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Measuring the Performance Efficiency of Hospitals: PCA – DEA Combined Model Approach

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Abstract

Objectives: The Government provides primary health care services to the poor and needy people through District Hospitals. Primary care is an ongoing care being focused on person over time that fulfils the health related requirements of people. This necessitates studying the efficient functioning of public hospitals. So the author concentrated and considered to analyse the performance of District Hospitals in the state of Tamil Nadu. **Methods/Statistical Analysis:** To attain the above objective one of the nonparametric methods namely Data Envelopment Analysis (DEA) has applied. It is a mathematical technique based on linear programming problem and it measures the relative efficiency of similar type of organizations termed as Decision Making Units (DMUs). In this study each DMU refers to the District Hospital in the state of Tamil Nadu. In DEA, normally there exists a significant correlation between the inputs and outputs. Inclusion of more number of inputs and /or outputs in DEA results in getting a more number of efficient units. Therefore, select the appropriate inputs and outputs based on the experts' advice who know their characteristics very well. A clear understanding of the characteristics of input/output variables will nullify the above deficiency and will really results with exact number of efficient units. An attempt is made to rectify the deficiency in introducing the technique viz., Principal Component Analysis. Principal Component Analysis helps the decision maker to reduce the data structure into certain principal components which are essential for identifying efficient DMUs. It is important to note that we have used the basic BCC model for the entire analysis. **Findings:** The author has considered 31 DMUs for the study using BCC model. The data results with 13 DMUs out of 31 DMUs as efficient. Subsequently with the same original data 3 PCs on input and output explains 98 percent of the total variance and this has resulted with only 3 DMUs as efficient. Then we have considered 2PCs on input and output. This has resulted with 90.31 percent variance in the case of input and 95.78 percent variance in the case of output. Normally in Principal Component analysis, if the variance lies between 80 percent to-90 percent it is judged as a meaningful one. The above computations result with 2 DMUs efficient. Finally, we have attempted to identify the efficient units with the above expected variance interval 80 percent to 90 percent. This has resulted with 90 percent variance explained with 2 input variable and one output variable with 89 percent explained variance. This also resulted with only 2 efficient DMUs. It is concluded that Principal Component Analysis plays an important role in the reduction of input output variables and helps in identifying the efficient DMUs and improves the discriminating power of DEA. **Application/Improvements:** Introduction of PCA with DEA is definitely is an improvement which helps the decision maker to apply in wider sense and to take proper managerial decision.

Keywords: Data Envelopment Analysis, Decision Making Unit, Principal Component Analysis, Principal Components

1. Introduction

Health care services and community services to the public by any Government becomes vital because it prevents many health related hazards and diseases. Therefore, it becomes the necessity of governments to develop population-oriented health care services to meet the primary health care of the people. Primary care is the provision of first contact, person-focused, ongoing care over time that meets the health-related needs of

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people, referring only those too uncommon to maintain competence, and coordinates care when people receive services at other levels of care. A primary care oriented system is important for improving health (improving effectiveness). In this contest it becomes essential to study the efficient functioning of hospitals, particularly public hospital. So this study has been taken up to identify the efficient district hospital (DMUs), find factors which causes inefficiency and fix the suitable ways to improve their efficiency. To serve the above purpose DEA is used. Data used here consist of 31 district public hospitals with 4 input and 4 output variables.

Generally, in applying DEA the problem arises, when the variables are more in number. The set with more number of variables, DEA reflects most of the DMUs efficient. So, the selection of appropriate number of variables is very much essential. For these objectives, here we combine principal component analysis with DEA.

Linear programming is a specific methodology which makes DEA more powerful among the other productivity management tools. The theoretical foundations of efficiency measurement are based on the seminal work of [1] that includes the measurement of technical and allocative efficiency using radial measures of distance to the production frontier. Developed DEA[2] to handle multiple inputs and multiple output situations. This model deals the Constant Return to Scale (CRS) situation. Modifications on DEA to handle Variable Return to Scale (VRS) categories were first described by [3].

The majority of health care researchers have analyzed the effect of regulatory changes on the efficiency of health care facilities using a Data Envelopment Analysis. States that DEA is by far the most common method for analyzing efficiency in health care sectors[4–6]. Analysed the technical efficiency of district hospitals and fixed benchmarking for inefficient hospitals[7]. Estimated the maximum possible demand that able to serve by the emergency department with a current available resources in Malaysia[8]. There is much other literature based on hospitals are[9–14].

The main aim of PCA-DEA model is data reduction and improving discriminatory power of DEA, it frequently fails when there is an excessive number of inputs and outputs when compared to Decision Making Units (DMUs). Proposed using Principal Component Analysis as a measure of weighting inputs and/or outputs and summarizing parsimoniously rather than selecting[15]. Develop Principal Component Analysis-Data Envelopment Analysis (PCA-DEA), a general statistical method used to reduce the dimensionality of the data set by expressing the variance structure of a matrix of data through a weighted linear combination of variables[16]. Each principal component (obtained from the weighted linear combination of original variables and ordered in decreasing order of percentage variance) accounts for maximal variance while remaining uncorrelated with the preceding principal components. Give a separate PCA-DEA mathematical formulation to obtain the efficiency estimates in which the principal components replace the original variables[17]. In this method, a percentage of the information is retained from each of the original variables, thus improving the discriminatory power of DEA. PCA-DEA concept is used by[18] with the aim of reducing the curse of dimensionality that occurs in DEA when there is an excessive number of inputs and outputs in relation to the number of decision making units. Extended the concept of PCA to all basic DEA models[19].

2. Objective of the Study

In this study we have the following Objectives

- To study the Relative Efficiency of 31 District hospitals.
- Application of Principal Component Analysis to reduce the number of variables without much loss of information.
- Identification and comparison of Efficient and Inefficient District Hospitals based on the Efficiency scores of DEA and PCA-DEA models.
- Benchmarking for inefficient hospitals based PCA-DEA model.
- Ranking of Decision Making Units.

3. DEA Methodology

DEA is one of the multi-factor productivity analysis model for measuring the relative efficiencies of an identical set of decision making units. In case of multiple inputs and outputs are preset, the efficiency is defined as:
Efficiency = Weighted sum of outputs / weighted sum of inputs.

Let there are n DMUs, each consumes m inputs and produce s outputs. The efficiency score of an observed DMU p is obtained by solving the following model proposed by[2]:

Mathematical Formulation

$$\begin{aligned} \text{Max} \quad & T - e^t s - e^t \sigma \\ \text{s.t} \quad & Y\lambda - s = TY^a \\ & -X\lambda - \sigma = -X^a \\ & e^t \lambda = 1 \\ & \lambda, s, \sigma \geq 0 \end{aligned} \qquad (1)$$

This maximizes the efficiency ratio for DMU p, subject to the following constraint. It restricts all the units being compared to secure not more than 100% efficiency, since the same set of *u* and v coefficients are applied.

$$st \qquad 0 \leq \frac{\sum_{k=1}^{s} v_k y_{ki}}{\sum_{j=1}^{m} u_j x_{ji}} \leq 1 \ \forall\, i$$

$v_k, u_j \geq 0 \ \forall\, k, j$

where,

$k = 1$ to s, $j = 1$ to m, $i = 1$ to n,

y_{ki} = amount of output k produced by DMU i,

x_{ji} = amount of input j utilized by DMU i,

v_k = weight given to output k, u_j = weight given to input j.

The above Mathematical programs are fractional. The above fractional program can be converted to a linear program by normalizing the denominator as shown.

$$\text{Max} \sum_{k=1}^{s} v_k y_{kp}$$

$$\text{s.t} \sum_{j=1}^{m} u_j x_{jp} = 1$$

$$\sum_{k=1}^{s} v_k y_{ki} - \sum_{j=1}^{m} u_j x_{ji} \leq 0 \quad \forall\, i$$

$v_k, u_j \geq 0 \ \forall\ k, j$

The model gives efficiency scores, weights of inputs and outputs. In general, a DMU is considered to be efficient if it obtains a score of 1 and a score of less than 1 implies that it is inefficient. CCR admits Constant Return to Scale (CRS) assumption, which is a rigid one, so the another flexible model due to author[3] which admits variable return to scale assumption (VRS) is given

$$\begin{aligned} & \text{Max } T - e^t s - e^t \sigma \\ & \text{s.t } Y\lambda - s = TY^a \\ & -X\lambda - \sigma = -X^a \, (1) \\ & e^t \lambda = 1 \\ & \lambda, s, \sigma \geq 0 \end{aligned}$$

where, λ represents a vector of DMU weights chosen by the linear program, e^t a transposed vector of ones, σ and s vectors input and output slacks respectively, X^a and Y^a are the input and output column vectors for DMU a, T represents a constant.

3.1 Principal Component Analysis

When more number of inputs and/or outputs used in DEA model it exhibits large number of efficient DMUs. Principle component analysis is used in order to aggregate all inputs and separately to all outputs. PCA can be used to aggregate inputs (and outputs) with minimum loss of information preferably keeping the ratio of number of inputs and outputs to the number of decision making units low.

A principal component analysis is concerned with explaining the variance-covariance structure of a set of variables through a few linear combinations of these variables. The main objective is data reduction, and it generally describes 80-90% of the variance in the data. Most of the population variance can be attributed to the first few components, and then they can replace the original variables with minimum loss of information.

Principal components are the uncorrelated linear combinations and ranked by their variances in descending order. Stated that the random vector $X = [X_1, X_2, \ldots X_p]$ (i.e., the original inputs and outputs chosen to be aggregated) have the correlation matrix C with eigenvalues $\lambda_1 \geq \lambda_2 \geq \ldots\ldots \lambda_p \geq 0$ and the normalized eigenvector $l_1, l_2, \ldots\ldots, l_p$[20]. Then the principal components are defined by

$$X_{PCi} = l_i^t X = l_{1i} X_1 + l_{2i} X_2 + \ldots\ldots\ldots + l_{pi} X_p, \ i = 1,2,\ldots\ldots,p$$

Generally, inputs and outputs used in DEA analysis are positive, whereas the result of the principal components can be negative. Stated that when additive model is used the data can be utilized without any change in the result[21]. Introduced a Semi Oriented Radial Measure (SORM) for modeling DEA with negative data[22]. Introduced a modified SORM model when flexible and negative data exist[23]. Author[24] stated the BCC model can be used without a change in the definition of efficient DMUs. Author[25] proves that BCC output oriented model is input translation invariant and vice versa. Presented a formulation, to adjust if we use PC scores instead of original data, and is given by [16,17],

$$\text{Max} \quad T - e^t s_0 - e^t L_y^{-1}(s_{PC}^{+} - s_{PC}^{-}) - e^t \sigma_0 - e^t L_x^{-1}(\sigma_{PC}^{+} - \sigma_{PC}^{-})$$

$$\text{s.t} \quad Y_0 \lambda - s_0 = TY_0^a,$$

$$Y_{PC} \lambda - s_{PC} = TY_{PC}^a \qquad (2)$$

$$- X_0 \lambda - \sigma_0 = - X_0^a,$$

$$-X_{PC} \lambda - \sigma_{PC} = - X_{PC}^a \qquad (3)$$

$$L_y^{-1}(s_{PC}^{+} - s_{PC}^{-}) \geq 0$$

$$L_x^{-1}(\sigma_{PC}^{+} - \sigma_{PC}^{-}) \geq 0 \qquad (4)$$

where, L_x^{-1} and L_y^{-1} represent the inverse matrix of input and output weights attained through the PCA. The slacks in the objective function ensure an equivalent problem/solution to that of the original linear program. The first and second constraints of the BCC model (1) are replaced with 2 and 3. Constraints (4) relate to the new slacks relevant only to the PC data. If all the PCs are used, we attain the same results as that achieved under the DEA formulation. If less information is used than the full information some of the explanatory power of the full data and may not achieve the exact results of the original data. Applied PCA-DEA[19] to all basic DEA models apart from additive model alone as mentioned in[16,17]. In this study PCA-DEA is applied to radial BCC model.

The disadvantages of PCA-DEA are that the data must be transformed and then, once the results are obtained, it must be transformed back to the original form in order to find the targets for improvement. Showed that the interpretations of the inefficiency rating, the targets and the efficient peers change under weights restrictions[26,27]. A similar phenomenon occurs under the PCA-DEA formulation.

4. Data Structure

The data Analyzed here is obtained from the Directorate of Medical Sciences, DMS complex, Chennai-6, for the year 2013-2014. Here 31District General Hospitals of Tamil Nadu state numbered from 1 to 31 is considered as DMU's. For each DMU four inputs are considered, they are,

- 1. Number of Hospitals 2. Number of Beds available 3. Number of Staff Nurses 4. Number of Surgeons

For each DMU the following four outputs are considered

- 1. Number of Outpatients treated 2. Number of Inpatients treated 3. Total Major Surgeries conducted 4. Total Deliveries performed.

5. Results and Discussion

5.1 Descriptive Statistics

Descriptive statistics for the district hospitals are presented in the Table 1. It indicates that in 2013-14 the hospitals, on average, outpatients treated on OP was 2217 thousands, ranging from 749 thousand to 4984 thousands with the standard deviation of 914 thousand, and also it indicates that the overall inefficiency in the operation of these facilities has increased. Similarly, on an average 62 thousand in patients were treated. The average number of major surgeries performed and total deliveries conducted were 7631 and 6241 respectively with their standard deviations 3757 and 3260.

Table 1. Descriptive statistics for district hospitals

	Mean	Std. deviation	Minimum	Maximum	N
OP1	2217337	914354.3	749148	4984101	31
OP2	61802.13	28335.97	23162	129192	31
OP3	7631.7097	3757.374	1393	17755	31
OP4	6241.9677	3260.317	1254	16517	31
IP1	9.8065	3.55358	3	18	31
IP2	780.6774	316.81429	207	1375	31
IP3	126.1935	45.67306	24	198	31
IP4	38.9355	17.03709	14	80	31

Source: DEA solver

5.2 Results of DEA and PCA-DEA

Table 2 shows the percentage of variance of all the principal components for input and output variables. For input data set first two components accounts for approximately 90% of the variance where as in output data first component itself accounts for approximately 90% of the variance. Therefore, based on 90% variance two PC's (pc1 and pc2) on input and one PC (pc1) on output explained most of the variance in the original data. Therefore, scores of first two PCs on input and first PC on output is used to estimate relative efficiency of hospitals.

Table 3 presents the efficiency scores calculated using BCC model. In the first model efficiency scores are calculated using the original data and the second model includes all input and output PCs along with the original data. It is observed that both these models reflect the same and similar number of DMUs efficient. This is due to the utilization of complete information. The next model includes 3 PCs related to input and output which explains approximately 98% of the variance reveals 3 DMUs efficient. Further it is observed that, 2 PCs in respect of input and output explains 90% and 96% of the variance respectively. These results 2 DMUs are efficient. It is found that, the result is similar to the previous one in the consideration of 2 PCs on input and one PC on output, which explains approximately 90% of the variance on both sides. It is observed that the results vary very much when complete information was not utilized. Specifically, the number of efficient units varies very much (model 3) when one PC is dropped. When we remove two PCs the result is similar (model 4) to that of model 3, except DMU number 11. So use of PCA in DEA helps us to differentiate

Table 2. PCA linear coefficients for input and output data

	Inputs L_x					Outputs L_y			
	PC1	**PC2**	**PC3**	**PC4**		**PC1**	**PC2**	**PC3**	**PC4**
No.of.hospitals	-0.402	0.891	0.205	0.0	Outpatients	-0.483	0.738	-0.453	0.131
no.of.Beds	-0.538	-0.146	-0.556	-0.617	Inpatients	-0.509	0.193	0.748	-0.379
no.of.nurses	-0.551	-0.143	-0.285	0.771	total. Surgeries	-0.499	-0.521	-0.465	-0.513
no.of.surgens	-0.496	-0.405	0.753	-0.151	total Deliveries	-0.509	-0.383	0.137	0.759
Proportion of Variance									
	74.31	15.99	7.07	2.62		88.99	6.79	2.62	1.60
Cumulative Proportion									
	74.31	90.31	97.38	100		88.99	95.78	98.40	100

Source: DEA solver

Table 3. Efficiency scores of PCA-DEA with adapted BCC model

S.NO	DMU	ORIGINAL DATA (Model 1)	ORIGINAL DATA AND 4 PC'S ON BOTH SIDE (Model 2)	3 PC'S ON BOTH SIDES (Model 3)	2 PC'S ON BOTH SIDES (Model 4)	2 PC'S IN INPUT AND 1 PC IN OUTPUT (Model 5)
1	ARIYALUR	1	1	0.814	0.814	0.814
2	COIMBATORE	1	1	0.847	0.837	0.834
3	CUDDALORE	1	1	0.767	0.767	0.767
4	DHARMAPURI	1	1	1	1	1
5	DINDIGUL	0.819	0.843	0.73	0.678	0.678
6	ERODE	0.671	0.679	0.429	0.429	0.429
7	KANCHEEPURAM	0.846	0.855	0.705	0.705	0.705
8	KANYAKUMARI	1	1	0.819	0.81	0.809
9	KARUR	0.819	0.828	0.636	0.629	0.629
10	KRISHNAGIRI	1	1	0.696	0.696	0.696
11	MADURAI	1	1	1	0.921	0.918
12	NAGAPATTINAM	0.994	0.996	0.67	0.67	0.670
13	NAMAKKAL	0.699	0.707	0.648	0.634	0.632
14	PERAMBALUR	1	1	0.699	0.695	0.695
15	PUDUKOTTAI	0.777	0.777	0.497	0.497	0.497
16	RAMANATHAPURAM	0.642	0.648	0.467	0.467	0.467
17	SALEM	0.761	0.797	0.793	0.733	0.729
18	SIVAGANGAI	0.561	0.581	0.529	0.525	0.524
19	THANJAVUR	0.768	0.768	0.643	0.643	0.643
20	THE NILGIRIS	0.682	0.682	0.239	0.239	0.239
21	THENI	0.949	0.96	0.754	0.753	0.752
22	THOOTHUKUDI	0.783	0.806	0.659	0.659	0.659
23	TIRUCHIRAPALLI	0.718	0.735	0.685	0.682	0.682
24	TIRUNELVELI	1	1	0.765	0.76	0.757
25	TIRUPPUR	0.816	0.834	0.671	0.662	0.662
26	TIRUVALLUR	1	1	0.976	0.972	0.972
27	TIRUVANNAMALAI	1	1	0.726	0.72	0.720
28	TIRUVARUR	0.803	0.806	0.638	0.638	0.638
29	VELLORE	1	1	1	1	1
30	VILLUPURAM	1	1	0.962	0.962	0.962
31	VIRUDHUNAGAR	0.742	0.742	0.526	0.526	0.526
	NO. OF EFFICIENT DMUs	**13**	**13**	**3**	**2**	**2**
	% OF EFFICIENT DMUs	**42**	**42**	**10**	**6**	**6**

Source: DEA solver

between efficient and inefficient DMUs and strengthens the discrimination power of DEA. It is interest to note that, removal of one or two PCs does not affect the results much. The above information is summarized in the Table 4.

6. Benchmarking and Ranking of Hospitals Based on Model - 4

In this study in order to improve the discriminatory power of DEA, PCA is applied to the complete set of variables (inputs and outputs separately). Here 2 principal components in input as well as in output explain 90% of the variance therefore model- 4 is taken for benchmarking. When we apply model- 4, benchmarking and ranking of hospitals based on this model can be carried out. A set of efficient DMUs act as a reference set or Peers for inefficient DMUs. For improvement, the inefficient DMUs can follow their reference set. Benchmarking for inefficient DMUs are done based on model- 4. Two DMUs (Dharmapuri and Vellore) are efficient in model- 4 and these DMUs act as a peer for remaining 29 inefficient DMUs.

Table 5 shows the reference set and corresponding weights of the inefficient Hospitals. Since only two hospitals namely hospital number 4 and 29 are efficient they act as peer set for remaining hospitals. For example, hospital number 1 is an inefficient and it has two reference hospitals 4 and 29 with weights 0.873 and 0.127 respectively. Hospital number 1 can follow any of these two hospitals for improving. Similarly, other inefficient hospitals have their reference set. Ranking the hospitals based on efficiency scores and peer counts to inefficient and efficient hospitals respectively. Therefore, first rank is given for efficient hospital which acts as a peer for maximum number of inefficient hospitals. Hospital 29 is peer for 29 inefficient hospitals, receives first rank and hospital number 4 is peer for 27 inefficient hospitals and is given second rank. Hospital number 20 stands last with the least efficient score 0.239. There may be some situations that tie of rank occurs among the DMU's. In such a situation to break up the tie the cross efficiency DEA model introduced[28], can be applied. Consequently, [29] proposed a new method for ranking extreme and non-extreme DMU's based on cross efficiency aggregate units.

Table 5. Benchmarking and ranking of hospitals

DMU No.	Reference Set	Peer weight	Peer count	Rank
1	4, 29	0.873, 0.127	0	07
2	4, 29	0.704, 0.296	0	06
3	29	1.000	0	09
4	4	1.000	27	02
5	4, 29	0.172, 0.828	0	18
6	29, 4	0.961, 0.039	0	30
7	4, 29	0.281, 0.719	0	14
8	4, 29	0.777, 0.223	0	08
9	29, 4	0.339, 0.661	0	25
10	4, 29	0.627, 0.373	0	15
11	4, 29	0.661, 0.339	0	05
12	4, 29	0.064, 0.936	0	19
13	4, 29	0.390, 0.610	0	24
14	4, 29	0.952, 0.048	0	16
15	4, 29	0.077, 0.923	0	28
16	4, 29	0.037, 0.963	0	29
17	29, 4	0.575, 0.425	0	12
18	4, 29	0.531, 0.469	0	27
19	4, 29	0.149, 0.851	0	22
20	4, 29	0.442, 0.558	0	31
21	29, 4	0.394, 0.606	0	11
22	4, 29	0.600, 0.400	0	21
23	4, 29	0.540, 0.460	0	17
24	4, 29	0.194, 0.806	0	10
25	4, 29	0.252, 0.748	0	20
26	4, 29	0.403, 0.597	0	03
27	4, 29	0.640, 0.360	0	13
28	4, 29	0.631, 0.369	0	23
29	29	1.000	29	01
30	4, 29	0.445, 0.555	0	04
31	29	1.000	0	26

Source: DEA solver

Table 4. Results of different models

Model	Number of DMUs efficient	efficient DMU number
Original data	13	1,2,3,4,8,10,11,14,24,26,27,29,30
Complete PC and original data	13	1,2,3,4,8,10,11,14,24,26,27,29,30
3PCs on input & output	3	4, 11, 29
2PCs on input & output	2	4, 29
2PCs on input & 1PC on output	2	4, 29

7. Conclusion

The present study aimed at analyzing the efficiency of District Hospitals of Tamil Nadu state for the year 2013-14. For this purpose, a non-parametric optimizing technique DEA is used. When there are more number of inputs and outputs in respect of the number of units then larger number of decision making units becomes efficient. In real life applications, it may not be possible to get less number of variables. To overcome this difficulty, an integrated PCA with DEA is used in this study. PCA is applied to all inputs and separately to all outputs. Efficiency of the hospital is found using variable return to scale output orientation by both traditional DEA and PCA-DEA. The key findings of this study are

- Out of 31 district hospitals 13 hospitals are efficient in traditional DEA model, whereas only 3 hospitals are efficient using PCA-DEA model with 3 PC on input and output case. This indicates when we have large number of variables PCA-DEA is preferable to discriminate between efficient and inefficient than traditional DEA. It may be concluded that the use of principal components can considerably improve the strength of DEA models.
- DMU number 4 and 29 are efficient in all the models whereas 11 other DMUs efficient in traditional DEA model are misclassified as efficient.
- DMU number 29 is peer for 29 inefficient hospitals it was given first rank and followed by hospital 4 second rank with 27 peers.
- Hospital 20 stands last (rank 31) with the least efficient score of 0.239 and it was preceded by hospital 6 with efficient score 0.429.

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