a particular sequence of length N, the error is below the threshold at the sample n, then termination of that sequence results into savings of $2M(N-n)$ multiplications. However, to calculate MSE, error for all input samples is needed. So, for subsequent samples, the error is assumed to be the same as that of the last sample where the sequence is terminated.

Figure 2(a-d) shows the results when additionally a sequence is terminated if the magnitude of the error becomes less than the threshold at a particular sample. The title of each subfigure in Figure 2 describes the threshold used, number of early terminated sequences and the number of operations reduced due to this strategy. As an example, Figure 2 (a) shows that if the threshold is fixed at 0.001, then 428 sequences out of 500 are early terminated and the number of operations saved is 2775360. Clearly, for a stricter threshold, the corresponding advantage will be lesser. Figure 2 (b), (c) and (d) show the results for the thresholds 10^{-4}, 10^{-5} and 10^{-6}. respectively.

4. Conclusion

Two problems associated with LMS algorithm, namely, slower convergence and large number of operations required in the training phase, are addressed in this paper. It is concluded that by varying initial weights, the convergence becomes faster. Moreover, if the sequence is terminated earlier in addition, then there will also be a reduction in the number of operations. The conclusion may be tested on other kind of inputs such as inputs from Gaussian distribution or from a first order Markov chain.



(a) Only First Time Initial Weights are Zeros and Threshold = 0.001

(b) Only First Time Initial Weights are Zeros and Threshold = 0.0001

(c) Only First Time Initial Weights are Zeros and Threshold = 0.00001

(d) Only First Time Initial Weights are Zeros and Threshold = 0.000001

Figure 2. Effect of varying initial weights and early termination of sequence.

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