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Earth Pressure behind a Retaining Wall under Linearly Varying Geotechnical Parameters

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Abstract

Objectives: The present study discuss estimation of active and passive earth pressure behind a smooth vertical retaining wall with horizontal backfill under linearly varying geotechnical properties, i.e., cohesion (c), angle of internal friction (φ) and unit weight (γ) of backfill vary linearly with depth.It is noted that the earth pressure behind the retaining wall is nonlinear and its variation can be established through regression analysis that follows 2nd order polynomial in the form of $p_a = -az^2+bz+c$; where z is the depth below the backfill surface, a, b & c are constants. **Methods/Statistical Analysis**: Regression analysis incorporating least square error approach option is available in Microsoft excel in data analysis tool pack. The total amount of thrust (active or passive) estimated through non-linear distribution of earth pressure behind retaining wall is compared with the corresponding values computed through most widely used Rankine's earth pressure theory in which average values of geotechnical parameters are taken. **Findings**: It is noted that there is 5-7% reduction in the active earth thrust although not much reduction is achieved in the moment. In passive case, the increase in passive thrust is almost 20%. **Application/Improvements:** Consideration of non-linear earth pressure distribution is useful in situations where base width of the retaining wall is controlled with respect to base shear. It will ultimately lead to reduction in the total cost of construction of the retaining wall and will help in achieving economy when the length of the retaining wall is large.

Keywords: Earth Pressure, Geotechnical, Regression,Retaining Wall

1. Introduction

Retaining wall is used to provide support to vertical or nearly vertical slopes of soil. In most of the cases, these retaining walls are provided against the existing natural ground or slopes subjected to active or passive condition. Phoon and Kulhawy[1] reported that geotechnical parameters of natural soil with depth is not always constant and it varies linearly or nolinearly with depth due to overburden and stress history. Estimation of total thrust (active or passive) is one of the critical aspects in the geometric design of retaining wall. Rankine's theory[2], where soil is assumed to follow Mohr-Coulomb failure criterion, is commonly used to calculate the earth pressure on retaining wall. Initially, Rankine assumed the backfill soil as dry, cohesionless along with other assumptions, such as, (i) back of the wall is smooth (ii) backfill soil is in the state of plastic equilibrium (iii) backfill surface is horizontal and back of the wall is vertical. In due course, several of the Rankine's assumptions were reconsidered and modifications in the original theory were proposed, such as, consideration of inclined backfill, inclined back of the wall, cohesive-frictional backfill etc. Detailed discussions on the topic are available in standard texts[3,4]. Later, other

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concepts and theories, such as, arching effect[5], nonlinear failure criteria[6] and finite element method[7] were also proposed to define the amount of lateral earth pressure behind a retaining wall. A brief review of the recent work done in the field of earth pressure theory is presented herewith.

Handy[8] replaced linear Mohr-Coulomb failure envelop by a catenary arch describing the path of minor principal stress. Soil arching effect due to rough wall was considered in two stages, i.e., triangular (exceeding those from Rankine and Coulomb theory) and curvilinear distribution (due to arching action) and the curved pressure distribution was used to obtain the shear and bending moment values. Williams[9] described, theoretically, the curve of minor principal stress behind a retaining wall catenary as well as circular. In the past, many experimental studies have confirmed that the active earth pressure distribution on a wall is non-linear. Wang[10] derived a differential equation of 1st order to estimate the active earth pressure on a retaining wall based on the Coulomb's concept and with the assumption that the sliding wedge that exerts pressure on a retaining wall passes through the bottom edge of the wall. Analytical expression proposed was verified through experimentation. Zhu *et al*[11] presented a numerical procedure with the help of limit equilibrium approach and concepts of critical slip field (CSF) for obtaining the distribution of earth pressure on retaining wall for homogeneous, cohesionless and sloping ground surface carrying with uniform surcharge. The results obtained through computer programming were verified with closed form solutions and its extension was suggested for non-homogeneous c-φ soil with complicated loading conditions. Goel and Patra[12] studied the effect of shapes of critical failure surface and arching of soils on lateral earth pressure. According to them shape of failure surface plays a very important role in calculating the values of lateral stress and the height (from the base of wall) at which they act. The results were also compared with field results and design charts were proposed for modified active earth pressure coefficient. Khosravi *et al*[13] confirmed the arching effect behind the retaining walls. They studied the behavior of granular soil from the *at rest* condition to the active condition experimentally using a set of precise miniature pressure cells and Particle Image Velocimetry (PIV). The results obtained from study were compared with the existing arch-action based theories.

In the estimation of earth pressure, representative values of geotechnical parameters of backfill soil, such as, unit weight (γ), cohesion (c) and angle of internal friction (φ) are obtained from soil tests and they are assumed as uniformly constant. It has been very well reported that geotechnical parameters varies linearly with depth due to overburden and past stress history. Hence, consideration of linear variation of geotechnical parameter provides a more realistic treatment to the *in situ* condition and also helps in achieving economy in designing as well as in the construction of retaining wall.

In the present study, expression for nonlinear variation of active or passive earth pressure (2nd order polynomial in the form of $-az^2+bz+c$; z = depth below backfill surface) is established when the geotechnical parameters, c (cohesion), φ (angle of internal friction) and γ (unit weight of the soil) are assumed to vary linearly with depth. The top of the backfill surface is taken as horizontal. Back of the wall is considered to be smooth and vertical. Effect of water table is neglected. Soil follows Mohr-Coulomb failure criteria and a state of plastic equilibrium is achieved in the backfill soil. Backfill soil is first considered as cohesionless and then the procedure is extended for cohesive-frictional soil to further consider depth of tension crack and unsupported cut as there phenomenon are relevant to cohesive-frictional backfill material under active condition. Similar calculation are performed for passive case considering cohesionless and cohesive-frictional backfill and results are discussed in the light of commonly used Rankine's theory of earth pressure.

2. Part A - Active Case

2.1 Cohesionless Soil

A typical case of a retaining wall of height H and variation of geotechnical parameters φ (angle of internal friction) and γ (unit weight) can be considered. It is noted that the values of φ and γ at the top (ϕ_0, γ_0) as well as at the bottom (φ_H, γ_H) of backfill soil are known through any subsurface exploration technique or procedure. The back of the retaining wall is assumed to be vertical and smooth. Also, backfill surface is considered as horizontal. Assuming that the backfill soil is in the state of plastic equilibrium and the backfill soil follows Mohr-Coulomb failure criterion, the active earth pressure coefficient (K_A) can be obtained as:

$$K_A = \frac{(1-\sin\theta)}{(1+\sin\theta)} \quad [1]$$

Similarly, the lateral active earth pressure (p_A) at any depth (z) from the backfill surface is then estimated as

$$p_A = K_A \gamma z \quad [2]$$

From equation (2), it is clear that the value of p_A will vary with depth as the φ and γ both varies with depth. Considering a typical case of retaining wall with the following parameters, i.e., H = 10m, $\phi_0 = 8°$, $\varphi_H = 35°$, γ_0=12.5 kN/m³, γ_H=26 kN/m³; the nonlinear variation of p_A with depth is obtained as shown in Figure 1. It can be noted that variation of p_A with depth is nonlinear and using regression analysis, option available in Microsoft excel, it was found that 2nd order polynomial in the form pA = $-az^2+bz$, as given by equation (3), best fit the trend with coefficient of determination (R^2) close to 1.0.

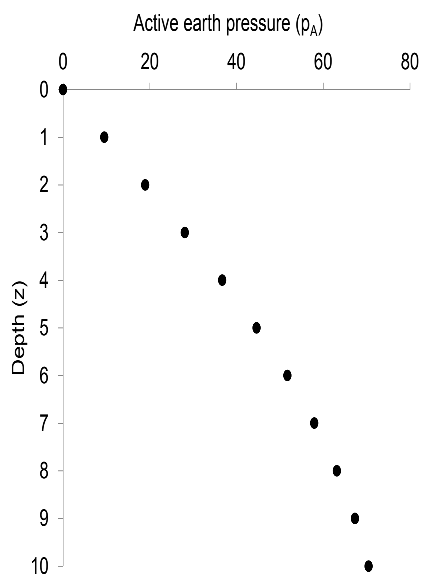

Figure 1. Non-linear variation of active earth pressure with depth.

$$p_A = -0.3513z^2 + 10.638z \quad ;$$

R2 (regression coefficient) = 0.9994 [3]

2.1.1 *Estimation of Total Active Thrust (P_A)*

A general form of equation obtained with regression analysis for active earth pressure is $p_A = -az^2+bz$. The amount of active thrust on the wall is obtained by computing the area below the earth pressure distribution line as shown in Figure 2. The force dP_A acting on the wall is obtained from the area of strip $p_A dz$; where $p_A = az^2+bz$. Total active thrust (P_A) can be calculated by integration with limit 0 to H as below:

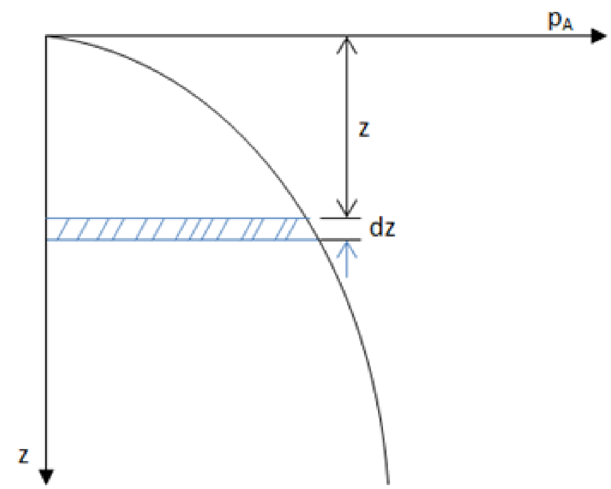

Figure 2. Estimation of total active thrust for non-linear variation of p_A.

$$\int_0^{P_A} dP_A = \int_0^H \left((-a)z^2 + bz\right)dz$$

$$P_A = {}_0^H\left[-a\left(\frac{z^3}{3}\right)_0^H + b\left(\frac{z^2}{2}\right)_0^H\right] + C$$

where C is constant of integration whose value is zero because $P_A = 0$ at $z = 0$.

$$P_A = -\frac{(a)H^3}{3} + \frac{(b)H^2}{2} \quad [4]$$

Line of action of P_A from the base of the wall $(\bar{X})$ is estimated as

$$\bar{X} = H - \frac{\int_0^H (P_A z)\, dz}{\int_0^H p_A\, dz}$$

$$\bar{X} = H - \frac{\left[-a\frac{z^4}{4} + b\frac{z^3}{3}\right]_0^H}{\left[-a\frac{z^3}{3} + b\frac{z^2}{2}\right]_0^H}$$

$$\bar{X} = \mathbf{H} - \frac{\left[-a\frac{H^4}{4} + b\frac{H^3}{3}\right]}{\left[-a\frac{H^3}{3} + b\frac{H^2}{2}\right]} \qquad [5]$$

The moment (M) acting on the wall

$$M = P_A \times \bar{X} \qquad [6]$$

2.1.2 Cohesive Frictional (c-φ) soil

Similarly, for cohesive –fractional backfill, geotechnical parameters c, φ and γ are assumed as linearly varying with values at top (χ_o, ϕ_o and γ_o) and bottom (c_H, φ_H, γ_H) of the back fill are known. The expression for active earth pressure behind a retaining wall in the form of $p_A=az^2+bz+c$ is obtained through regression analysis.

For the same problem, the total active thrust from Rankine's theory is computed by using the following equation (7):

$$P_A = \frac{1}{2} K_A \gamma H^2 \qquad [7]$$

where, K_A is obtained from equation (1). Rankine assumed linear variation of p_A with depth (z) which is justified when the geotechnical parameters φ and γ are assumed as uniformly constant. Determination of representative values of φ and γ is a daunting task in situations where these parameters vary linearly. Generally, average values of φ and γ are considered for estimating the total active thrust on the wall. It is also imperative that the total active thrust (P_A) acts at height H/3 from the base of the wall.

Table 1 shows comparison of results for different values of height of retaining wall. In this study, the input geotechnical parameters are taken as $\phi_0 = 8°$, $\varphi_H = 35°$, γ_0=12.5 kN/m^3, γ_H=26 kN/m^3. Figure 3 shows nonlinear active earth pressure distribution for different heights of retaining wall along with 2nd order polynomial (regression) equation and coefficient of determination values that best fit the data. From Table 1, it can be noted that there is 6.7% reduction in the total active thrust when nonlinear variation in p_A is taken into consideration. Not much difference is observed in the moment (M) when the results of the present study are compared with Rankine's theory.

Table 2 shows the comparison of results (active earth thrust and moment) for H= 10 m with due consideration of different rate of variation of angle of internal friction (φ) and unit weight (γ). Figure 4 shows nonlinear active earth

Table 1. Comparison of results for different height of retaining walls

Height (H)	Rankine's Theory			Present Study			% (Reduction) In P_A
	P_A(kN)	$\bar{X}$ (m)	M(kN-m)	P_A (kN)	$\bar{X}$ (m)	M(kN-m)	
2.5	24.741	0.83	20.617	23.073	0.89	20.535	6.7%
5.0	98.96	1.67	165.93	92.292	1.79	164.93	6.7%
7.5	222.67	2.5	556.67	207.66	2.68	556.64	6.7%
10	395.85	3.33	1319.5	369.143	3.57	1319.35	6.7%

Table 2. Comparison of results for different rate of variation of φ and γ (H = 10 m)

φ (°)	γ (kN/m^3)	Rankine's Theory			Present Study			% Red. (P_A)
		P_A (kN)	$\bar{X}$ (m)	M (kN-m)	P_A (kN)	$\bar{X}$ (m)	M (kN-m)	
8^0-25^0	12.5-21	466.99	3.33	1555.08	453.38	3.43	1555.11	2.91%
8^0-30^0	12.5-23.5	457.92	3.33	1524.87	435.82	3.50	1525.37	4.83%
8^0-35^0	12.5-26	446.22	3.33	1485.89	414.89	3.58	1485.41	7.01%
22°-38°	12.5-18.5	258.07	3.33	859.37	245.62	3.51	862.13	4.82%

pressure distribution along with 2nd order polynomial (regression) equation and coefficient of determination (R^2) values that best fit the data. It can be observed that with increase in the rate of variation of geotechnical parameters (φ&γ), there is increased reduction in the total active thrust. Hence, consideration of nonlinear variation of active earth pressure will be beneficial when the height of the retaining wall is large where a reduction of 7% or more may be expected. It is noted that not much variation in expected in the applied moment (M) values. The last row of Table 2 indicates more realistic treatment of geotechnical parameters and it can be observed that a reduction of 4.82% is achieved when nonlinear variation of active earth pressure is taken into consideration. The study suggests that reduction in the active earth thrust may help in reducing the base with of the retaining wall in a situation where it is governed by base shear. This will lead to cost cutting and economy when the length of the retaining wall is large.

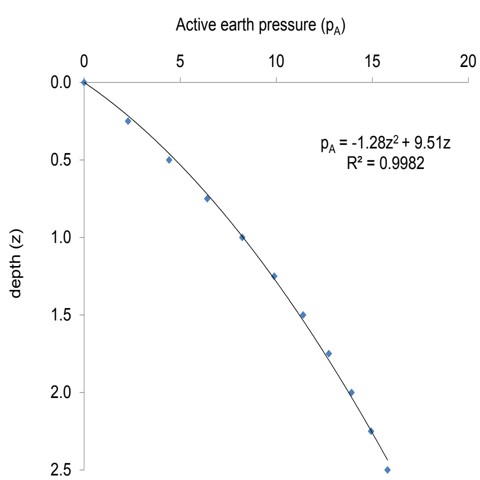

Figure 3 (a) Height of RW = 2.5 m.

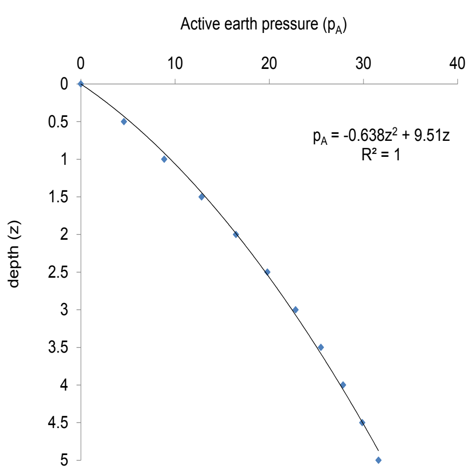

Figure 3 (b) Height of RW = 5.0 m.

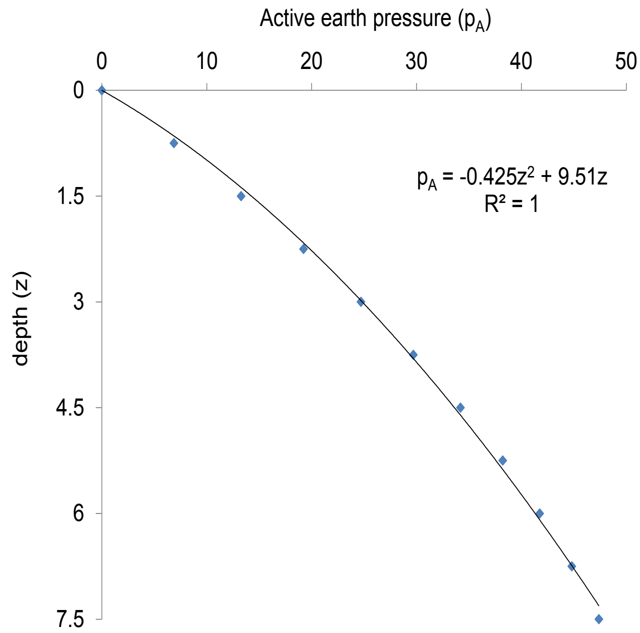

Figure 3 (c). Height of RW = 7.5 m.

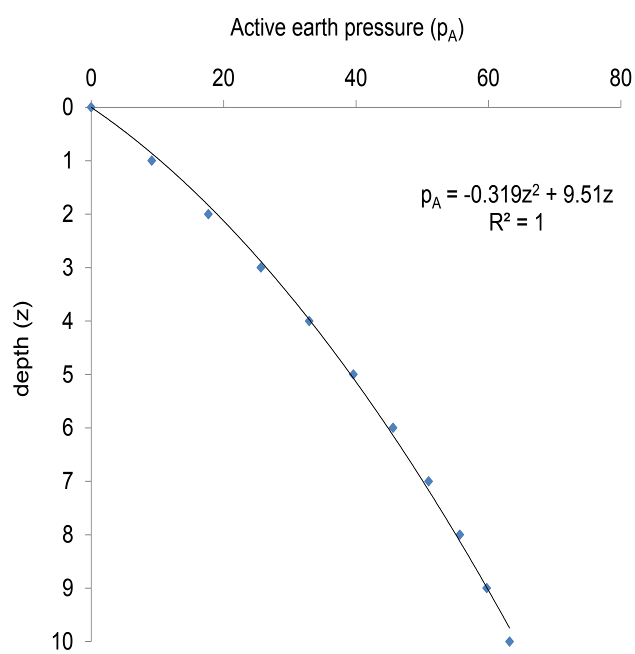

Figure 3 (d). Height of RW = 10 m.

Figure 3. Nonlinear pressure distribution (2nd order polynomial equation) for different heights.

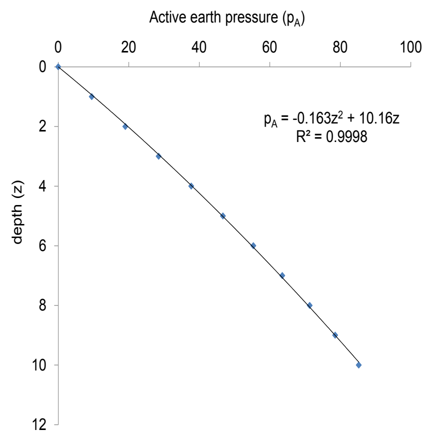

Figure 4(a). Height H = 10 m, linear variation of φ = 8°-25° and γ = 12.5-21 kN/m³.

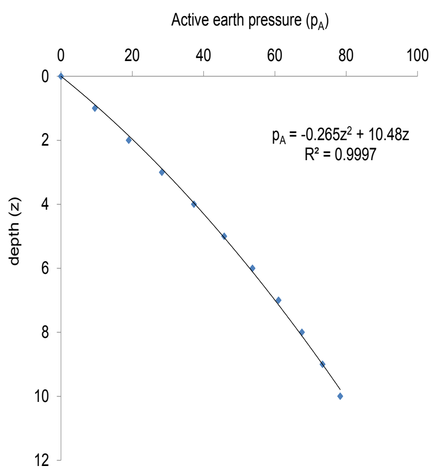

Figure 4(b). Height H = 10 m, linear variation ofφ = 8°-30° and γ = 12.5-23.5 kN/m³.

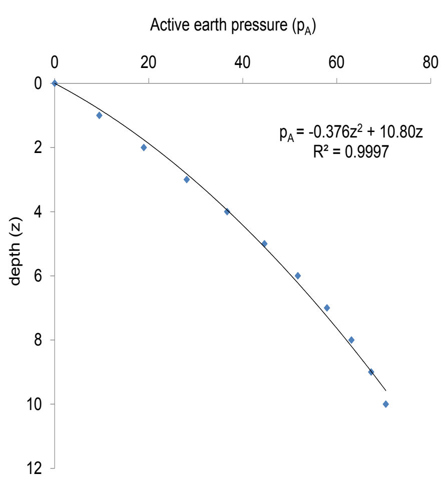

Figure 4(c). Height H = 10 m, linear variation ofφ = 8°-35° and γ = 12.5-26 kN/m³.

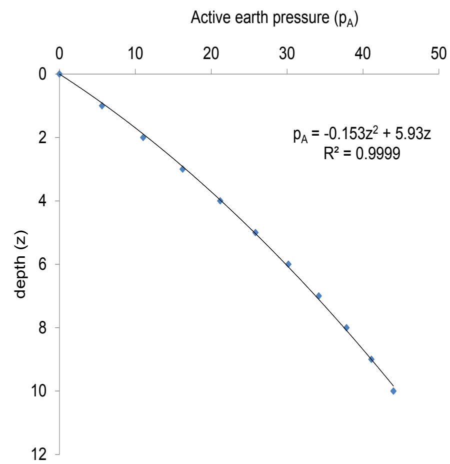

Figure 4(d). Height H = 10 m, Linear of φ = 22°-38°and γ = 12.5-18.5 kN/m³.

Figure 4. Nonlinear pressure distribution (2nd order polynomial) with different rate of variation in geotechnical properties.

2.1.3 Depth of Tension Crack

Considering a typical case in which the height of the retaining wall (H) is assumed as 10 m and geotechnical parameters c (cohesion), φ (friction angle) and γ (unit weight) are assumed to be linearly varying with depth as indicated in Table 3. The values of geotechnical parameters at top and bottom are c_o = 20 kPa and c_H = 10 kPa; ϕ_o = 25° and φ_H = 37°; γ_o = 20 kN/m³ and γ_H = 18 kN/m³ and the average values of these parameters are obtained as c_{avg} = 15 kPa, ϕ_{avg} = 31° and unit weight (γ) = 17 kN/m³. The quadratic equation for active earth pressure distribution (p_A) behind retaining wall is given as

Table 3. Typical values of geotechnical parameters and its variation with depth

Depth (m)	Cohesion (c)	Friction angle (φ)	Unit weight (γ)
0	20	25.0	16.0
1	19	26.2	16.2
2	18	27.4	16.4
3	17	28.6	16.6
4	16	29.8	16.8
5	15	31.0	17.0
6	14	32.2	17.2
7	13	33.4	17.4
8	12	34.6	17.6
9	11	35.8	17.8
10	10	37.0	18.0

$$p_A = -0.21z^2 + 7.62z - 25.42 \quad [8]$$

where z = depth below the surface of backfill soil. The distribution of active earth pressure with depth is shown in Figure 5. The depth of tension crack (z_o) is obtained where pA = 0 and it is obtained as 3.40 m. To obtain the depth of unsupported cut (z_d), the area under pressure distribution diagram between 0 to z_o and z_o to z_d is equated.

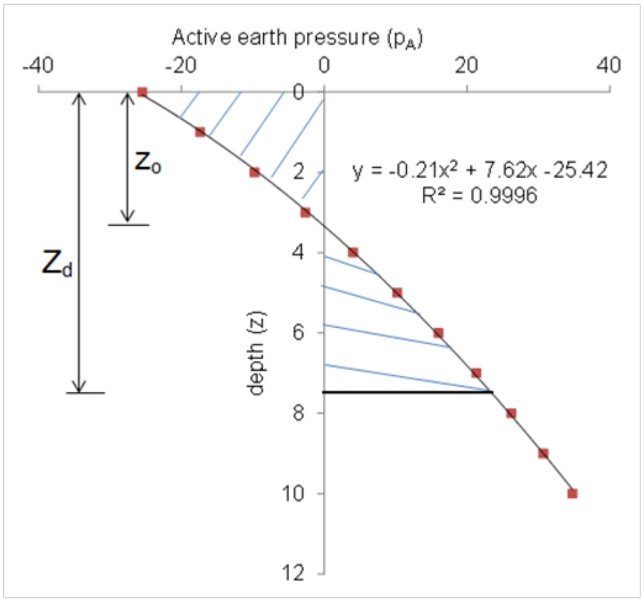

Figure 5. Nonlinear active earth pressure distribution for cohesive – frictional soil.

$$\int_0^{zo=3.4}(-0.21z^2 + 7.62z - 25.42)dz + \int_{zo=3.4}^{zd}(-0.21z^2 + 7.62z - 25.42)dz = 0$$

Solving the equation, the value of unsupported cut is obtained as 7.23 m.

The total active thrust (P_A) with consideration of tension crack is obtained as

$$P_A = \int_{zo=3.4}^{H=10}(-0.21z^2 + 7.62z - 25.42)dz$$

P_A = 125.2 kN/m length of wall

The results, i.e., depth of tension crack, depth of unsupported cut and active earth thrust (with tension crack) are compared with the Rankine's theory (with average values of geotechnical parameters) as indicated in Table 4. It can be noted that not much difference is observed in total active thrust but the values of depth of tension crack and depth of unsupported cut are different. Consideration of nonlinear active earth pressure provides larger value of depth of tension crack and depth of unsupported cut when the respective value is compared with Rankine's theory. Obtaining higher value of depth of unsupported cut with due consideration of linear variation in geotechnical parameters will ensure economy in the construction as one can be on slightly higher side while dealing with unsupported excavation. Similarly, estimation of higher value of depth of tension crack is advantageous during analysis and design of earth retaining structure under special circumstances when it is assumed that the tension crack is filled with water. One can be on the safer size by estimating the larger value of hydrostatic pressure.

Table 4. Comparison of results for cohesive friction backfill soil

	Rankin's theory	Present study
Depth of tension crack (m)	3.12 m	3.40
Depth of unsupported cut (m)	6.24 m	7.23
Active thrust (with consideration of tension crack) kN/m of wall	128.76	125.2

3. Part B: Passive Case

3.1 Cohesionless Backfill (c = 0)

In passive condition, the earth pressure distribution behind the retaining wall is computed as:

$$P_p = K_p gz \quad [9]$$

Where, passive earth pressure coefficient is com puted as

$$K_P = \frac{(1+\sin\theta)}{(1-\sin\theta)} \quad [10]$$

As followed in the active case, the passive earth pressure distribution behind a retaining wall under linearly varying geotechnical parameters is established as varying nonlinearly with depth. Through regression analysis, 2nd order polynomial is established.

Table 5. Comparison of passive earth thrust for cohesive and cohesive-frictional soil

Soil type	Rankine's Theory			Present Study			% Increase in passive force	% increase in moment
	Pp (kN)	X (m)	M (kN-m)	Pp (kN)	X (m)	M (kN-m)		
Cohesionless backfill	2325.0	3.33	7742.25	2844.83	2.72	7737.94	22.36%	≈ 0
Cohesive-frictional backfill	2844.6	3.64	10348.08	3439.15	3.12	10730.15	20.90%	3.7%

3.2 Cohesive – Frictional Backfill (c-φ soil)

Similar procedure can be followed for cohesive frictional soil and nonlinear passive earth pressure distribution can be established through regression analysis.

Table 5 provides the comparison of results of calculation of passive thrust on the wall of 10 m height. Both cohesive and cohesive-frictional backfill cases are considered. Geotechnical parameters are taken from Table 3. It can be noted that there is appreciable amount of increase in passive force (20-22%) when nonlinear distribution of passive earth pressure is considered. In case of cohesive-frictional backfill, the passive moment is also increased by 4%, slightly.

4. Conclusion

- Nonlinear variation of active earth pressure behind a retaining wall is to be considered is a situation where geotechnical properties, i.e., angle of internal friction and unit weight of cohesionless backfill varies linearly with depth.
- Through regression analysis, it is established that best fit curve is 2nd order polynomial in the form $p_A = az^2+bz+c$ with coefficient of determination (R^2) close to one.
- Consideration of nonlinear active earth pressure distribution indicates 5-7% reduction in the active earth thrust when the results are compared with the most widely used Rankine theory with averaged geotechnical properties. Although, there is no reduction in the moment calculations.
- In case of cohesive –frictional back fill, estimation of higher values of depth of unsupported cut and depth of tension crack is advantageous and geotechnical applications. Although, the total active thrust value (with tension crack) is obtained nearly same as obtained from Rankine's theory.

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