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A Fuzzy Approach in Finding an Optimal Solution of a Fuzzy Reliability Problem

T. Nagalakshmi[1*] and G. Uthra[2]

[1]Department of Mathematics, S.A. Engineering College. Chennai - 77, Tamil Nadu, India; nagalakshmit@saec.ac.in, nagalakshmi.1979@gmail.com
[2]P. G. and Research Department of Mathematics, Pachaiyappa's College, Chennai - 30, Tamil Nadu, India; uthragopalsamy@yahoo.com

Abstract

Objectives: The mathematical technique of optimizing a sequence of interrelated decisions over a period of time with fuzzy parameters is called Fuzzy Dynamic Programming. A typical characteristic of this proposed approach is the ambiguity and vagueness in Reliability models is effectively eliminated by FDP. **Methods/ Statistical Analysis:** In this paper, a new fuzzy approach is made to find an optimal solution of a fuzzy reliability problem. The values of Reliability are assumed as GTrFNs (Generalized Trapezoidal Fuzzy Numbers). **Findings:** Fuzzy Forward recursive equations as well as Fuzzy Backward recursive equations are framed to solve the numerical example of a fuzzy reliability problem. Now-a-days, many problems arise in the real world which involve decision making in multi stages. Sometimes, the parameters will not be known exactly due to some unmanageable components. Or else, some important data would be lost if the obtained results are taken as crisp values. Classical mathematics has proved its inadequate in handling many optimization problems that involve large number of decision variables along with inequality constraints. FDP furnishes an orderly process to determine the synthesis of decisions in order to maximize the effectiveness on the whole. Hence it has wide range of applications in the future. **Application/Improvement:** The proposed approach to solve a fuzzy reliability problem by fuzzy dynamic programming can be made use in the field of problems which involves various decision making situations. This will assist researchers currently engaged in fuzzy optimization to stimulate new areas of research in other DP models. This also gives Decision makers new tools and ideas on how to make decisions in fuzzy environment in optimization problems in current real life situations.

Keywords: Fuzzy Dynamic Programming, Fuzzy Reliability Problem, Fuzzy Recursive Equations, Generalized Trapezoidal Fuzzy Numbers, Optimal Solution

1. Introduction

Dynamic programming is mathematical technique which deals with the optimization of multistage decision problems. A multistage decision problem can be separated into a number of sequential steps and stages, which may be accomplished in one or more ways. The solution of each stage is a decision and the sequence of decisions for all the stages constitutes a decision policy. Each decision is associated with some return in the form of costs or benefits. The objective of dynamic programming is to select a decision policy, i.e., a sequence of decisions, so as to optimize the returns.

Nowadays, Dynamic Programming[1] is an effective tool in dealing with Multi stage decision making problems that arise in all human activities. But even now factors such as uncertainty, imprecision and vagueness prevail in the analysis of multistage decision making problems. Zadeh[2] has made fuzzy theory as an important tool in dealing with uncertainty and vagueness in real world problems. Bellman and Zadeh[3] applied fuzzy set theory in decision making process. Since then fuzzy concepts have been applied in all fields. The process of solving optimization problems of Dynamic programming with fuzzy parameters is termed as FDP. Many researchers were attracted by the concept of FDP and they contributed

** Author for correspondence*

enormous papers in these fields. Main developments and applications of FDP are discussed by Kacprzyk[4,5], Esogbue and Kacprzyk[6], and Esogbue et al[7]. Other excellent applications appear in the reference[8-14].

The purpose of this paper is to find an optimal solution to a fuzzy Reliability problem with uncertain and vague parameters. More specifically, the values of reliability R_i are taken as GTrFNs. A generalized trapezoidal fuzzy number $(a_1, a_2, a_3, a_4 \; ; \; w)$ is a four value judgment where w is the height of the GTrFN, $0 < w \leq 1$. The fuzzy forward and backward recursive equations are framed and the optimal solution is obtained.

The paper is formulated as follows. In the Second section, the basic preliminaries and arithmetic operations of GTrFN are given. In the Third section, a general fuzzy reliability problem is presented. In the Forth section, the forward and backward recursive equations are framed in a fuzzy environment. In the Fifth section, an illustrative example is provided and in the Sixth section, the concluding remarks are provided.

2. Definition and Preliminaries

2.1Fuzzy Set

If X is a collection of objects denoted generically by x then a fuzzy set $\tilde{A}$ in X is a set of ordered pairs:

$$\tilde{A} = \left\{ \left(x, \mu_{\tilde{A}}(x) \right) / x \in X \right\}$$

where $\mu_{\tilde{A}}(x)$ is called the membership function.

2.2 Trapezoidal Fuzzy Number(TFN):

Let $\tilde{A}$ be a TFN expressed by (a_1, a_2, a_3, a_4). It is described with the following membership function

$$\mu_{\tilde{A}}(x) = \begin{cases} 0\,, & x \leq a_1 \\ \dfrac{x - a_1}{a_2 - a_1}\,, & a_1 \leq x \leq a_2 \\ 1\,, & a_2 \leq x \leq a_3 \\ \dfrac{a_4 - x}{a_4 - a_3}\,, & a_3 \leq x \leq a_4 \\ 0\,, & x \geq a_4 \end{cases}$$

2.3 Generalized Fuzzy Number (GFN)

Let $\tilde{A} = (a_1, a_2, a_3, a_4 \; ; \; w)$ be a fuzzy number defined on the universal set R. R is the set of all real numbers. It is said to be GFN if its membership function $\mu_{\tilde{A}}(x)$ possess the following qualities:

- $\mu_{\tilde{A}}(x)$ ranges from R to $[0,1]$ is continuous.
- $\mu_{\tilde{A}}(x)$ is equal to 0 for all $x \in (-\infty, a_1] U \, [a_4, \infty)$
- $\mu_{\tilde{A}}(x)$ strictly increases on the closed interval $[a_1, a_2]$ and strictly decreases on the closed interval $[a_3, a_4]$.
- $\mu_{\tilde{A}}(x)$ is equal to w for all $x \in [a_2, a_3]$, where $0 < w \leq 1$.

2.4 Generalized Trapezoidal Fuzzy number (GTrFN):

Let $\tilde{A}$ be a GFN with the fuzzy argument $(a_1, a_2, a_3, a_4 \; ; \; w)$. It is defined as a GTrFN[15] if its function of membership is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0\,, & x \leq a_1 \\ w\dfrac{x - a_1}{a_2 - a_1}\,, & a_1 \leq x \leq a_2 \\ w\,, & a_2 \leq x \leq a_3 \\ w\dfrac{a_4 - x}{a_4 - a_3}\,, & a_3 \leq x \leq a_4 \\ 0\,, & x \geq a_4 \end{cases}$$

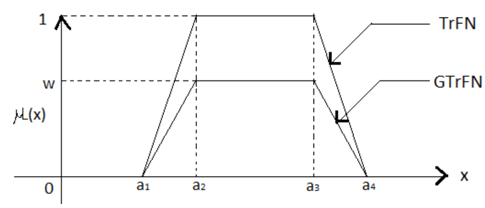

Figure 1. Comparison between membership function of TrFN and GTrFN.

2.5 Properties of GTrFN (Generalized Trapezoidal Fuzzy number)

Let $\tilde{A} = (a_1, a_2, a_3, a_4 \; ; \; w_1)$ and $\tilde{B} = (b_1, b_2, b_3, b_4 \; ; \; w_2)$ be two GTrTNs with the membership function defined above, then its properties are discussed below:

- **Equality of two GTrFNs**

$\tilde{A} = \tilde{B}$ if and only if

$a_1 = b_1, a_2 = b_2, a_3 = b_3, a_4 = b_4 and w_1 = w_2$

- **Addition of two GTrFNs**

$\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4 \; ; w)$

where $w = \min(w_1, w_2)$

- **Subtraction of two GTrFNs**

$\tilde{A} - \tilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1 \; ; w)$

where $w = \min(w_1, w_2)$

- **Multiplication of two GTrFNs**

$\tilde{A} * \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4 ; w)$ where
$w = \min(w_1, w_2)$

$\tilde{A} \div \tilde{B} = \left(\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1} ; w\right)$ where
$w = \min(w_1, w_2)$

- **Scalar Multiplication of GTrFN**

$k\tilde{A} = (ka_1, ka_2, ka_3, ka_4 ; w)$ where
$w = \min(w_1, w_2)$ if $k \geq 0$
$k\tilde{A} = (ka_4, ka_3, ka_2, ka_1 ; w)$ where
$w = \min(w_1, w_2)$ if $k < 0$

2.6 Ranking method:

Let $\tilde{A_1} = (a_1, b_1, c_1, d_1; w_1)$ be the GTFN then the crisp value given by the method[16] is R $(\tilde{A_1}) = w_1 (a_1+b_1+ c_1+ d_1)/4$.

Section-3

3.1 Fuzzy Reliability Problem

The linear programming problem of Fuzzy Reliability becomes

Maximize $Z = \tilde{R}_1(k_1) * \tilde{R}_2(k_2) * \tilde{R}_3(k_3) * \ldots \tilde{R}_N(k_N)$

Subject to the constraints,

$k_1 + k + k_3 + \cdots . k_N = C \quad and \quad k_j \geq 0 \ , \quad j = 1, 2, 3, \ldots . N$

The basic elements of the DP model are (1) stages, (2) states at each stage and (3) alternatives involving decision at every stage. FRP (Fuzzy Reliability Problem)is solved by Fuzzy Forward or Fuzzy backward recursive equations. In fuzzy forward recursion, the computations start at first stage and ends at the last stage and vice-versa in fuzzy backward recursion.

4. Fuzzy Recursive Equations

4.1 Fuzzy Forward Recursive Equations

The DP model consist of

- C be the total Capital available for the design of the device.
- Let x_j be the capital allocated to the stage j where j = 1,2,……N.
- Let $\mathbf{k}_j$ be the number of parallel units assigned to main component j.
- Let $R_j(k_j)$ be the Reliability of the j[th] component which is taken as GTrFNs.
- Let $C_j(k_j)$ be the Cost of the j[th] component (j=1, 2, 3).
- Let $\tilde{f}_j(x_j)$ be the Reliability of components (stages) given that o$\leq$ $x_j \leq$ c.
- Let $\tilde{f}_j(x_j)$ be the optimal fuzzy value of stages 1,2,….j given the state x_j.

The fuzzy forward recursive equations are thus given as

$\tilde{f}_1(x_1) = Max \{\tilde{R}_1(k_1)\}$ where $c_1(k_1) \leq x_1$

$\tilde{f}_j(x_j) = Max \left\{\tilde{R}_j(k_j) * \tilde{f}_{j-1}(x_j - c_j(k_j))\right\}$

where

$c_j(k_j) \leq x_j$ and j=1,2,3…N

4.2 Fuzzy Backward Recursive Equations

The DP model consist of

- C be the total Capital available for the design of the device.
- Let x_j be the capital allocated to the stage j where j = 1,2,……N.
- Let $\mathbf{k}_j$ be the number of parallel units assigned to main component j.
- Let $R_j(k_j)$ be the Reliability of the j[th] component which is taken as GTrFNs.
- Let $C_j(k_j)$ be the Cost of the j[th] component (j=1, 2, 3).
- Let $\tilde{f}_j(x_j)$ be the Reliability of components (stages) given that o $\leq$ $x_j \leq$ c.
- Let $\tilde{f}_j(x_j)$ be the optimal fuzzy reliability of the components j, j+1, … N for the given capital x_j.

Similarly the fuzzy backward recursive equations are given as

$\tilde{f}_N(x_N) = Max \{\tilde{R}_N(k_N)\}$ where $c_N(k_N) \leq x_N$
$\tilde{f}_j(x_j) = Max \left\{\tilde{R}_j(k_j) * \tilde{f}_{j+1}(x_j - c_j(k_j))\right\}$
where
$c_j(k_j) \leq x_j$ and j=1,2…N-1

Where $\tilde{f}_j(x_j)$ is the optimal fuzzy value of stages j, j+1,……..N given the state x_j.

5. Illustrative example

The design of an electronic device which consists of 3 main components is considered. We can arrange the components in a series in such a way that its failure will cause the entire device to fail. We can install a stand by device in every component to improve the reliability (the probability of no failure). Three units can be included in parallel for each component. For the design of the device, we can make use of the total capital available which is \$10,000. Let $R_j(k_j)$ be the Reliability of the j[th] component which is taken as GTrFNs. Let $C_j(k_j)$ be the Cost of the component j where j=1, 2, 3. The reliability and cost values are given in the Table 1. Let k_j be the parallel units allocated to the stage **j**. The determination of the number of parallel units k_j in component j that will maximize the Reliability of the device without exceeding the allocated capital is carried out in this proposed approach.

Solution by fuzzy backward recursive equations

The elements of the FDP model are given in the Table-2.

Stage 3:

$\tilde{f}_3(x_3) = Max\ \{\tilde{R}_3(k_3)\}$ where $c_3(k_3) \leq x_3$

Stage 2:

$\tilde{f}_2(x_2) = Max\ \{\tilde{R}_2(k_2) * \tilde{f}_3(x_2 - c_2(k_2))\}$ where $c_2(k_2) \leq x_2$

Stage 1:

$\tilde{f}_1(x_1) = Max\ \{\tilde{R}_1(k_1) * \tilde{f}_2(x_1 - c_1(k_1))\}$ where $c_1(k_1) \leq x_1$

The optimal (total) value is (0.12,0.21,0.336,0.504;0.1) and the optimal solution is attained from the following table.

$(k_1^*, k_2^*, k_3^*) = (2, 1, 3)$.

The optimal combination is (2, 1, 3) with fuzzy reliability (0.12,0.21,0.336,0.504;0.1)

Using the ranking of GTrFN Total value is given by

$$R[(0.12, 0.21, 0.336, 0.504;0.1)] = 0.1\left(\frac{0.12+0.21+0.336+0.504}{4}\right) = 0.0293$$

Table 1. Reliability and Cost of the Components

k_j	Component 1		Component 2		Component 3	
	R_1	C_1	R_2	C_2	R_3	C_3
1	(0.3,0.4,0.5,0.6;0.1)	1	(0.4,0.5,0.6,0.7;0.1)	3	(0.2,0.3,0.4,0.5;0.1)	2
2	(0.5,0.6,0.7,0.8;0.1)	2	0.5,0.6,0.7,0.8;0.1)	5	(0.4,0.5,0.6,0.7;0.1)	4
3	(0.6,0.7,0.8,0.9;0.1)	3	(0.6,0.7,0.8,0.9;0.1)	6	(0.6,0.7,0.8,0.9;0.1)	5

Table 2. Elements of FDP Model

Stages	States	Alternatives
Stage 3 is Component 3	x_3 = Capital Amount allocated to stage 1	k_1 = 1, 2, 3.
Stage 2 is Component 2	x_2 = Capital Amount allocated to stages 2 and 3	k_2= 1, 2, 3.
Stage 1 is Component 1	x_1= Capital Amount allocated to stages 1, 2 and 3.	k_3 = 1, 2,3.

Table 3. Stage-3

x_3	k_3= 1	k_3 = 2	k_3 = 3	$\tilde{f}_3(x_3)$	k_3^*
2	(0.2,0.3,0.4,0.5;0.1)	-	-	(0.2,0.3,0.4,0.5;0.1)	1
3	(0.2,0.3,0.4,0.5;0.1)	-	-	(0.2,0.3,0.4,0.5;0.1)	1
4	(0.2,0.3,0.4,0.5;0.1)	(0.4,0.5,0.6,0.7;0.1)	-	(0.4,0.5,0.6,0.7;0.1)	2
5	(0.2,0.3,0.4,0.5;0.1)	(0.4,0.5,0.6,0.7;0.1)	(0.6,0.7,0.8,0.9;0.1)	(0.6,0.7,0.8,0.9;0.1)	3
6	(0.2,0.3,0.4,0.5;0.1)	(0.4,0.5,0.6,0.7;0.1)	(0.6,0.7,0.8,0.9;0.1)	(0.6,0.7,0.8,0.9;0.1)	3

Table 4. Stage-2

x_2	k_2 = 1	k_2 = 2	k_2 = 3	$\tilde{f}_2(x_2)$	k_2^*
5	(0.08,0.15,0.24,0.35;0.1)	-	-	(0.08,0.15,0.24,0.35;0.1)	1
6	(0.08,0.15,0.24,0.35;0.1)	-	-	(0.08,0.15,0.24,0.35;0.1)	1
7	(0.16,0.25,0.36,0.49;0.1)	(0.1,0.18,0.28,0.4;0.1)	-	(0.16,0.25,0.36,0.49;0.1)	1
8	(0.24,0.35,0.48,0.63;0.1)	(0.1,0.18,0.28,0.4;0.1)	(0.12,0.21,0.32,0.45;0.1)	(0.24,0.35,0.48,0.63;0.1)	1
9	(0.24,0.35,0.48,0.63;0.1)	(0.2,0.3,0.42,0.56;0.1)	(0.12,0.21,0.32,0.45;0.1)	(0.24,0.35,0.48,0.63;0.1)	1

Table 5. Stage-1

x_1	$k_1 = 1$	$k_1 = 2$	$k_1 = 3$	$\tilde{f}_1(x_1)$	k_1^*
10	(0.072,0.14,0.24,0.378;0.1)	(0.12,0.21,0.336,0.504;0.1)	(0.096,0.175,0.288,0.441;0.1)	(0.12,0.21,0.336,0.504;0.1)	2

Table 6. Optimal Solution

x_1	k_1^*	$x_2 = x_1 - c_1(k_1^*)$	k_2^*	$x_3 = x_2 - c_2(k_2^*)$	k_3^*
10	2	10 – 2 = 8	1	8 – 3 = 5	3

6. Conclusion

In this paper, a new technique has been proposed for solving fuzzy reliability problem in which the values of Reliability are taken as GTrFNs. The solution is obtained by using the fuzzy backward recursive equations which is an optimal solution. Many problems arising in the present scenario involve multistage and sequential decision making. Sometimes, the parameters may be fuzzy due to some unmanageable components. If the values are crisp, then we may lose some useful data. Fuzzy Dynamic Programming is a significant optimization procedure that is specifically useful to many complicated problems which involves a order of interrelated decisions in a fuzzy circumstances in the future. Therefore, it has wide range of applications.

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