

# Fusion of SONAR Image using Enhanced Multi-Scale Transform and Sparse Representation Method

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## Abstract

**Objective:** The main goal of this paper is to enhance fusion of sonar image thereby achieve the better entropy, standard deviation and PSNR value. **Methods:** Multi-Scale Transforms (MST) and Sparse Representation (SR) methods are the two well-known methods used in image and signal representation theory. The novel image fusion framework is proposed in this paper by the combining enhanced MST method and SR based image fusion. The proposed scheme consists of three phase; first de-noised the sonar image using DTCWT with mean filter; second select the obtained pixels or features from sonar image using Novel PCA method; third obtained fusion image using Enhanced MST with SR structure. **Findings:** It is good at suppressing noise, especially for images with a higher noise level. The advantage of the proposed enhanced MST with SR technique than conventional MST with SR method is different level of decomposition using four popular MST methods; DWT, DTCWT, CVT and NSCT. The proposed method obtained better result in terms of entropy, standard deviation compared to conventional method. **Applications:** To realize earth surfaces with focus on underwater applications like depth sounding, sea-bed imaging and fish echolocation the SOund Navigation And Ranging (SONAR) technology is used.

**Keywords:** Denoising, Image Fusion, Multi-Scale Transforms, Sparse Representation, Sonar Image

## 1. Introduction

In image processing field, image fusion has to be important in recent years. The objective of the image fusion is to produce a composite image by combining the correlative information from multiple source images of the same scene. It is one of the most effective technology to process multi-sensor image and it can be applicable in the field of medical science, military affairs<sup>1</sup>, remote sensing and other fields.

Multi-Scale Transform (MST) is familiar in image fusion areas like multi-focus image fusion, visible-infrared image fusion and multimodal medical image fusion Ratio of low-pass Pyramid (RP)<sup>2</sup>, MST-based fusion includes pyramid-based methods such as Laplacian Pyramid (LP)<sup>3</sup> and Stationary Wavelet Transform (SWT)<sup>4</sup>, Gradient Pyramid (GP)<sup>5</sup>, Discrete Wavelet Transform (DWT)<sup>6</sup> and Non-Sub Sampled Contourlet Transform (NSCT)<sup>7</sup> and dual-tree complex wavelet transform (DTCWT)<sup>8</sup> and

Multi-scale Geometric Analysis (MGA)-based ones like Curvelet Transform (CVT)<sup>9</sup> discrete wavelet transforms and fuzzy combination was implemented in multi-focus image fusion method<sup>10</sup>. Enhancing the fuzzy function performance at every level of decomposition was done by DWT and Fuzzy function is used to speed up the fusion process. This approach obtained an entropy of 7.2, standard deviation of 49.8.

Pixel- Level image fusion method was proposed<sup>11</sup> where different methods such as averaging method, Multiplicative method, Brovey method and DWT method were implemented. This method obtained entropy for the above different methods are 5.23, 6.26, 5.93, 6.26 and 5.12. Sparse Representation (SR) is first introduced<sup>12</sup> into image fusion method. The image fusion method was implemented by Sliding window technique is introduced to make the image fusion process into very robust and effective one. Sparse representation based image fusion approach was implemented<sup>13-15</sup>.

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Integrating MST with SR methods is applicable in various image fusion techniques. It has some drawbacks and it is described in next section. To overcome the conventional method problem, we propose a scheme for image fusion method using Enhanced MST with SR using four popular multi-scale transform methods such as DWT, DTCWT, CVT and NSCT with decomposition level. The last decomposition is obtained from NSCT methods it is explained in following section; it provides high and low pass band. The high pass band techniques are integrated with max absolute rule whereas low pass band are integrated with SR technique. Finally fused image is obtained using Enhanced MST with SR technique.

The rest of this paper is organized as follows. The fusion framework is elaborated in Section 2. The proposed framework is explained in Section 3. The experimental results are given in Section 4. Section 5 summarizes some main conclusions of this paper.

## 2. Related Work

Multiscale Image Fusion using Undecimated Wavelet Transform with Spectral Factorization and Non-orthogonal Filter Banks was presented<sup>16</sup>. In this method the image fusion framework is enhanced using Undecimated Wavelet Transform method where decomposition level is splitting by two successive filter operations. The experimental result shows that IUWT method provides better result than DTCWT and NSCT.

Stationary Wavelet transform is presented<sup>17</sup>, where detect the edges of input image using level 1 and level 2. Spatial frequency measurement method is employed to combine the two edge image and final fusion image is produced. This method is combined with HAAR wavelet transform in terms of PSNR value, the results reveals that stationary wavelet transform with spatial frequency measurement methods provides lower distortion in obtained fusion image.

Fusion image is done by PCA and DWT is implemented<sup>18</sup>, first DWT is used to decompose the source and reference image, fusion image is obtained from above decomposition using choose-maximum rule. Second PCA method is applied on source and reference image- finally obtained the fusion image from PCA parameters from both the image. Results reveal that DWT method provides higher contrast for fusion image than PCA.

A Tutorial of wavelet-based image fusion methods are discussed<sup>19</sup> where is consists of inclusive correlation of

different pyramid combining methods, different wavelet families and different resolution levels are presented. Three examples for fusion were provided, such as multi-focus images, multispectral-panchromatic remote sensing images, and functional- anatomical medical images<sup>20</sup>.

## 3. Methodology

This paper implements an image fusion algorithm based on Multi-Scale Transform (MST) method. First, Dual Tree Complex Wavelet Transform (DT-CWT) is used for two images separately for the removal of noise in sonar images. The two images were dissolved into low frequency and high frequency by DTCWT. The mean filter is applied to LL, LH, HL and HH level to check the pixels and remove the noise in the obtained low and high frequency pixel. Novel PCA method is applied to obtain the selected features for two images separately. Finally obtained two images are overlapped or fused together using MST method. The proposed framework block diagram is shown in Figure 1.

### 3.1 Preprocessing

The preprocessing is a vital role in image processing field it is useful to predict the better accuracy of image fusion.

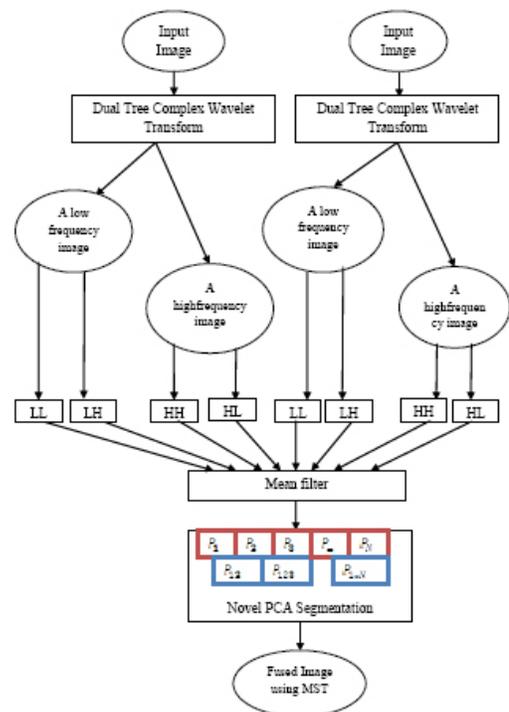


Figure 1. Block diagram of proposed framework.

The sonar image is affected by noise based on acquisition settings then the noisy pixel from sonar image has to be removed and it useful for further processing in proposed framework. In this paper, preprocessing is consisting of DT-CWT, mean filter and Inverse ID-CWT. DT-CWT dissolve the sonar image into low and high frequency band and the mean filter is applied to both low and high frequency band. Finally, Inverse DT-CWT is used to regenerate the denoised image. The preprocessing framework is explained in the following subsection elaborately.

### 3.1.1 Improved Dual Tree Complex Wavelet transform (DT-CWT) for Denoising

The normal wavelet transform suffers from four fundamental limitation ssuch as Aliasing, Shift Invariance, Oscillation, and Lack of directionality. To overcome these DWT short comings, complex wavelet transform is introduced. The main thing in complex wavelet transform is Fourier Transform (FT), because FT is not suffered from these problems. Advantages of FT over these four shortcomings are: First, FT does not oscillate in positive and negative. Second, magnitude of the FT gives correct shift invariant techniques.

Third, FT does not reconstruct the signal with aliasing, because FT coefficients are not aliasing method. Fourth, FT has higher directional plane waves in their sinusoids. But CWT based on wavelets does not perfectly possess the analytical signal properties. So CWT not perfectly overcomes these four drawbacks of DWT. The key challenge of this drawback is designing of Dual Tree wavelet transform(DT); it is based on the separable Filter Bank (FB) trees<sup>21</sup>. DT-CWT has following properties to overcome the drawback of normal wavelet transform or DWT: Approximate shift invariance; Perfect reconstruction; directional selectivity is good; redundancy is limited; efficient order-N computation

DT-CWT<sup>22</sup>oneoftheefficientmethodsforimplementing an analytical wavelet transform. Figure 2 illustrates the framework of DT-CWT, where DT-CWT consists of 2 real DWT; the real part and imaginary part of the transform is given by first and second part of the DWT.

Two different set of filters was used by two real wavelet transforms of DWT and each one satisfies the reconstruction condition. In the above framework  $g_0(n)$  and  $g_1(n)$  demotes the low pass and high pass filter pair for lower separable filter bank while  $h_0(n)$  ,  $h_1(n)$  are low pass and high pass filter pair for upper separable filter bank.

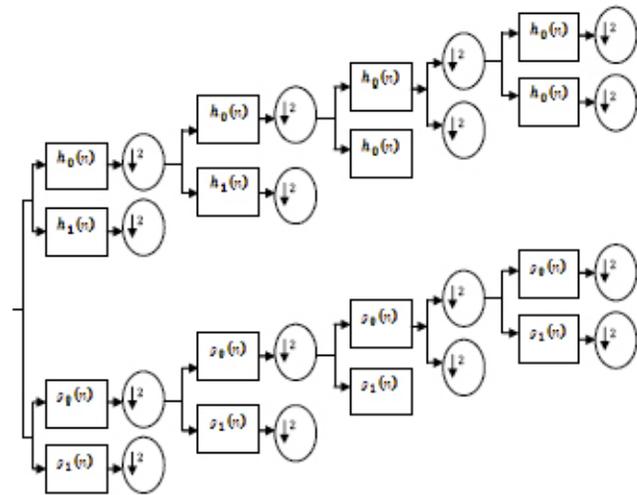


Figure 2. Framework of Dual Tree Complex Wavelet Transform.

Let us consider  $\psi_h(t)$  and  $\psi_g(t)$  are the two real wavelet associated with each of the real wavelet transform. Perfect reconstruction condition is satisfied by designing of complex wavelet transform are given in below equation:

$$\psi(t) = \psi_h(t) + j\psi_g(t) \tag{1}$$

where,  $\psi_g$  denotes approximate Hilbert transform of the wavelet associated with the lower DWT of  $\psi_h$ . When designed in this way, the DT-CWT is nearly shifting invariant contrast to normal wavelet transform or DWT.

### 3.1.2 Mean Filter

The goal of the mean filters is to replace the noise pixel by average value of its total pixel. It is simple to implement and reduces the intensity between one pixel and the next pixel. Finally, Inverse DT-CWT algorithm is applied for reconstruction of decomposition denoised image. The block diagram of DT-CWT with mean filter is shown in Figure 3.

### 3.2 Novel PCA Method

In this paper, Novel PCA method is applied to obtain the brightness information of the original two images. Novel PCA method is implemented by pixel by pixel calculation of source image. For example, let us consider that original image size is  $250 \times 250$  , then first apply the PCA method for pixel  $1 \times 1$  , after finding the Eigen vector of  $1 \times 1$  pixel, then apply PCA method to  $2 \times 2$  Pixel. This process is continued until  $250 \times 250$  pixel.

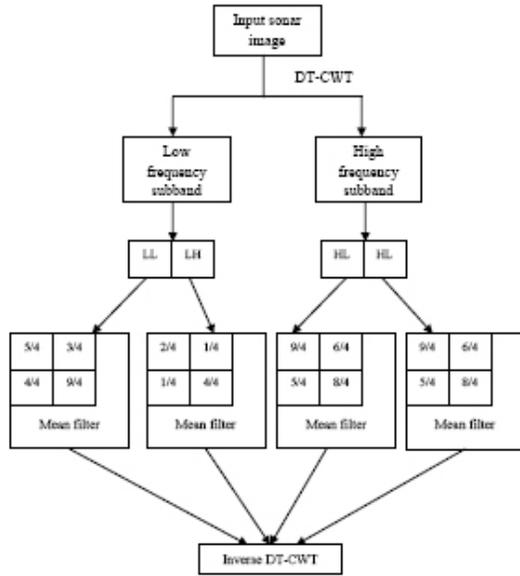


Figure 3. Block diagram of DT-CWT with mean filter.

The PCA<sup>23</sup> is a standard method for statistical pattern recognition, signal processing for data reduction and extracts the features of the given image/signal<sup>24</sup>.

First step of the PCA algorithm is to find average for original image is calculated using below equation:

$$x_k = \frac{1}{N} \sum_{m=1}^N F_m, k = 1, 2 \quad (2)$$

where, m is the sum of pixels in an image

For source image a data matrix is formed is given in below equation:

$$X = (x_1, x_2)^N = \begin{pmatrix} x_{11} & \dots & x_{1m} \\ x_{21} & \dots & x_{2m} \end{pmatrix} \quad (3)$$

The covariance matrix C for data matrix X is evaluated as given below:

$$C = \frac{1}{m} \sum_{i=0}^{m-1} (x_{i,j} - \bar{x}_i)(x_{i,j} - \bar{x}_i) \quad (4)$$

In the above equation,  $\bar{x}_i$  defines the average gray value of  $i^{th}$  image.

Let us consider eigenvector  $e_i$  of  $(x_{i,j} - \bar{x}_i)(x_{j,j} - \bar{x}_i)$  such that

$$(x_{i,j} - \bar{x}_i)(x_{j,j} - \bar{x}_i) e_i = \mu_i e_i \quad (5)$$

The above equation is multiplied by  $(x_{i,j} - \bar{x}_i)$  is given in below

$$(x_{i,j} - \bar{x}_i)(x_{i,j} - \bar{x}_i)(x_{j,j} - \bar{x}_i) e_i = \mu_i e_i \quad (6)$$

In the above equation  $(x_{j,j} - \bar{x}_i) e_i$  defines the eigenvector and  $\mu_i$  defines the eigenvalue.

### 3.3 Enhanced Multi-Scale Transform with SR

Figure 4 illustrates the decomposition of different wavelet transform of MST structure. Figure 5 illustrates the proposed framework of MST with SR structure. Conventional MST Structure have drawbacks of loss of contrast and difficult to select the level of decomposition. To alleviate this problem, in this paper proposed enhanced MST with Sparse representation framework, the proposed framework consists of four popular MST algorithms are DWT, DTCWT, CVT and NSCT. This paper proposes the image fusion method by the integration of MST and SR method. First low pass and high pass coefficients are given by DWT algorithm and it given to DTCWT algorithm for further process where DTCWT gives low pass and high pass coefficients to CVT algorithm. Like DTCWT, CVT algorithm obtain coefficients of low pass and high pass and it is given as an input to NSCT algorithm. Finally, NSCT algorithm obtains low pass and high pass coefficients where the high-pass bands are fused based on absolute values of coefficients whereas the low-pass bands are integrated with SR-based fusion technique. The Multilevel wavelet decomposition from DWT, DTCWT, CVT and NST provides the information about frequency components

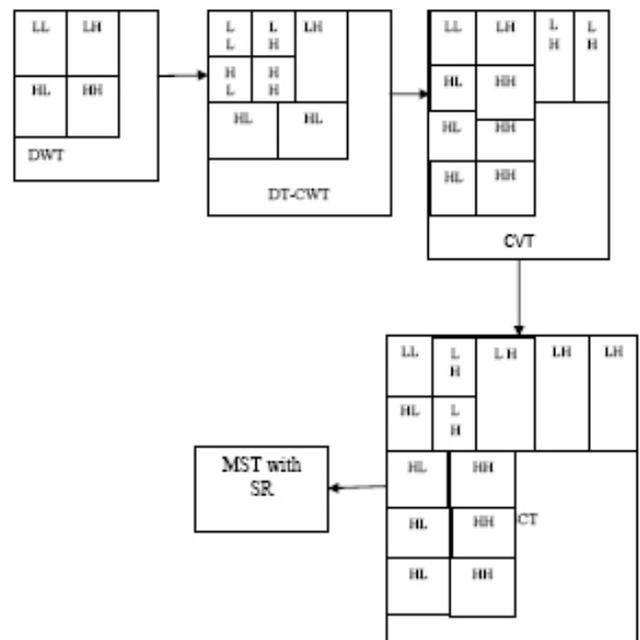


Figure 4. Proposed Framework of Multi-scale Transform.

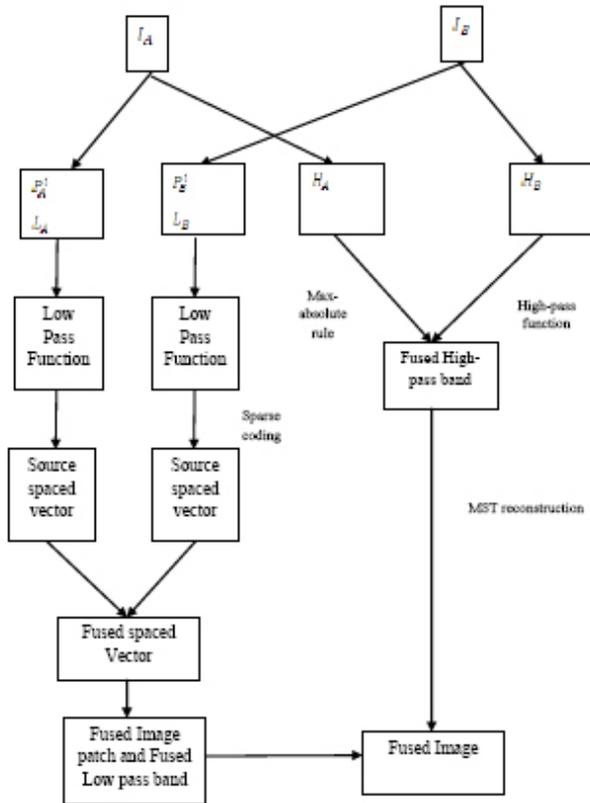


Figure 5. Framework of MST-SR.

present and enhances the information about the signal or image for further processing also preserve the contrast of the image.

Step 1: Obtain low pass band  $\{L_A, L_B\}$  and high pass band  $H_A, H_B$  from last level of decomposition from NSCT algorithm on two source images  $\{I_A, I_B\}$ .

Step 2: Divides the low band image into several image patches with size of  $\sqrt{N} \times \sqrt{N}$  using sliding window technique from upper left to lower right with a step length of  $s$  pixels. If there are  $T$  patches in  $L_A, L_B$  and it is represented as  $\{p_A^i\}_{i=1}^T$  and  $\{p_B^i\}_{i=1}^T$ .

Step 3: For each position  $i$ , reorganize  $\{p_A^i, p_B^i\}$  into column vectors  $\{V_A^i, V_B^i\}$  then normalize each vector's mean value to zero and obtain  $\{\hat{V}_A^i, \hat{V}_B^i\}$  using below equation:

$$\hat{V}_A^i = V_A^i - \bar{v}_A^i \cdot \mathbf{1} \tag{7}$$

$$\hat{V}_B^i = V_B^i - \bar{v}_B^i \cdot \mathbf{1} \tag{8}$$

In the above equation, value 1 represents the all-one valued  $n \times 1$  vector, mean value of all elements are denoted as  $\bar{v}_A^i$  and  $\bar{v}_B^i$ .

Step 4: Sparse coefficient vectors are calculated using the orthogonal matching pursuit (OMP) algorithm<sup>25</sup> and it is defined in below equation:

$$a_A^i = \arg \min_a \|a\|_0 \text{ s.t. } \|\hat{V}_A^i - Da\|_2 < \varepsilon \tag{9}$$

$$a_B^i = \arg \min_a \|a\|_0 \text{ s.t. } \|\hat{V}_B^i - Da\|_2 < \varepsilon \tag{10}$$

In this above equation  $D$  represents the learned dictionary

Step 5: Max-L1 rule is used to merge the sparse coefficients vectors.

$$\alpha_F^i = \begin{cases} \alpha_A^i & \text{if } \|\alpha_A^i\|_1 > \|\alpha_B^i\|_1 \\ \alpha_B^i & \text{otherwise} \end{cases} \tag{11}$$

Finally the fused result of  $V_A^i$  and  $V_B^i$  is calculated using below equation:

$$V_F^i = D\alpha_F^i + \hat{v}_F^i \cdot \mathbf{1} \tag{12}$$

In the above equation,  $\hat{v}_F^i$  is obtained using below equation:

$$\hat{v}_F^i = \begin{cases} \hat{v}_A^i & \text{if } \alpha_F^i = \alpha_A^i \\ \hat{v}_B^i & \text{otherwise} \end{cases} \tag{13}$$

Step 6: The above process is continued for all source image patches to obtain the fused vectors.

Step 7: Max-Absolute rule is used to merge the  $H_A$  and  $H_B$  to obtain fused high-pass band  $H_F$ .

## 4. Experimental Results

The proposed image fusion is evaluated using several images collected from the online websites to assess the Sonar image effectively. The effectiveness of the pre-processing method is performed using performance metrics such as PSNR. The objective evaluation metrics for proposed system is defined:

Standard deviation (SD) for obtained fusion image is calculated using the equation below:

$$SD = \sqrt{\frac{1}{M \times N} \sum_{x=1}^M \sum_{y=1}^N (F(x,y) - \mu)^2} \tag{14}$$

where,  $\mu$  denotes the mean of the fused image,  $M$  and  $N$  relates the size of the two source image. The overall contrast of the fusion image is evaluated based on standard deviation.

Entropy (EN): The entropy of the fused image is calculated using the equation below:

$$EN = - \sum_{i=0}^{L-1} p_F(i) \log_2 p_F(i) \quad (15)$$

where, L relates the number of grey level,  $p_F(i)$  represents the histogram of fused image. I denotes the size and is set to 256. Entropy is to calculate the amount of information in the fused image.

PSNR: Peak Signal to Noise Ratio (PNSR) is an objective metric for evaluating the performance of images. PSNR 1 denotes the ratio of pixel gray value between the distortion image and the source image. The ratio of pixel gray value between the distortion image and the reference image is given by PSNR 2. Finally take Root Mean square between PSNR1 and PSNR 2 to obtain the final fusion PSNR, if the PSNR value is bigger value then smaller the distortion. A source image and reference image is employed to check the effectiveness of the proposed fusion framework and is shown in Figures 6 and 7. Figure 8 illustrates the denoised reference image using improved DTCWT algorithm is the combining of DTCWT and mean filter. Figure 9 illustrates the final fused image using proposed method of multi-level decomposition.

### 4.1 Experimental Results on Four Popular MSTs

Four popular Multi-Scale Transforms, Dual-Tree Complex Wavelet Transforms(DTCWT),Discrete Wavelet Transform

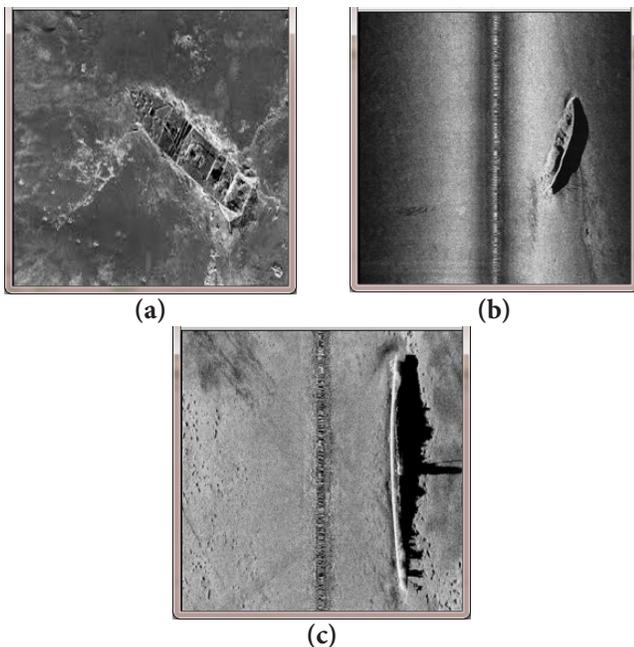


Figure 6. Source image used in our experiment.

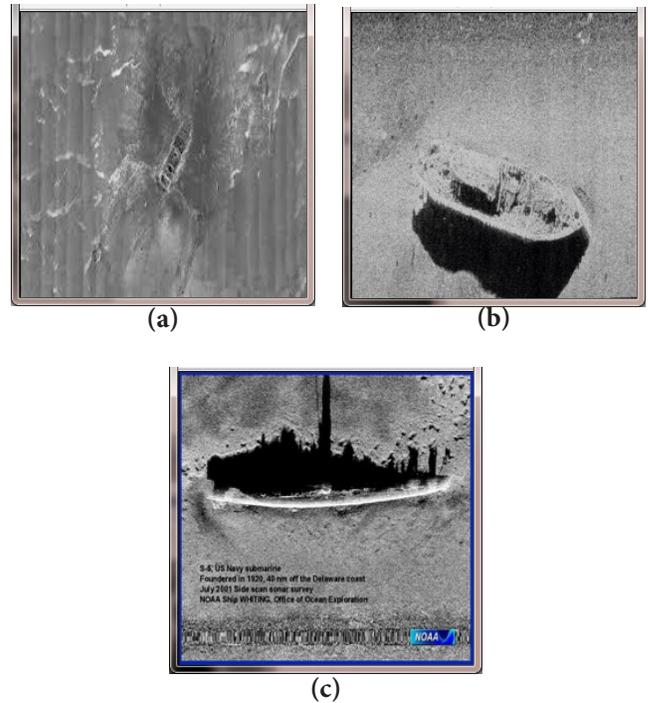


Figure 7. Reference image used in our experiment.

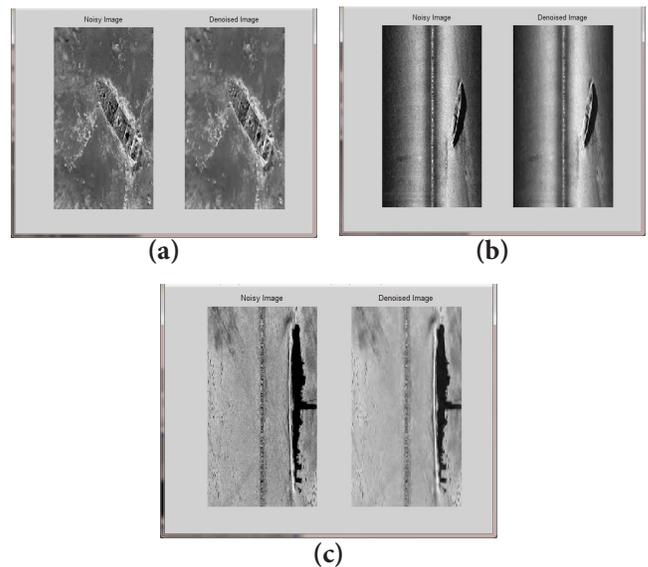
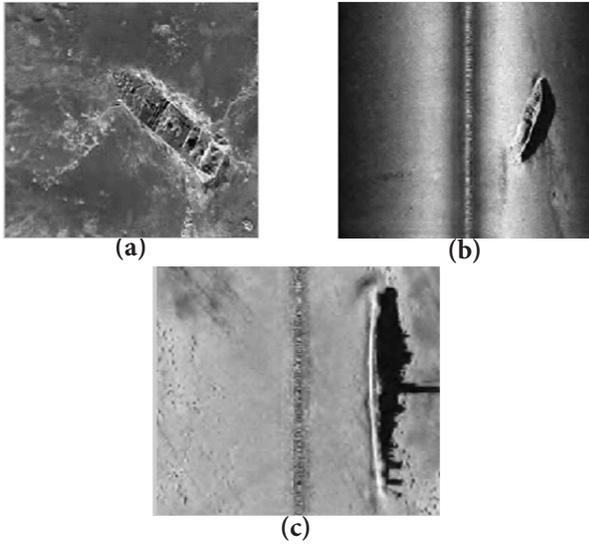


Figure 8. Denoised image for source image.

(DWT), Non-Subsampled Contourlet Transform (NSCT) and Curvelet Transform (CVT) are used in this proposed framework. The proposed image fusion effectiveness is verified by comparing the proposed method to above MST with the decomposition level of increasing from 1 to 4. The comparison is taken as DWT-SR with proposed method MSTs, DWT-SR-1 it defines the combination of DWT and SR with decomposition level is 1.



**Figure 9.** Final fusion image.

**a) Comparison of DWT, DTCWT, CVT, NSCT and proposed method MSTs.**

Table 1 gives the list of objective assessment of DWT, DTCWT, CVT, NSCT and proposed method MSTs. From the given table analyzed that proposed method provides better results than other methods.

Table 2 gives the comparison of Entropy, SD, and PSNR for DWT-SR, DTCWT-SR, CVT-SR, NSCT-SR and proposed method MSTs. Compared to Table 1 results, Table 2 provides better result because of combination of methods with SR. In Table 2; proposed method provides better result in PSNR, SD, Entropy compared to DWT-SR, DTCWT-SR, CVT-SR and NSCT-SR. Proposed MST method is combined with SR and provides better results of higher PSNR value, Entropy value and standard deviation. So from the table and results analyzed that proposed MST with SR obtains higher information of fused image, higher contrast of an image.

Table 3 lists the objective assessments of DWT-SR-1, DTCWT-SR-1, CVT-SR-1, and NSCT-SR-1 and Proposed method MSTs-SR-1. Compared to table 1 & 2, the objective assessments of DWT-SR-1, DTCWT-SR-1, CVT-SR-1, NSCT-SR-1 and Proposed method-1 provides better result because of 1 level decomposition. Tables 4, 5 and 6 lists the objective assessments of DWT-SR, DTCWT-SR, CVT-SR, NSCT-SR and proposed method MSTs-SR with decomposition level of 2, 3&4. Compared to 1-level decomposition results 2, 3 & 4 provides better results in terms of high contrast and collection of fusion image with low level distortion. Proposes method of MSTs with SR-1 level, MSTs with SR-2 level, MSTs with SR-3 level & MSTs with SR-4

**Table 1.** Comparison of objective assessment of DWT, DTCWT, CVT, NSCT and proposed method

Methods	SD	EN	PSNR
DWT	18	10	8
DTCWT	18.5	10.1	8.012
CVT	18.2	10.23	8.123
NSCT	19	10.01	8.23
Proposed MST method	20	10.45	8.42

**Table 2.** Comparison of objective assessment of DWT-SR, DTCWT-SR, CVT-SR, NSCT-SR and proposed method

Methods	SD	EN	PSNR
DWT-SR	19.7	18	8.3
DTCW-SR	20.6	20	8.4
CVT-SR	28.1	24	8.42
NSC-SR method	28.16	26	8.50
Proposed method MST-SR	29.47	56	8.59

**Table 3.** Comparison of objective assessment of DWT-SR-1, DTCWT-SR-1, CVT-SR-1, NSCT-SR-1 and proposed method-1

Methods	SD	EN	PSNR
DWT-SR-1	49.1	19	8.41
DTCW-SR-1	49.6	22	8.5
CVT-SR-1	49.63	28	8.52
NSC-SR method-1	49.9	35	8.55
Proposed method-1	52.0	59	8.67

**Table 4.** Comparison of objective assessment of DWT-SR-2, DTCWT-SR-2, CVT-SR-2, and NSCT-SR-2 and proposed method of MSTs with SR-2

Methods	SD	EN	PSNR
DWT-SR-2	49.34	22	8.51
DTCW-SR-2	49.72	23	8.65
CVT-SR-2	49.81	28.34	8.70
NSC-SR method-2	49.99	35.07	8.72
Proposed method-2	52.56	59.12	8.79

**Table 5.** Comparison of objective assessment of DWT-SR-3, DTCWT-SR-3, CVT-SR-3, and NSCT-SR-3 and proposed method of MSTs with SR-3

Methods	SD	EN	PSNR
DWT-SR-3	49.89	22.56	8.67
DTCW-SR-3	49.99	23.12	8.78
CVT-SR-3	50	28.67	8.81
NSC-SR method-3	50.12	35.23	8.89
Proposed method-3	52.89	59.45	9

**Table 6.** Comparison of objective assessment of DWT-SR-4, DTCWT-SR-4, CVT-SR-4, and NSCT-SR-4 and proposed method of MSTs with SR-4

Methods	SD	EN	PSNR
DWT-SR-4	50	22.89	8.90
DTCW-SR-4	50.10	23.30	8.92
CVT-SR-4	50.34	28.89	8.94
NSC-SR method-4	50.56	35.45	8.99
Proposed method-4	53.11	60.02	9.06

level provides better results compared to DWT, DTCWT, CVT and NSCT with decomposition level of 1, 2, 3 and 4.

## 5. Conclusion

This paper proposes an image fusion algorithm for sonar image based on Improved DTCWT, Novel PCA method and Enhanced MSTs with SR techniques.

- For reducing the distortion on source image, Improved DTCWT algorithm is implemented in this work. In this proposed Improved DTCWT algorithm, mean filter is used to remove the noise in decomposition result of DTCWT.
- Novel PCA method is implemented in this paper to recover or preserve the selected features from decomposition results of DTCWT. Compared to conventional PCA method, novel PCA method is check each pixel in that image and finally obtained the Eigen value results.
- Enhanced MST method with SR technique is employed in this paper to improve the fusion image contrast. Compared to conventional method of MSTs with SR technique, proposed enhanced MSTs with SR techniques provides better results in terms of higher contrast, lower distortion and obtains the abundant information, visual effects so on from sonar image.

The experimental result reveals that proposed sonar image fusion method is effective and better compared to conventional image fusion algorithms.

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