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Application of Discrete Multi-wavelet Transform in Denoising of Mammographic Images

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Abstract

Objectives: Clinical and surgical procedures heavily depend on acquiring and transmitting medical information by electronic devices and media in the form of medical images. In this process the quality of these images are degraded by noise, making denoising mandatory. **Methods/Statistical Analysis:** In this paper de noising algorithms in wavelet domain for mammographic images (used for detection of breast cancer in women) are considered. A modified approach for de noising of mammographic images using Multi Wavelet Transform has been proposed with four different thresholding techniques. These Multi wavelets possess important properties like orthogonality, symmetry and compact support simultaneously. Performance of denoising methods improves considerably due to these properties. **Findings:** Proposed method is applied on large data set of digital mammographic images freely available as mammographic data base by MAIS. DMWT based denoising scheme with four different types of thresholding estimates, namely Bayesian shrink, Visu shrink, neighborhood shrink and modified neighborhood shrink is applied on data set and compared the performance of proposed denoising algorithm with dwt based existing denoising methods. Higher the value of σ (noise variance) for Gaussian noise considered lower is the value of PSNR, obtained, whatever is the transformation and techniques applied. Even among the DMWT: GHM, CL and SA4, SA4 gives the best response for all the cases. **Applications/Improvements:** The results are compared on the basis of Peak Signal to Noise Ratio (PSNR) in dB. Results clearly indicate the superiority of the proposed method in all the four cases over exiting wavelet based method. This new improved method will help a lot in diagnosis of breast cancer more accurately and the correct prognosis can save many lives.

Keywords: Discrete Multi Wavelet Transform, Denoising, Mammographic Images, Multiwavelet Transform

1. Introduction

In the concept of health net, diagnosis and treatments of many diseases, starting from early stages of identification to complex stages of surgery depend heavily on medical images and different modalities of medical imaging. Popular modalities are X-Ray scan, Magnetic Resonance Imaging (MRI), Computed Axial Tomography (CAT), Ultrasonography (USG), Electrocardiography (ECG), Intra-Vascular Ultra Sound (IVUS) and Mammography. During the process of acquisition, transmission and reception of medical images they get corrupted by the noise. Image denoising is still a challenging problem in the field of image processing because the performance of image denoising algorithms greatly affects the accuracy and speed of clinical diagnosis.

Mammography is low energy X ray of breast, used for early detection of Breast Cancer in women. World Health Organization report 2013[1], indicates breast cancer as one of the main reason of death in women. Prognosis is the first step towards best cure for breast cancer. Motivated by this fact for accurate diagnosis, we try to modify and improve the pre processing or denoising algorithms for Mammographic images. Main objective of these denoising algorithms are to acquire best estimate of original image from the corrupted image. Spatial filters like Weiner Filters, Median Filters etc when used, results in blurred edges and textures.

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Denoising algorithms should focus on removing the noise and preserving the boundaries, edges, contrasts and textures. Multi Resolution Analysis is more successful in preserving the originality while removing the noise. In this paper, focus is on Multi resolution Analysis based denoising algorithms i.e. in transform domain using Discrete Multi Wavelet Transform. Wavelet transform has amazing capability of capturing the energy of the signal as well as compacting it in a very few transform coefficients. Wavelet transform coefficients with smaller values are equivalent to noisy part of the signal while coefficients with larger magnitude are representing important detailed features of the signal. Now by adopting some nonlinear thresholding technique, we can very easily differentiate between the noise and signal and eliminate noise[2-4]. In this way important attributes of the image are preserved.

In the last two decades, the use of wavelet in medical imaging has tremendously increased[5-7]. In almost every sphere of image processing like compression, enhancement, denoising, feature extension, segmentation and registration wavelets are used. Recently not the scalar but multi wavelets have been more popular in usage. Multi wavelets are designed with more than one scaling function (multiplicity r, no. of scaling function and wavelet function is r).

These Multi wavelets possess some important properties like orthogonality, symmetry and compact support simultaneously but at the cost of extra computing time and complexity[8-11]. They provide more degree of freedom and better energy compactness. Performance of denoising methods improve considerably due to these properties[12,13].

We have developed an improved denoising algorithm using Discrete Multi Wavelet Transform along with four different thresholding estimate methods namely Bayesian Shrink, Visu Shrink, Neighborhood Shrink and Modified Neighborhood Shrink method. Organization of this paper is follows as in Section 2, mathematical background and notations associated with wavelet bases and the idea of Discrete Multi Wavelet Transform is given. In Section 3, filter banks and implementation of DWT (Discrete Wavelet Transform) and DMWT (Discrete Multi Wavelet Transform) using filter banks is discussed. In section 4 proposed algorithm is discussed. Section 4 elaborates the denoising schemes with different type of thresholding estimates. These four thresholding methods are used along with DWT and DMWT to denoise mammography images. In section 5, a comparison of different methods with different transforms is done on the basis of Peak Signal to Noise Ratio (PSNR). After applying these algorithms on a large set of data and analyzing the results obtained, conclusions are made.

2. Mathematical Background

Analyzing the signals in wavelet domain, means representing the signal with another function known as wavelets, having short duration and finite energy. This type of transformation of signal under consideration is called wavelet transform. A large number of wavelet functions (mother wavelet) are available for signal analysis. Choice of a particular wavelet function mostly depends on application considered. Once a particular wavelet function chosen can be manipulated with the help of two parameters namely translation and scaling. The translation means the shifting of the mother wavelet along the time axis and the scaling means either the expanding or compressing the mother wavelet in time domain. If this process of translation and scaling are done in continuous mode it is known as Continuous Wavelet Transform and if they are done in discrete steps then it's called Discrete Wavelet Transform[14].

By using two variables representation, a sufficient amount of redundancy is used to retain the local properties of original signal. A fast computation of DWT is possible by implementing the wavelet function equations and scaling function equation in different subspaces with the help of convolution and filter banks [14, 15].

$$\psi(t) = \sqrt{2}\sum_{k} H_k\, \psi[2t-k], \tag{1}$$

$$\Psi(t) = \sum_{k} G_k\, \psi[2t-k], \tag{2}$$

Here H (k) is low pass filter coefficients and G (k) is high pass filter coefficients.

2.1 Multiwavelet

These are the natural extension of wavelet B. K. Alpert proposed sparse matrix representation of multiwavelet bases . Thereafter, Geronimo, Hardin and Massopust proposed Matrix representation of GHM multiwavelet functions coefficients known as scaling and wavelet function coefficients[10,16]. Multiwavelet of multiplicity r =2 , will be represented by equation(3) and (4).

$$\left\{\Phi(t)=\begin{bmatrix} f_1(t) \\ f_2(t) \end{bmatrix} = \begin{matrix} H_0\Phi(2t)+H_1\Phi(2t-1) \\ +H_2\Phi(2t-2)+H_3\Phi(2t-3) \end{matrix}\right\} \quad (3)$$

$$\left\{\Psi(t)=\begin{bmatrix} \Psi_1(t) \\ \Psi_2(t) \end{bmatrix} = \begin{matrix} G_0\Phi(2t)+G_1\Phi(2t-1) \\ +G_2\Phi(2t-2)+G_3\Phi(2t-3) \end{matrix}\right\} \quad (4)$$

For CL multi wavelets, its scaling functions have short support of [0, 2]. Scaling functions and wavelet functions are symmetric and anti symmetric ortho normal pairs. SA4 multi wavelet is another such example.

Graphical representation of scaling and wavelet functions for GHM, CL and SA4are given in Figure 1, 2 and 3 respectively.

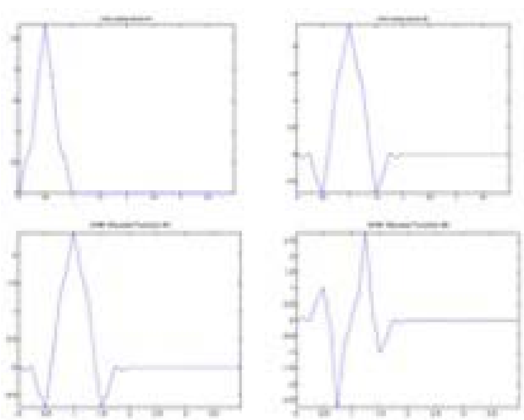


Figure 1. Scaling functions and wavelet functions for GHM multi-wavelet.

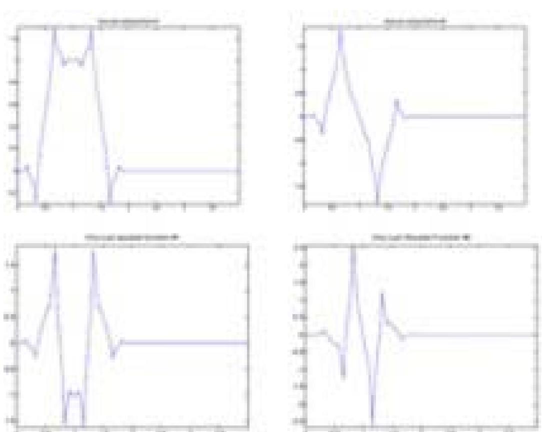


Figure 2. Scaling functions, wavelet functions for CL Multi wavelet.

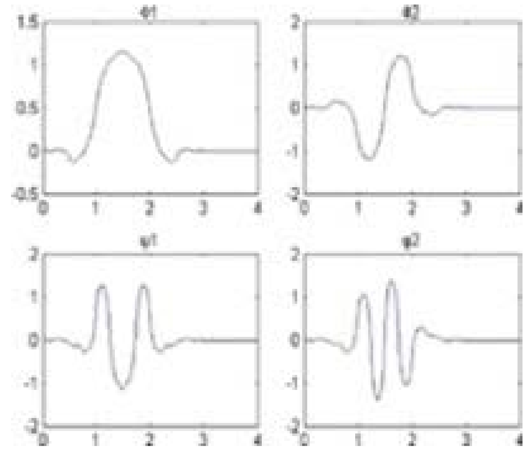


Figure 3. Scaling functions and wavelet functions for SA4 multi-wavelet.

Before applying filter, scalar valued input signal is converted into vector valued signal, called preprocessing or pre filtering[8,17] as shown in Figure 4. During reconstruction, post filtering is done.

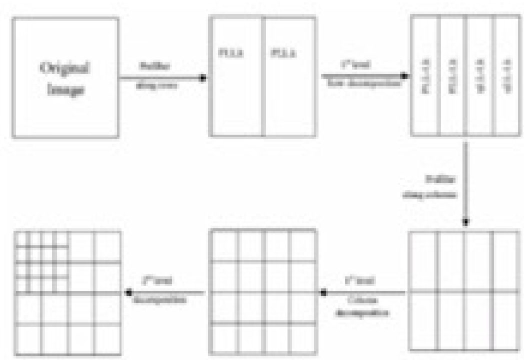


Figure 4. Pre filtering and multi-level decomposition.

For GHM, CL and SA4 Pre filter coefficients (matrix P) are given in Table 1. Inverse of the P matrix gives Post filter coefficients[18-21].

Table 1. Co-efficient for pre filter

Multi wavelets	Pre filter coefficients
GHM	$P=\begin{bmatrix} 0.11942 & -0.00598 \\ 0.99158 & 0.99158 \\ 0.99158 & 0.99158 \\ 0.99158 & 0.99158 \end{bmatrix}$
CL	$\begin{bmatrix} 0.25 & 0.25 \\ 0.2713 & 0.2713 \end{bmatrix}$
SA4	$P=\begin{bmatrix} 0.7071 & 0.7071 \\ -0.7071 & 0.7071 \end{bmatrix}$

Low pass filter coefficients H_k's and high pass filter coefficients G_k's for multi wavelets are represented in form of a matrix as they are vectors.

For GHM, these coefficients are listed below as equation (5) and equation (6)

$$H_0=\begin{bmatrix} 3/5 & 4\sqrt{2}/5 \\ -1/10\sqrt{2} & -3/10 \end{bmatrix},\quad H_1=\begin{bmatrix} 3/10 & 0 \\ 9\sqrt{2}/40 & 1/2 \end{bmatrix} \tag{5}$$

$$H_2=\begin{bmatrix} 0 & 0 \\ 9\sqrt{2}/40 & -3/20 \end{bmatrix},\quad H_3=\begin{bmatrix} 0 & 0 \\ -\sqrt{2}/40 & 0 \end{bmatrix} \text{ and}$$

$$G_0=\begin{bmatrix} 3/10 & 2\sqrt{2}/5 \\ -\sqrt{2}/40 & -3/20 \end{bmatrix}\quad G_1=\begin{bmatrix} 3/10 & 2\sqrt{2}/5 \\ -\sqrt{2}/40 & -3/20 \end{bmatrix}$$

$$G_2=\begin{bmatrix} 3/10 & 2\sqrt{2}/5 \\ -\sqrt{2}/40 & -3/20 \end{bmatrix}\quad G_3=\begin{bmatrix} 3/10 & 2\sqrt{2}/5 \\ -\sqrt{2}/40 & -3/20 \end{bmatrix}$$

For CL multi wavelet the low pass filter coefficients H_k's and high pass filter coefficients G_k's are listed below as equation (7) to equation (8).

$$H_0=\begin{bmatrix} 1/4 & 1/4 \\ -\sqrt{7}/8 & -\sqrt{7}/8 \end{bmatrix}\quad H_1=\begin{bmatrix} 1/2 & 0 \\ 0 & 1/4 \end{bmatrix}\quad H_2=\begin{bmatrix} 1/4 & -1/4 \\ -\sqrt{7}/8 & -\sqrt{7}/8 \end{bmatrix} \tag{7}$$

$$G_0=\begin{bmatrix} -1/4 & -1/4 \\ 1/8 & 1/8 \end{bmatrix},\quad G_1=\begin{bmatrix} 1/2 & 0 \\ 0 & \sqrt{7}/20 \end{bmatrix},\quad G_2=\begin{bmatrix} -1/4 & 1/4 \\ -1/8 & 1/8 \end{bmatrix} \tag{8}$$

SA4 multiwavelet, coefficients has been given as

$$S=\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix},\quad A=\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix},\quad \alpha=0.749423 \tag{9}$$

$$\beta=\alpha^2+1$$

and

$$\begin{aligned} H_0&=\begin{bmatrix} 1/\beta & \alpha/\beta \\ 1/\beta & -\alpha/\beta \end{bmatrix} & G_0&=-H_3*A \\ H_1&=\begin{bmatrix} 1/\beta & \alpha/\beta \\ 1/\beta & -\alpha/\beta \end{bmatrix} & G_1&=H_2*A \\ & & G_2&=-H_1*A \\ H_2&=S*H_1*S & G_3&=-H_0*A \\ H_3&=S*H_0*S \end{aligned} \tag{10}$$

Fulfill such important properties simultaneously was not possible in case of scalar wavelets. Filter coefficients in multi wavelet cases are vectors and represented by matrices. More degree of freedom is possible in designing and applications as a result better performance is expected than scalar wavelets[16].

As discussed, Multi wavelet is the extension of scalar wavelet, thus it is also satisfying multi resolution analysis. Decomposition of image using scalar wavelet at single level results in a vector consisting of 4 sub-bands named LL, LH, HL and HH as shown in Figure 5(a). LL sub-band represents low frequency components, obtained by applying low pass filter along rows and then along columns of input vector. LH indicates detail coefficients in vertical direction, obtained by applying low pass filter along rows and then high pass filter along columns. Similarly, HL gives detail coefficients in horizontal direction and HH gives details along diagonal.

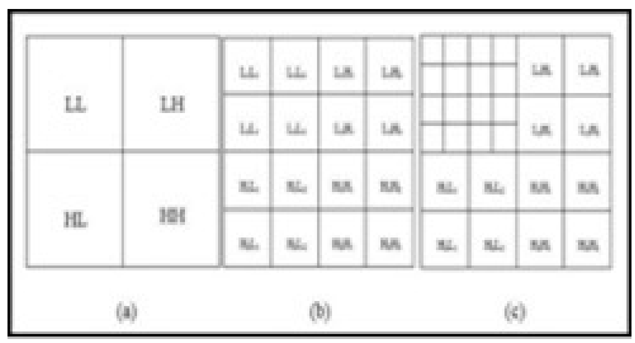


Figure 5. (a) Image after Prefilters (b) Image after 1st level 2D DMWT (c) Image after 2nd level 2D DMWT.

For single level decomposition of image using multi-wavelet, again these four sub bands are filtered using appropriate transformation matrix. L_1H_2 is obtained from second sub-band high pass filter in horizontal direction and first sub-band low pass filter in vertical direction[14]. This process results in 16 sub-bands as shown in Figure 5(b). Figure 5(c) represents 2nd level Discrete Multi Wavelet Transformed vector or image.

3. Filter and Filter Banks

3.1 Filter

A filter is linear time invariant system or operator. It process the input vector. The output vector is convolution of x with a fixed vector, gives the output vector. The vector h are the filter coefficients:

h(0), h(1), h(2),.......h(n). Our filter are digital, not analog, so the coefficients h(n) come at discrete times t = nT. The action of a filter in time and frequency is the foundation on which signal processing is built[14,15].

3.2 Filter Bank

- A filter bank is a set of filters. The analysis bank has two filters, low pass and high pass. These filters separate the input signal in to low and high frequency bands.
- Compression of these sub signals is much more efficient than the original signal. At any time the signals can be recombined (by the synthesis bank).
- It is not necessary to preserve the full outputs from the analysis filters. Normally they are down sampled. We keep only the even components of the low pass and high pass filter outputs.
- Two sets of coefficients h(k) and g(k) in scaling function representation are acting as low pass and high pass filter coefficients respectively[14,15].
- Multi level Discrete Multiwavelet Transform (MLDMWT) that decomposes the signal to be transmitted into different frequency components by successively applying low pass and high pass filters based on MultiFilters or Multiwavelets as shown in Figure 6.

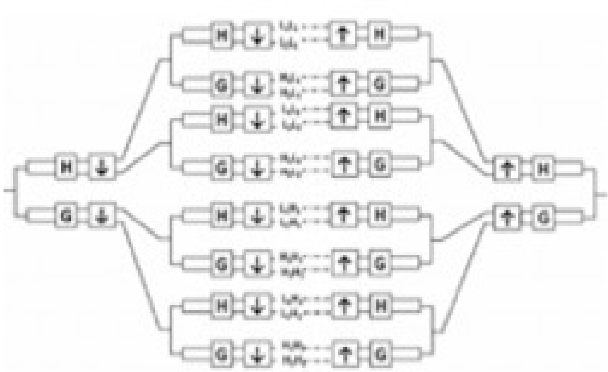


Figure 6. Multi Wavelet Filter Bank for one level of composition and reconstruction.

4. Proposed Algorithm

Proposed algorithm can be implemented in five different phases as in Figure 7.

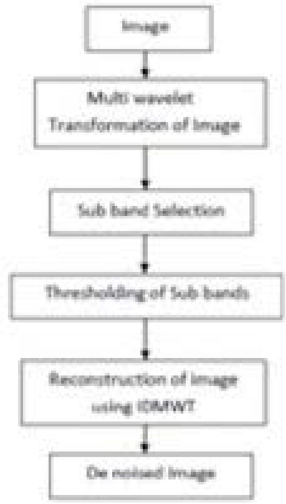
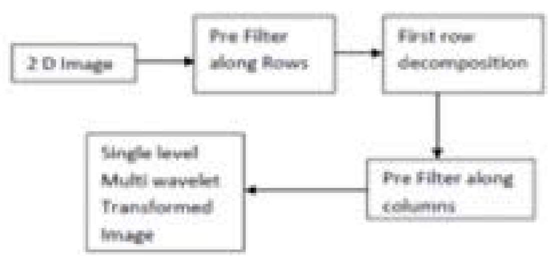


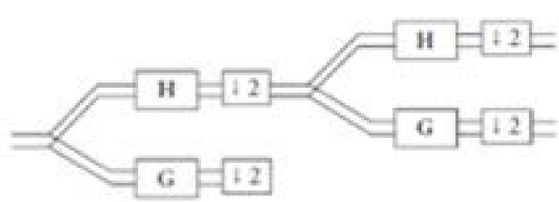
Figure 7. Flow chart of the proposed Algorithm.

4.1 Discrete Multivavelet Transformation (DMWT) of Image

For taking DMWT, first thing is to pre filter the image to convert it in to vector form. Pre-filtering is done here by repeated row processing. Decomposition of image and DMWT Transform implementation through filter bank is shown in Figure 8(a) and (b).



(a)



(b)

Figure 8. Decomposition of image using DMWT.

4.2 Selection of Important Subbands

Multi wavelet transformed image has 16 sub bands after first level of decomposition and 256 more after second level of decomposition while wavelet gives only 4 sub bands after first level and 16 sub bands more after second level. We keep 16 sub-bands of the first level of decomposition and 16 most significant sub bands of low pass region of second level of decomposition. These most significant sub-bands are selected on the basis of their variances σ_k denote the variance of the k^{th} sub band where k is from 1, ...M.

M =16 for first level of decomposition and M =256 for second level of decomposition. Let

$$P_i = \frac{\frac{1}{M}\sum_{k=1}^{M}\sigma^2{}_k}{\left(\prod_{k=1}^{M}\sigma^2{}_k\right)^{\frac{1}{M}}} \tag{11}$$

This parameter is generally greater than one. Greater the value better is to select that sub band. We select only 32 bands and then further all the processing is done on these selected sub bands only[18,19]. Threshold will be estimated for every sub-band, as given in section 4.3.

4.3 Thresholding of Coefficents of Selected Subbands

The key parameter in all the methods of thresholding is the estimate of "t", the threshold value. Optimal thresholding occurs when the thresholding parameter is set to the noise level i.e. σ = t. Setting t<σ, will allow unwanted noise enter the estimate while setting t>σ will destroy the main information that really belongs to underlying image. Thus best possible thresholding or de noising occurs when t = σ[22,23].

- Hard Thresholding**:** To suppress the noise we apply the following non linear transform to the wavelet coefficients. Hard threshold is to either kill or keep approach and appealing[8,14].

$$F(x) = x.I(|x| \rangle\ t) \tag{12}$$

- Soft Thresholding: Soft Threshold shrinks coefficients above the threshold in absolute value. Difference between the hard and the soft thresholding procedure is in choice of the nonlinear transform on the wavelet coefficients.

$$S(x) = \mathrm{sgn}(x)(|x|-t)I(|x| \rangle\ t) \rangle\ t) \tag{13}$$

4.3.1 Techniques Applied

Bayesian or Normal Shrink calculates the threshold value (T_N),

$$T_N = \frac{\beta\sigma^2}{\sigma_y} \tag{14}$$

which is adaptive to different sub band characteristics. Where the scale parameter β is computed once for each scale using the following relation:

$$\beta = \sqrt{\log(\frac{L_K}{J})} \tag{15}$$

L_k is the length of the sub band at kth scale, σ^2 is the noise variance, which is estimated from the sub band HH1, using the

$$\hat{\sigma}^2 = \left[\frac{\mathrm{medain}(|Y_{ij}|)}{0.675}\right]^2, Y_{ij} \in \text{subband } HH_1 \tag{16}$$

Where $\hat{\sigma}_y{}^2$ is the standard deviation of the sub band under consideration?

Visual shrink combination of soft thresholding and universal threshold is Visu Shrink. Threshold T can be calculated using the formulae,

$$T = \sigma\sqrt{2\log n^2} \tag{17}$$

This method outperforms under a number of applications because wavelet transform has the compaction property of having only a small number of large coefficients. Most of the wavelet coefficients are very small. This algorithm provides the advantages of smoothness and adaptation. However visual artifacts are visible[5-7].

Neighbour Shrink Let d(i,j) denotes the wavelet coefficients of interest then

d(i,j) = d(i,j) * B(i,j) where

$$B(i,j) = (1 - \frac{\tau^2}{s^2(i,j)}) and\ s^2 = \sum d^2(i,j) \tag{18}$$

Modified Neighbour Shrink is same as Neighbour Shrink except

$$B(i,j) = (1 - 0.725\frac{\tau^2}{s^2(i,j)}) and\ s^2 = \sum d^2(i,j) \tag{19}$$

4.4 Inverse Multiwavelet Transform and Reconstruction of Denoised Image

After thresholding and the selection of sub bands, we reconstruct the image from those sub bands using IDMWT and after post filtering we get the denoised image[2,7].

4.5 Evaluation Criteria

The objective quality of the reconstructed image is measured by

$$\mathrm{PSNR} = 10\log_{10}\frac{255^2}{\mathrm{MSE}} \quad (20)$$

where MSE is mean square error between original(x) and denoised image($\hat{x}$)

$$\mathrm{MSE} = \frac{1}{MxN}\sum_{i=1}^{M}\sum_{j=1}^{N}[x(i,j)-\hat{x}(i,j)]^2 \quad (21)$$

5. Results and Conclusions

On a set of mammographic data base by MAIS (link of Mammographic Image Analysis homepage is http://www.mammoimage.org/database/)[24], proposed denoising algorithms are tested using Matlab. The experiments are conducted on images of size 256 × 256, at different noise levels. Results for different values of σ are tabulated in Table 2 and Table 3. Figure 9 and Figure 10 give the comparison of PSNR values for different schemes used for denoising respectively for σ = 10 and for σ = 20.

Table 2. Filtering results for noise variance = 10

Wavelet	Thresholding Techniques, PSNR in dB, for σ = 10			
	Modified neighbourhood	Bayesian	Neighbourhood	Visu
SA4	33.91	31.96	29.95	28.03
GHM	31.97	31.01	27.46	26.87
CL	31.02	30.09	27	26.04
DWT	24.7	24.9	23.38	21.03

Table 3. Filtering results for noise variance = 20

Wavelet	Thresholding Techniques PSNR in dB, for σ = 20			
	Modified neighbourhood	Bayesian	Neighbourhood	Visu
SA4	32.01	30.97	28.25	27
GHM	30.97	30.01	26.76	24.97
CL	30.02	29.09	26	25.17
DWT	24.7	24.9	23.38	21.03

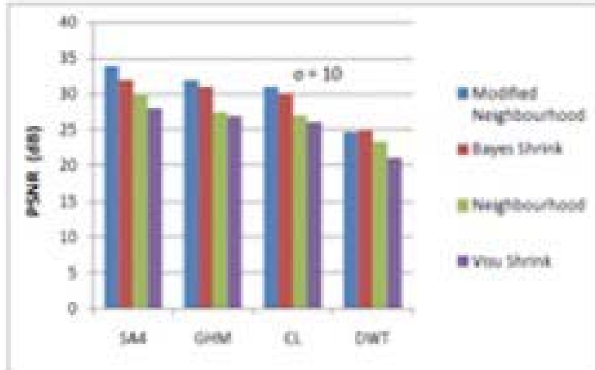


Figure 9. Comparison of PSNR values for Denoising for different thresholding techniques in DWT and DMWT for σ = 10.

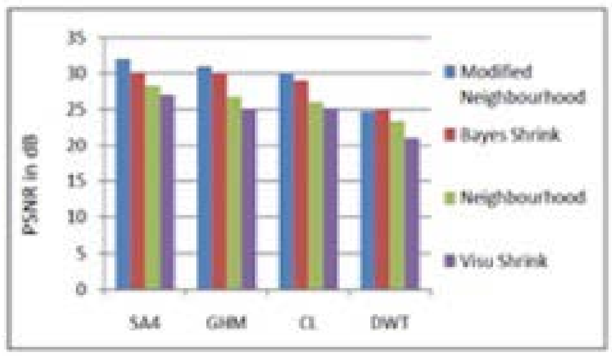


Figure 10. Comparison of PSNR values for Denoising for different thresholding techniques in DWT and DMWT for σ = 20.

Using DMWT and DWT for standard deviation of noise σ = 10 and σ =20, of random noise, all four methods of thresholding namely Bayesian Shrink, Visu Shrink, Neighbourhood Shrink and Modified Neighbourhood Shrink are implemented. PSNR is calculated in each case. Results are tabulated in two different tables with a set of arbitrary images selected from data base. On analyzing these results it is clearly indicated that the PSNR in case of DMWT SA4 is improved by a considerable amount in each image for every combination of thresholding method and standard deviation of additive random noise.

Next observation is the higher the value of σ, lower is the value of PSNR, obtained, whatever is the transformation and techniques applied. Even for different multi wavelet considered SA4 gives the best results. Figure 11 to Figure 15 show original, noised and result of denoised images for different wavelets.

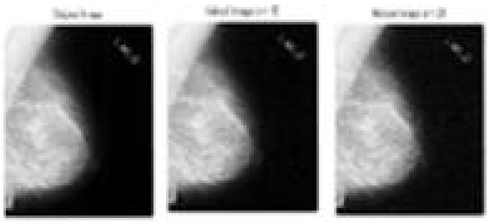


Figure 11. Images-original, for σ = 10, and σ = 20.

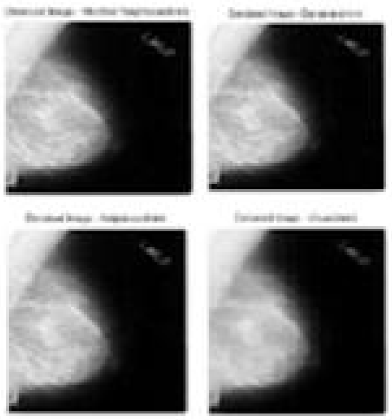


Figure 12. Denoised images for σ = 10, DWT.

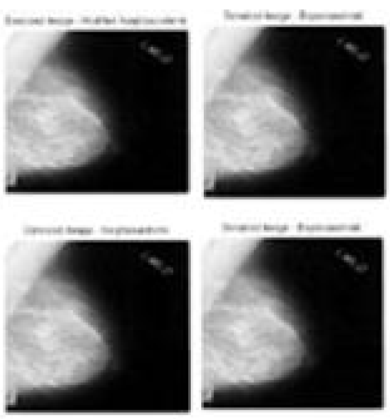


Figure 13. Denoised image for σ = 10, DMWT SA4.

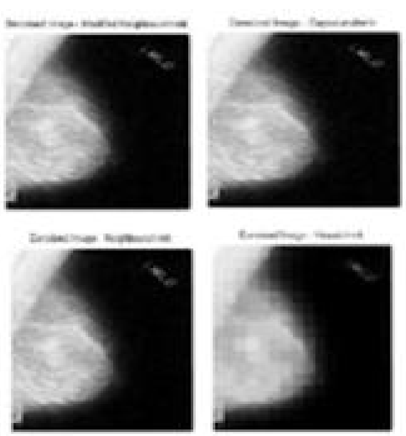


Figure 14. Denoised images for σ = 20, DWT.

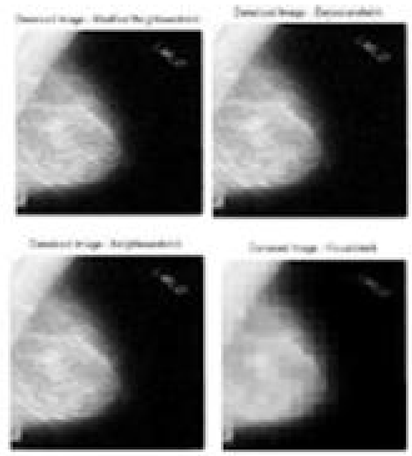


Figure 15. Denoised image for σ = 20, DMWT SA4.

The proposed improved algorithm can help in better detection and diagnosis of breast cancer in women than already existing methods[23] due to improved quality of denoised images.

6. Acknowledgment

Freely available medical images provided as mammographic data base by MAIS are used. Mammographic image analysis homepage link is http://www.mammoimage.org/database/. We are thankful to MAIS for providing such a big data base for use.

7. References

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