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Study on Binary Equivalent Decimal Edge Graceful Labeling

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Abstract

Let G (V(G), E(G)) be a graph with n vertices is said to be Binary Equivalent Decimal Edge Graceful Labeling (BEDE) graph if the vertices are assigned distinct numbers from 0,1,2,...,(n-1) such that the labels induced on edges by the values obtained using binary coding of end vertices for each edge which are distinct. This paper deals with graphs such as cycle graph, path graph and middle graph of above said graphs are BEDE graceful labeling.

Keywords: BEDE, Binary, Graceful, IBEDE, Incident, Labeling, Middle Graph

1. Introduction

This paper deals with finite, simple, connected graphs only. A Labeling of graph is an assignment of labels to the vertices or edges or both by some specific rule. Labeling plays an important role in Communication network addressing, Circuit design, Data base management etc. A useful survey on Graph labeling by J.A. Gallian (2010) can be found in[1]. To any Graph G there corresponds a $v \times \varepsilon$ matrix called incident matrix of G[3]. Let us denote the vertices of G by $v_1, v_2, \dots v_v$ and edges by $e_1, e_2, \dots e_\varepsilon$. Then the incident matrix of G is the matrix B(G)=$[b_{ij}]$ where b_{ij} is the number of times that v_i and e_j are incident. The Middle Graph M(G)[4] of a graph G is the graph whose vertex set is $V(G) \cup E(G)$ and in which two vertices are adjacent if and only if either they are adjacent edges of G or one is vertex of G and the other is an edge incident with it.

2. Binary Equivalent Decimal Edge Graceful Labeling

2.1 Incident Binary Equivalent Decimal Edge Graceful Labeling

2.1.1 Definition

Let G = (V(G), E(G)) be a graph with n vertices is said to be Incident Binary Equivalent Decimal Edge Graceful labeling (IBEDE), if f is a bijective mapping from vertices to the set of integers $\{0,1,2,\dots,(n-1)\}$ such that the induced map f^* from edge to the set of integers which is defined as

$f:$ V(G) $\rightarrow \{0,1,2,\dots,(n-1)\}$

f^*: E(G) $\rightarrow \{1,2,3,4,5,\dots\}$ and the edges are labeled with the values obtained from binary equivalent decimal coding. It is also equivalent to

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$e_k = (i, j) = 2^{n-i-1} + 2^{n-j-1}$ where $k = \{1, 2, 3, \ldots, n\}$ and i, j are finite positive integer labeled for end vertices of e_k and n is number of vertices in G.

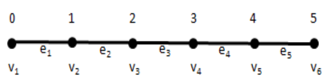
2.1.2 Example

Binary Equivalent Decimal Calculation (for the edges) for Figure 1.

$e_1 = 1x2^5 + 1x2^4 + 0x2^3 + 0x2^2 + 0x2^1 + 0x2^0 = 32 + 16 = 48$

Equivalent Calculation using formula for Figure 1.

$e_1 = (0, 1) = 48$
$e_2 = (1, 2) = 24$
$e_3 = (2, 3) = 12$
$e_4 = (3, 4) = 6$
$e_5 = (4, 5) = 3$



	0	1	2	3	4	5	
e1	1	1	0	0	0	0	48
e2	0	1	1	0	0	0	24
e3	0	0	1	1	0	0	12
e4	0	0	0	1	1	0	6
e5	0	0	0	0	1	1	3

Figure 1. Path Graph P_6 .

2.1.3 Example

Equivalent Calculation using formula for Figure 2.

$e_1 = (0, 1) = 384$
$e_2 = (1, 2) = 192$
$e_3 = (2, 3) = 96$
$e_4 = (3, 4) = 48$
$e_5 = (4, 5) = 24$
$e_6 = (5, 6) = 12$
$e_7 = (6, 7) = 6$
$e_8 = (7, 8) = 3$

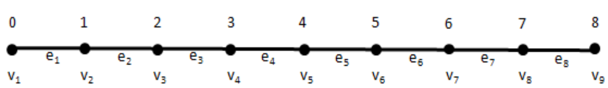


Figure 2. Path Graph P_9 .

2.1.4 Example

Equivalent Calculation using formula for Figure 3

$e_1 = (0, 1) = 24$
$e_2 = (1, 2) = 12$
$e_3 = (2, 3) = 6$
$e_4 = (3, 4) = 3$
$e_5 = (4, 5) = 17$

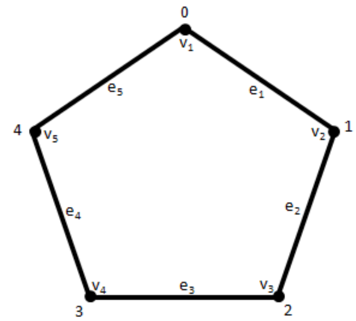


Figure 3. Cycle Graph C_5 .

2.1.5 Example

Equivalent Calculation using formula for Figure 4

$e_1 = (0, 1) = 48$
$e_2 = (1, 2) = 24$
$e_3 = (2, 3) = 12$
$e_4 = (3, 4) = 6$
$e_5 = (4, 5) = 3$
$e_6 = (0, 5) = 33$

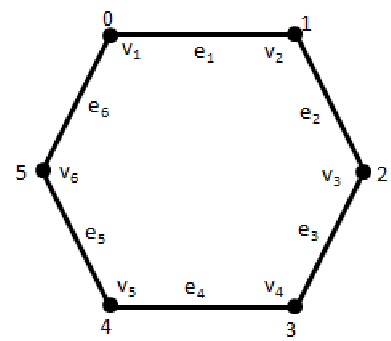


Figure 4. Cycle Graph C_6 .

2.1.6 Theorem

Every cycle graph C_n $(n \geq 3)$ is IBEDE graceful labeling graph.

Proof:

Let the vertices of C_n be $v_1, v_2, \ldots v_n$.

The labeling of vertices of C_n is as follows,

Define a bijective mapping $f :$ V(C_n) $\rightarrow \{0, 1, 2, \ldots, (n-1)\}$

$f(v_1) = 0$

$f(v_i) = f(v_{i-1}) + 1$ *for* $i = 2, 3, \ldots, n$

Now the vertices are labeled with distinct integers from 0 to n-1.

Now we define an induced function

f^*: E(C_n) {1, 2, ..., m} (m-finite)

Edges are labeled with the binary equivalent decimal coding obtained from the incident vertex. It is also equivalent to $e_k = (i, j) = 2^{n-i-1} + 2^{n-j-1}$ where $k = \{1, 2, 3, \ldots, n\}$ and i, j are finite positive integer labeled for end vertices of e_k.

This vertex labeling induces a edge labeling which are distinct.

Therefore every cycle graph C_n is IBEDE graceful labeling graph.

2.2 Incident Binary Equivalent Decimal Edge Graceful Labeling for Middle Graph

From the definition of middle graph $M(G)$[4], $M(P_n)$ and $M(C_n)$ are proved as Incident Binary Equivalent Decimal Edge Graceful Labeling.

2.2.1 Example

Equivalent Calculation using formula for Figure 5

$e_1 = (0, 1) = 384$	$e_5 = (4, 5) = 24$	$e_9 = (1, 3) = 160$
$e_2 = (1, 2) = 192$	$e_6 = (5, 6) = 12$	$e_{10} = (3, 5) = 40$
$e_3 = (2, 3) = 96$	$e_7 = (6, 7) = 6$	$e_{11} = (5, 7) = 10$
$e_4 = (3, 4) = 48$	$e_8 = (7, 8) = 3$	

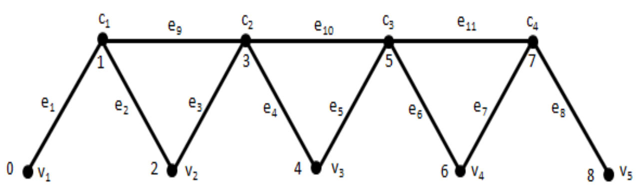


Figure 5. Middle Graph $M(P_5)$.

2.2.2 Example

Equivalent Calculation using formula for Figure 6

$e_1 = (0, 1) = 96$	$e_5 = (4, 5) = 6$
$e_2 = (1, 2) = 48$	$e_6 = (5, 6) = 3$
$e_3 = (2, 3) = 24$	$e_7 = (1, 3) = 40$
$e_4 = (3, 4) = 12$	$e_8 = (3, 5) = 10$

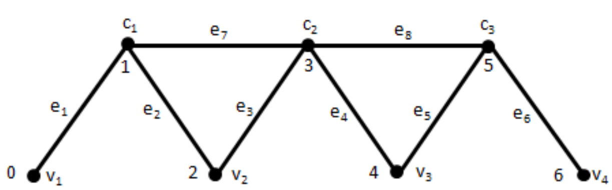


Figure 6. Middle Graph $M(P_4)$.

2.2.3 Example

Equivalent Calculation using formula for Figure 7

$e_1 = (0, 4) = 136$	$e_5 = (2, 6) = 34$	$e_9 = (4, 5) = 12$
$e_2 = (1, 4) = 72$	$e_6 = (3, 6) = 18$	$e_{10} = (5, 6) = 6$
$e_3 = (1, 5) = 68$	$e_7 = (3, 7) = 17$	$e_{11} = (6, 7) = 3$
$e_4 = (2, 5) = 36$	$e_8 = (0, 7) = 129$	$e_{12} = (4, 7) = 9$

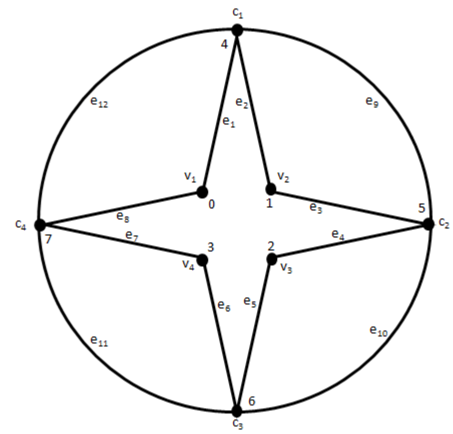


Figure 7. Middle Graph $M(C_4)$.

2.2.4 Example

Equivalent Calculation using formula for Figure 8

$e_1 = (0, 5) = 528$	$e_6 = (3, 7) = 68$	$e_{11} = (5, 6) = 24$
$e_2 = (1, 5) = 272$	$e_7 = (3, 8) = 66$	$e_{12} = (6, 7) = 12$
$e_3 = (1, 6) = 264$	$e_8 = (4, 8) = 34$	$e_{13} = (7, 8) = 6$
$e_4 = (2, 6) = 136$	$e_9 = (4, 9) = 33$	$e_{14} = (8, 9) = 3$
$e_5 = (2, 7) = 132$	$e_{10} = (0, 9) = 513$	$e_{15} = (5, 9) =17$

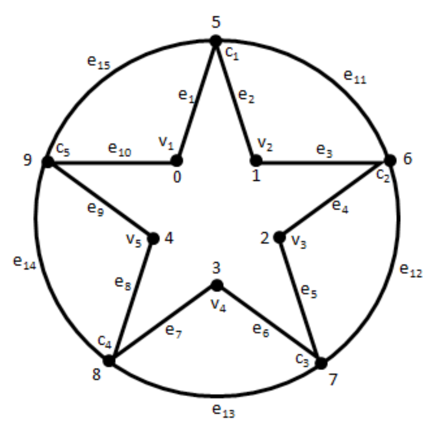


Figure 8. Middle Graph $M(C_5)$.

2.2.5 Theorem

If P_n $(n \geq 2)$ is a path graph then the middle graph $M(P_n)$ of P_n is IBEDE graceful Labeling.

Proof :

Let $V = v_1, v_2, v_3, \ldots, v_n, c_1, c_2, \ldots, c_{n-1}$ be the vertex set and $E = E_1 \cup E_2 \cup E_3$ be the edge set of middle graph $M(P_n)$ where

$E_1 = c_i v_i, 1 \le i \le n-1, E_2 = c_i v_{i+1}, 1 \le i \le n-1, E_3 = c_i c_{i+1}, 1 \le i \le n-2$

Let the total number of vertices of middle graph $M(P_n)$ be $2n-1$

Define a bijective mapping

$f : V(M(P_n)) \to \{0,1,2,\ldots,2(n-1)\}$

$f(v_i) = 2(i-1) \text{ for } 1 \le i \le n$

$f(c_j) = 2j-1, \text{ for } 1 \le j \le n-1$

Now we define an induced function

$f^{*}: E(M(P_n)) \to \{1, 2, \ldots, m\}$

Such that the edges are labeled with the values obtained from binary equivalent decimal coding or using $e_k = (i,j) = 2^{2n-i-2} + 2^{2n-j-2}$ where $k = \{1,2,3,\ldots(3n-4)\}$ and i,j are finite positive integer labeled for end vertices of e_k.

This labeling gives IBEDE graceful labeling for middle graph $M(P_n)$.

2.2.6 Theorem

If C_n $(n \ge 3)$ is a cycle graph then the middle graph $M(C_n)$ of C_n is IBEDE graceful Labeling.

Proof :

Let $V = v_1, v_2, v_3, \ldots, v_n, c_1, c_2, \ldots, c_n$ be the vertex set and $E = E_1 \cup E_2 \cup E_3 \cup E_4$ be the edge set of middle graph $M(C_n)$ where

$E_1 = c_i v_i, 1 \le i \le n, E_2 = c_i v_{i+1}, 1 \le i \le n-1, E_3 = c_i c_{i+1}, 1 \le i \le n-1$

and $E_4 = v_1, c_n, c_n, c_1$

Let the total number of vertices of middle graph $M(C_n)$ be 2n.

Define a bijective mapping

$f : V(M(C_n)) \to \{0,1,2,\ldots,(2n-1)\}$

$f(v_1) = 0$

$f(v_i) = f(v_{i-1}) + 1, i = 2, 3, \ldots n$

$f(c_1) = n$

$f(c_j) = f(c_{j-1}) + 1, \quad 2 \le j \le n$

Now we define an induced function

$f^{*}: E(M(C_n)) \to \{1, 2, \ldots, m\}$

Such that the edges are labeled with the values obtained from binary equivalent decimal coding or using $e_k = (i,j) = 2^{2n-i-1} + 2^{2n-j-1}$ where $k = \{1,2,3,\ldots 3n\}$ and i,j are finite positive integer labeled for end vertices of e_k.

This labeling gives IBEDE graceful labeling for middle graph $M(C_n)$.

3. Conclusion

In this paper some of the graphs such as cycle graph, path graph and middle graph of all the above said graphs are proved as Incident Binary Equivalent Decimal Edge Graceful Labeling with examples.

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