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# Free Vibration Analysis of various Viscoelastic Sandwich Beams

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## Abstract

**Background/Objectives:** Constrained Layer Damping (CLD) is an effective passive damping technique to suppress the vibrations using to analyze the vibration behaviour of viscoelastic sandwich beams. A Sandwich beam contains two face layers at top and bottom, one core layer of viscoelastic material. **Methods/Statistical Analysis:** In this paper free vibration analysis has been carried out on various viscoelastic sandwich beams likes Al-NR-Al, Al-NeR-Al, MS-NR-MS and MS-NeR-MS under four edge conditions viz., clamped-free, clamped - clamped, clamped-simply supported and simply supported-simply supported. Analytical solutions are to be carried out using Euler-Bernoulli's theory and Newton-Raphson method has to be adopted to investigate the natural frequencies of various sandwich beams. **Findings:** The beam's natural frequencies for different mode numbers with face material as aluminium and core as polyurethane rigid for analysis of fixed free sandwich beam and observed that as mode number increases natural frequencies increases due to non dimensional number increases. And found that the higher natural frequencies obtained for clamped-clamped condition of Al-NR-Al sandwich model for various edge conditions such as conditions like clamped-free, clamped - clamped, clamped -SS and SS-SS. As mode numbers increase the modal behaviour shows diverging nature because of the effect of eigenvalue. The maximum percentage variation in natural frequency from fixed-fixed and fixed-free condition is 26.35. **Improvements:** The higher natural frequencies are obtained when mild steel is used as face material. The natural frequencies were reduced when neoprene rubber was used as core material.

**Keywords:** CLD, Free Vibration, Natural Frequency, Passive Damping, Sandwich Beam, Viscoelastic

## 1. Introduction

A beam is an important slender structural member in engineering structures such as supporting members in buildings, railways, bridges, satellites, robot arms, airplane wings, etc. A viscoelastic sandwich beam is a layered fabricated structure prepared from two strong thin and solid face sheet materials joined to a less weight viscoelastic core material to create light weight and strong structural element. The development of sandwich structures will continue to be demand for the need of light weight, excellent strength under compression, providing outstanding stiffness to mass and strength to mass features. Different damping mechanism developed to suppress vibration. In[1] investigated the natural frequencies, vibration characteristics and corresponding complex loss factor of a sandwich beam. He derived an equation to find the core layer effective thickness. Required natural frequencies and corresponding loss-factors were found by him by applying boundary conditions to the loading uniform freely vibrating beam. In[2] studied the viscoelastic core sandwich beams free vibration. They identified high values calculated for beams of different material are capable for vibration damping applications. They presented that damping effort increased with core part of the beam as damping material. The applicability of the structural theory to find the natural frequencies is verified by them with experimental backup programme. In [3]used an energy technique in the primary mode of a viscoelastic core simply-supported sandwich beams to study the effect of damping. They developed a theory to find damped natural frequency and

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transverse damped vibration expressing in logarithmic decrement in a beam with known sizes and core material modulifrequency characteristics. This theory extends by them for higher discrete modes analysis. In[4] investigated flexural vibration features of non-linear sandwich beam using finite element displacement method. He studied various displacement models to examine their effect of natural frequencies on fixed-fixed non-linear sandwich beams. First three modes shear deformation effect on fixed-fixed non-linear sandwich beams is very small found by him. In[5] derived the governing equations for a dual core unsymmetrical sandwich beam vibrations by considering the both rotatory and longitudinal translator inertia effect. He used approximate methods to show the frequencies of flexural modes are significantly affected at higher frequencies. Vibration modes of four families which are divided as per the displacement ratios are obtained by him. In[6] developed two formulations for damping analysis of a partial sandwich beam using simplified exact technique. They derived a formulation for modal system loss factor using first formulation for a period of harmonic motion and Rayleigh-Ritz technique is used for analysis. First formulation gave modal system loss factors while classical and Rayleigh-Ritz techniques gave both modal system loss factors and associated resonance frequencies were obtained by them. In[7] carried to explain damping mechanism in three types of five layered partially covered beam using strain energy theory. They observed that energy dissipation of shear strain depends on constrained layer tensile stiffness. Three types of beam dynamic characteristics are confirmed by them with experimental results. In[8] presented mathematical model for both linear and transverse vibrations of a multilayered sandwich structures by random edge conditions. The Hamilton's principle and energy techniques were used by them to get equation of motion by applying random boundary condition. Matrix equation was obtained using governing equations of motion to develop system natural frequencies and system loss factors. They studied thickness effect, support position and damping properties based on structure resonance frequency and modal loss factor. In[9] presented one dimensional theory using thin viscoelastic layer to damp the beams. He derived the beam formulation based on continuity conditions at the interface using linear displacement and shear stress field. Presented theory found to be simple and effective to estimate the dynamic response of beam. In[10] implemented to fulfil continuity conditions at the interface using linear displacements and shear stresses. He found loss factors of damped viscoelastic simply supported beam using three layer sandwich beam theory. Constrained damping treatment on existing systems are examined by him are not apt when the centre layer is thick. In[11] presented a technique of free vibration analysis for three layer sandwich viscoelastic or elastic core beams by different edge conditions. Free vibration characteristic formulation was derived using Green function. They evaluated elastic or viscoelastic core layer thickness effectiveness and sandwich beam shear modulus expressing in natural frequency and loss factor. In[12] predicted dynamic behaviour of constrained viscoelastic layer sandwich beams expressing in loss factors and natural frequencies. The Rayleigh-Ritz technique was applied to sandwich viscoelastic beams by any edge conditions. They used polynomials to substitute in sandwich beams strain energy and kinetic energy in simple expression. In[13] carried symmetric three layered sandwich beams free vibration analysis with dynamic stiffness method. He used Hamilton's principle to derive governing partial differential equations for free natural vibration coupled with axial and bending deformation. Evaluate sandwich beam natural frequencies and mode shapes using Wittrick-Williams algorithm by him. In[14] presented basic concept for non-linear vibrations in viscoelastic sandwich beams. They joined one mode Galerkin's analysis with harmonic balance technique to get non-linear free vibration response was governed by two complex numbers. They extended the basic technique with finite element procedure to huge structures. They discussed influence of temperature and edge conditions on vibrations. In[15] developed a dynamic stiffness theory to investigate free vibration features of three layered sandwich beam. They derived governing equations of motion using Hamilton's principle on various viscoelastic sandwich beams. They found natural frequencies and mode shapes for a symmetric sandwich beam using Wittrick–Williams algorithm. In[16] developed dynamic stiffness model to investigate free vibration characteristics of uneven thicknesses in three layered sandwich beam using Timoshenko beam theory. They developed precise dynamic stiffness matrix for responses with corresponding amplitudes obtained by harmonically varying loads. They carried an impulse hammer test on three different sandwich beam models. In[17] estimated a time-domain formulation is called Golla–Hughes method for a mathematical model of viscoelastic material. They developed GHM by means of Laplace transformation. In[18] investigated functionally graded sandwich beam's free vibrations. He presented formulation for two dimensional elasticity problems with Galerkin method. Penalty technique was used to model sandwich beam. In[19] developed sandwich beams to compute flexural rigidity and dynamic properties. They investigated various models of multi layer cores and cells sandwich beams having different shape and orientation of holes in its cores. Results of natural frequencies, mode shapes and static deflection of sandwich beams were found by them using ANSYS and compared with analytical values. In[20] investigated free vibration of multi-layered symmetric sandwich beam having masses in harmonic nature using finite element procedure and dynamic stiffness method. They determined the closed form analytical solutions of governing equations of motion using Hamilton's principle. They developed dynamic stiffness matrix for beam by changing each mass in harmonic nature with an effective spring on edge condition. Effect of mass and stiffness of the mass in harmonic nature was examined on natural frequencies. In[21] presented a new Neural Network approach to control the vibration with active suspension system in vehicles for improving comport to passengers. They used PID controller for developing Neural Network to simulate the vibration control in active suspension system. In[22] identified that thermal impacts are intermittently disregarded in a large portion of the mechanical structures so far they must be taken into concern. They presented the impact of bi-explanatorily variation in temperature of an orthotropic rectangular plate. They derived frequency equation with a two-term deflection function by utilizing Rayleigh–Ritz method. In[23] demonstrated the reliance of the slurry pump life on the vibration parameters for building up the diagnostics calculation and express-technique to assess the working condition of the slurry pump in the pressure driven transportation framework. They were measured the pump vibration qualities with Prufnechnik convenient indicative framework utilizing independent encoders Vibscanners. They designed water powered transportation systems to acquire the vibration-based diagnostics of slurry pumps utilized for pulp-pumping to improve working existence of pumping gear.

## 2. Mathematical Modelling

A solid elastic beam with uniform cross section is considered for analysis. $w(x,t)$ represents transverse vibration equation of motion of a beam with the same cross section and homogeneous material at any point $x$ and time $t$. $f(x,t)$ represents transverse force per unit span using Euler-Bernoulli theory and the rotation of the element is insignificant. Euler-Bernoulli theory is applicable only in the case of slenderness ratio (length/height) of the beam is greater than 10. Shear deformation and rotary inertia effect are negligible.

Equation of motion for free vibration of the beam when external excitation is zero

$$\frac{\partial^4 w}{\partial x^4} + \frac{\rho A}{EI}\frac{\partial^2 w}{\partial t^2} = 0 \tag{1}$$

The solution of the Equation (1) is solved by the method of variable and separation, one is depending on the position and the other is depending on time, as follows

$$w(x,t) = X(x)T(t) \tag{2}$$

where $X(x)$ *and* $T(t)$ are independent of position $x$ and time $t$ correspondingly.

Substitute Equation (2) in Equation (1)

Variables are separated and each side of above equation should equal to a constant $\beta^4$.

$$\frac{X^{IV}}{X} = -\frac{\rho A}{EI}\frac{T^{II}}{T} = \beta^4 \tag{3}$$

If the time variable '$t$' is separated from Equation (3)

$$T^{II} + \omega^2 T = 0 \tag{4}$$

$$\beta^2 = \omega\sqrt{\frac{\rho A}{EI}} \tag{5}$$

General solution of Equation (4) is

$$T(t) = C_1 Sin\omega t + C_2 Cos\omega t \tag{6}$$

where $C_1$ and $C_2$ are constants.

Similarly if the position variable '$x$' is separated from Equation (3)

$$X^{IV} - \beta^4 X = 0 \tag{7}$$

General solution of Equation (7) is

$$w(x) = C_3 Cosh\beta_1 x + C_4 Sinh\beta_1 x + C_5 Cos\beta_2 x + C_6 Sin\beta_2 x \tag{8}$$

where $C_3$ . . . . $C_6$ are constants, *Sinh and Cosh* are hyperbolic functions.

By multiplying Equation (6) with Equation (8) upon solving and substituting initial and boundary conditions, six combined constants are obtained.

The natural frequency of the beam is calculated from Equation (5) as,

$$f_n = \frac{\omega}{2\pi} \tag{9}$$

## 3. Particular Solution

### 3.1 Clamped-Free Beam

By substituting the boundary conditions in a clamped-free viscoelastic sandwich beam, the following characteristic equation is obtained as

$$Cos\beta l\, Cosh\beta l = -1 \tag{10}$$

The Eigen values are obtained by calculating the first four roots of Equation (10) using Newton-Raphson method.

$$\beta_n l = \left(\frac{2n-1}{2}\right)\pi \quad where\ n=1,2,3....... \tag{11}$$

$$\omega_n = \left(\frac{2n-1}{2}\pi\right)^2 \frac{1}{l^2}\sqrt{\frac{EI}{\rho A}} \qquad where\ n=1,2,3....... \tag{12}$$

$$\beta_1 l = 1.8751, \beta_2 l = 4.6941, \beta_3 l = 7.8547,\ \beta_4 l = 10.9956 \tag{13}$$

### 3.2 Clamped-Clamped Beam

By substituting the boundary conditions in a clamped-clamped viscoelastic sandwich beam, the following characteristic equation is obtained as

$$Cos\beta l\, Cosh\beta l = 1 \tag{14}$$

The Eigen values are obtained by calculating the first four roots of Equation (14) using Newton-Raphson method.

$$\beta_n l = \left(\frac{2n+1}{2}\right)\pi \quad where\ n=1,2,3... \tag{15}$$

$$\omega_n = \left(\frac{2n+1}{2}\pi\right)^2 \frac{1}{l^2}\sqrt{\frac{EI}{\rho A}} \qquad where\ n=1,2,3....... \tag{16}$$

$$\beta_1 l = 4.73, \beta_2 l = 7.8532, \beta_3 l = 10.9965,\ \beta_4 l = 14.731 \tag{17}$$

### 3.3 Clamped-Simply Supported Beam

Substitution of edge conditions in clamped-simply supported viscoelastic sandwich beam is to obtain characteristic equation as

$$\tanh \beta_n l = \tan \beta_n l \tag{18}$$

The Eigen values are obtained by calculating the first four roots of Equation (18) using Newton-Raphson method.

$$\beta_n l = \left(\frac{4n+1}{4}\right)\pi \quad where\ n=1,2,3... \tag{19}$$

$$\omega_n = \left(\frac{4n+1}{4}\pi\right)^2 \frac{1}{l^2}\sqrt{\frac{EI}{\rho A}} \qquad where\ n=1,2,3....... \tag{20}$$

$$\beta_1 l = 3.9266, \beta_2 l = 7.06686, \beta_3 l = 10.212,\ \beta_4 l = 13.3518 \tag{21}$$

### 3.4 Simply Supported-Simply Supported Beam

By substituting the edge conditions in a simply supported-simply supported viscoelastic sandwich beam, the following characteristic equation is obtained as

$$Sin\beta l\, Sinh\beta l = 0 \tag{22}$$

Eigen values obtained by calculating first four roots of Equation (22) using Newton-Raphson method.

$$\beta_n l = n\,\pi \quad where\ n=1,2,3... \tag{23}$$

$$\omega_n = (n\pi)^2 \frac{1}{l^2}\sqrt{\frac{EI}{\rho A}} \qquad where\ n=1,2,3....... \tag{24}$$

$$\beta_1 l = 3.1416, \beta_2 l = 6.2832, \beta_3 l = 9.4248,\ \beta_4 l = 12.5664 \tag{25}$$

Material properties, length, width and thickness of all layers are same for a symmetric sandwich structure as shown in Figure 1.

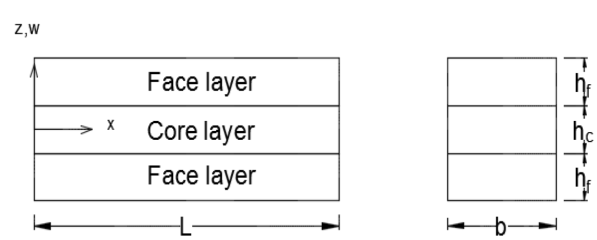


**Figure 1.** Sandwich beam.

The flexural rigidity of sandwich structure with single core[19]

$$D = 2\frac{E_f b h_f^3}{12} + \frac{E_c b h_c^3}{12} + \frac{E_f b h_f}{2}(h_f + h_c)^2 \quad (26)$$

Dynamic characteristics of a fixed free beam[19]

- The equivalent mass, $m_e = b\,(2h_f\rho_f + h_c\rho_c)$ (27)
- The equivalent stiffness, $K_e = \frac{3*D}{L^3}$ (28)
- The equivalent natural frequency,

$$f_{ne} = \frac{1}{2\pi}\left[\frac{K_e}{(.25m_e)}\right]^{0.5} \quad (29)$$

The frequency of various modes evaluated using the equation

$$f_i = \frac{(\beta_i)^2}{2\pi}\left[\frac{K_e}{(.25m_e)}\right]^{0.5} \quad (30)$$

where $(\beta_i)^2$, a constant depends on edge conditions and i = 1, 2, 3,.........., n

## 4. Results and Discussion

Free vibration analysis was carried out on various viscoelastic sandwich beams of rectangular cross section using different edge conditions like clamped-free, clamped-clamped, clamped-Simply supported and simply supported-Simply supported boundary conditions. The materials properties are considered for face and core layers are listed below.

Material properties of aluminium and polyurethane rigid foam[13]

Modulus of elasticity of face material ($E_f$) = 6.89 × $10^{10}$ N/m$^2$; modulus of elasticity ($E_c$)=0.00952 GPa; thickness of faces $h_f$ = 0.4527 mm; thickness of core $h_c$ = 12.7 mm; density of core material ($\rho_c$) = 32.8 kg/m$^3$; density of face material ($\rho_f$) = 2680 kg/m$^3$; Width of beam b = 1.0099 mm; beam length L = 0.9144 m.

Material properties of aluminium (Al) and mild steel (MS) as face layers, natural rubber (NR) and neoprene rubber (NeR) as core layers are[19]: modulus of elasticity of aluminium $(E_f)_{Al}$ = 6.89×$10^{10}$ N/m$^2$; modulus of elasticity of mild steel $(E_f)_{MS}$ = 210 GPa; modulus of elasticity of natural rubber $(E_c)_{NR}$ = 0.00154 GPa; modulus of elasticity of neoprene rubber $(E_c)_{NeR}$ = 0.0008154 GPa; thickness of faces ($h_f$) = 2mm; thickness of core $h_c$ = 2 mm; density of aluminium $(\rho_f)_{Al}$= 2680 kg/m$^3$; density of mild steel $(\rho_f)_{MS}$ = 7850 kg/m$^3$; density of natural rubber $(\rho_c)_{NR}$= 950 kg/m$^3$; density of neoprene rubber $(\rho_c)_{NeR}$= 960 kg/m$^3$; width of beam b = 30 mm; beam length L = 400 mm.

Sandwich Beam's natural frequencies for different mode numbers with face material as aluminium and core as polyurethane rigid foam is shown in Figure 2. Material properties of aluminium and polyurethane rigid foam were considered for analysis of fixed free sandwich beam. It is observed from this figure that as mode number increases natural frequencies increases, this may be because of non dimensional number increases. Figure 3 represents the mode number vs natural frequency of Al-NR-Al sandwich model for various edge conditions such as clamped-free, clamped-clamped, clamped-simply supported and simply supported-simply supported. It is observed from this figure the higher natural frequencies are obtained for fixed-fixed condition this may be because there is no effect of displacement and bending moment at fixed position. The maximum variation in natural frequency from fixed-fixed and fixed-free condition is 25.36%. The variation of natural frequencies for various boundary conditions of Al-NeR-Al at different modes is shown in Figure 4. As the mode numbers are increasing the curves are having of divergence nature because of the effect of eigenvalue. The maximum percentage variation in natural frequency from fixed-fixed and fixed-free condition is 26.35. Figure 5 represents the mode number vs natural frequency of MS-NR-MS sandwich model for various boundary conditions. It is noticed that higher natural frequencies are obtained when mild steel is used as face material. The maximum variation in natural frequency at sixth mode from fixed-fixed and fixed-free condition is 27.27 %.The variation of natural frequencies against modes for various boundary conditions of MS-NeR-MS is shown in Figure 6. The percentage variation in natural frequency is maximum from fixed-fixed and fixed-free condition is 28.11%.

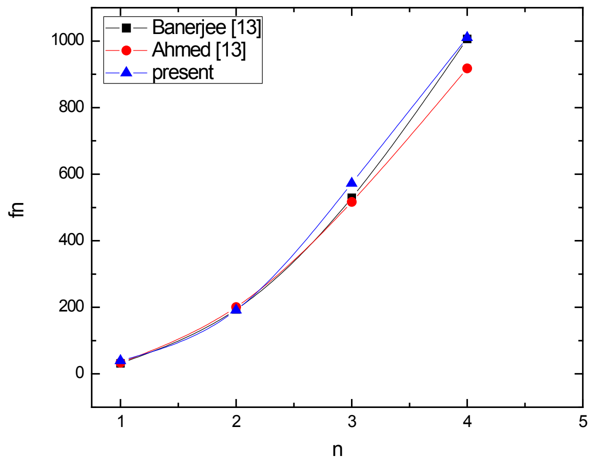


**Figure 2.** Natural frequencies of fixed free sandwich beam with face material as aluminium and core as polyurethane rigid foam.

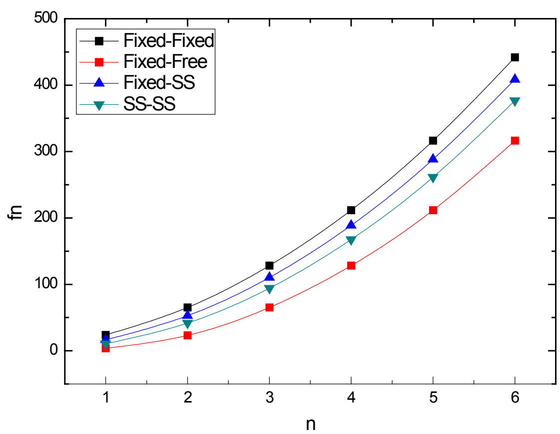


**Figure 3.** Mode number vs natural frequency of Al-NR-Al model for various boundary conditions.

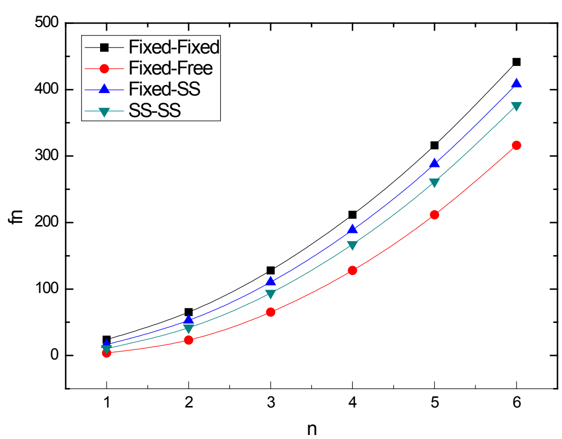


**Figure 4.** Mode number vs natural frequency of Al-NeR-Al model for various boundary conditions.

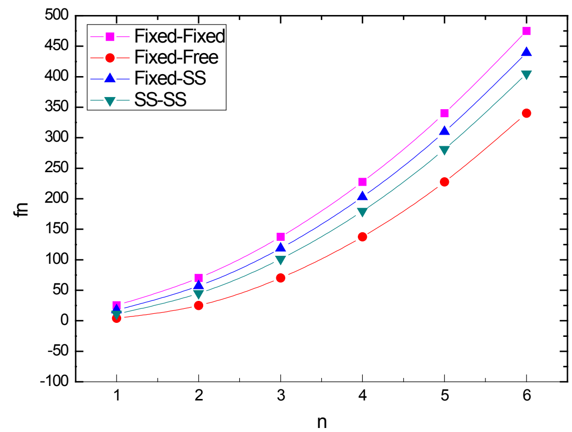


**Figure 5.** Mode number vs natural frequency of MS-NR-MS model for various boundary conditions.

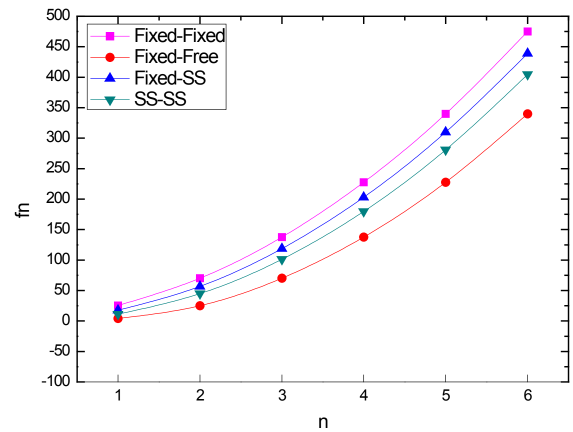


**Figure 6.** Mode number vs natural frequency of MS-NeR-MS model for various boundary conditions.

## 5. Conclusions

Present work deals with free vibration behaviour on various viscoelastic sandwich beams were studied at various edge conditions such as clamped-free, clamped - clamped, clamped-simply supported and simply supported-simply supported. Euler Bernoulli theory was considered for modelling the four viscoelastic sandwich beams with aluminium and mild steel as face material and natural rubber and neoprene rubber as core material. High natural frequencies were obtained for mild steel as face material rather than aluminium. Natural frequencies were reduced when neoprene rubber was used as core material. The analysis values were obtained using MATLAB are found in close agreement with available literature.

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## Nomenclature

L = Length of solid elastic beam / viscoelastic sandwich beam.

b = Width of solid elastic beam / viscoelastic sandwich beam.

h = Thickness of solid elastic beam / viscoelastic sandwich beam/ Total beam height ($2h_f+h_c$).

ρ = Mass density.

E = Modulus of elasticity.

u, v and w = Components of deformation in x, y, z directions respectively.

$f(x,t)$ = Transverse force per unit span.

$I$ = Area moment of inertia of the beam cross section.

$\rho A$ = Mass per unit length.

$EI$ = Flexural rigidity.

$dx$ = Beam finite element length.

ω = Frequency in rad/sec.

$\omega_n$ = Natural frequency in rad/sec.

$f_n$ = Natural frequency in Hz.

βl = Eigen value.

$D$ = Bending stiffness (or) flexural rigidity.
$E_f$ = Face material modulus of elasticity.
$E_c$ = Core material modulus of elasticity.
$h_f$ = Face material thickness.
$h_c$ = Core material thickness.
$m_e$ = Equivalent mass.
$K_e$ = Equivalent stiffness.
$f_{ne}$ = Equivalent natural frequency.
n = mode number.