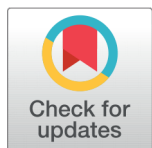


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Solving the cubic complex Ginzburg-Laundau equation by Homotopy analysis method

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Abstract

Objectives: This paper obtains the series solution of the cubic complex Ginzburg-Laundau equation, by means of homotopy analysis method (HAM). **Methods:** In addition to the homotopy analysis method, homotopy perturbation and Adomian decomposition methods are applied to determine approximation solution of the cubic complex Ginzburg-Laundau equation and advantage of using HAM. Also a theorem is proved to guarantee the convergence of the HAM to solve this equation. **Findings:** Three examples are solved to illustrate the efficiency of the proposed method, this method is compared with other analytical approximate methods such as homotopy perturbation method (HPM) and Adomian decomposition method (ADM) and it can be seen that these methods have the same results for this equation. **Application:** Homotopy analysis method as a reliable and valid scheme can be used to work out the cubic complex Ginzburg-Laundau equation which is nonlinear partial differential equation.

Keywords: Homotopy analysis method; Ginzburg-Laundau

1 Introduction

The Ginzburg-Laundau equation is one of the partial differential equations which occurs in chemical reactions, fluid mechanic, and many other sciences. The cGL is the general amplitude model which is describing the slow phase and amplitude modulations of a spatially distributed assembly of coupled oscillators close to its Hopf bifurcation⁽¹⁾. The cubic cGL equation has been applied to investigate various applied issues such as chemical turbulence, Poiseuille flow, Taylor-Coutte flow, Rayleigh-Benard convection, reaction-diffusion systems, nonlinear optics, and hydrodynamical stability problems. It shows rich dynamics and has been a model for the transition to spatio-temporal chaos. The cGL can be considered

as an usual form for a Hopf bifurcation in different types of spatially extended systems⁽²⁾.

We consider cubic complex Ginzburg-Laundau equation(cGL) as follows.

$$\frac{\partial w}{\partial t} = (1 + ia)\frac{\partial^2 w}{\partial x^2} + Rw - (1 + ib)|w|^2w, w(x, 0) = f(x), i^2 = -1 \quad (1)$$

where a, b, R are real constants and $w = w(x, t)$ is a complex unknown function and t is a nonnegative real quantity, also x is real^(3–7). The complex field w describes the modulations of the oscillator field b and R are two real control parameters. In point of fact, the amplitude w explains slow modulations in space and time of the underlying bifurcating spatially periodical pattern⁽⁸⁾, also some other details about other types of this equation can be considered in^(9–12). Wazwaz studied this equation by using the separation of variables method in⁽¹³⁾. In this work, the HAM, HPM, ADM are considered in order to obtain the approximate solution of Equation (1). HAM is a strong analytical method to solve the nonlinear topics and was first introduced and applied⁽¹⁴⁾. Lately, this method has been well used to work out plenty types of problems in different branches of science and engineering^(15–25). Homotopy analysis method has an auxiliary parameter h which gives us an easy approach to regulate the convergent region and the rate of convergence of the series solution. HAM mostly generates a very fast convergence of the solution series, usually just a few iterations attendant sufficient approximate solution, as well⁽²⁴⁾. Likewise, a theorem will be proved which illustrates the convergence of HAM. Also HPM and ADM present satisfactory results^(26,27). Total explanation of the present paper is as follows: In section 2, some preliminaries are given, and in section 3, the main idea of this paper is explained. In section 4, the convergence theorem is proved, and finally in section 5 three examples are solved by all three methods, and h -curves are plotted to show the region of convergence.

2 Preliminaries

Let the following partial differential equation:

$$N[w(x, t)] = 0$$

where N is a nonlinear operator, x and t define the independent variables and w is an unknown function. Via HAM, the zeroth-order deformation equation is:

$$(1 - q)L[\Phi(x, t, q) - w_0(x, t)] = qhH(x, t)N[\Phi(x, t, q)] \quad (2)$$

Where $q \in [0, 1]$ is the embedding parameter, $h \neq 0$ is an auxiliary parameter, L is an auxiliary linear operator and $H(x, t)$ is an auxiliary function. $\Phi(x, t, q)$ is an unknown function and $w_0(x, t)$ is an initiative approximation of $w(x, t)$. It is obvious, if $q = 0$ and $q = 1$ then:

$$\Phi(x, t, 0) = w_0(x, t), \Phi(x, t, 1) = w(x, t)$$

respectively. Therefore, when q increases from 0 to 1, the solution $\Phi(x, t, q)$ varies from $w_0(x, t)$ to the exact solution $w(x, t)$. Via Taylor's theorem, it can be expanded $\Phi(x, t, q)$ in a power series of the embedding parameter q as comes:

$$\Phi(x, t, q) = w_0(x, t) + \sum_{m=1}^{\infty} w_m(x, t)q^m \quad (3)$$

where

$$w_m(x, t) = \frac{1}{m!} \frac{\partial^m \Phi(x, t, q)}{\partial q^m} \Big|_{q=0} \quad (4)$$

Let the initiative approximation $w_0(x, t)$ the auxiliary linear operator L , the nonzero auxiliary parameter h and the auxiliary function $H(x, t)$ be correctly selected so that the power series Equation (3) converges at $q = 1$, then, it can be seen:

$$w(x, t) = w_0(x, t) + \sum_{m=1}^{\infty} w_m(x, t) \quad (5)$$

which must be the solution of the main nonlinear equation. Here, we consider the following set of vectors:

$$\vec{w}_n = \{w_0(x, t), w_1(x, t), \dots, w_n(x, t)\} \quad (6)$$

By differentiating the zeroth order deformation Equation (2), m times with regarding the embedding parameter q and then putting $q = 0$ and ultimately dividing by $m!$. we will have the following m th order deformation equation:

$$L[w_m(x, t) - \chi_m w_{m-1}(x, t)] = hH(x, t)R_m(\vec{w}_{m-1}) \quad (7)$$

where

$$R_m(\vec{w}_{m-1}) = \frac{1}{(m-1)!} \frac{\partial^{m-1} N[\Phi(x, t, q)]}{\partial q^{m-1}} \Big|_{q=0} \quad (8)$$

and

$$\chi_m = \begin{cases} 0 & m \leq 1 \\ 1 & m > 1 \end{cases} \quad (9)$$

It should be mentioned that $w_m(x, t)$ for $m \geq 1$ is governed by the linear Equation (7) with linear boundary situations that arise from the main problem. In order to study more about the HAM, ⁽²⁸⁾ is suggested.

3 Main Idea

In this section, we apply the HAM, HPM and ADM to solve Equation (1).

3.1 Homotopy analysis method

We consider Equation (1) as follows:

$$\frac{\partial w}{\partial t} = (1 + ia) \frac{\partial^2 w}{\partial x^2} + R w - (1 + ib) w^2 \bar{w}, w(x, 0) = f(x), i^2 = -1 \quad (10)$$

and

$$L[\Phi(x, t, q)] = \frac{\partial \Phi(x, t, q)}{\partial t}, L(c) = 0 \quad (11)$$

where c is a real constant,

$$N[\Phi(x, t, q)] = \frac{\partial \Phi(x, t, q)}{\partial t} - (1 + ia) \frac{\partial^2 \Phi(x, t, q)}{\partial x^2} - R\Phi(x, t, q) + (1 + ib)\Phi^2(x, t, q)\bar{\Phi}(x, t, q) \quad (12)$$

and $H(x, t) = 1$. The zeroth-order deformation equation is:

$$(1 - q)L[\Phi(x, t, q) - w_0] = qhN[\Phi(x, t, q)] \quad (13)$$

Also, the m th-order deformation equation:

$$L[w_m - \chi_m w_{m-1}] = hR_m(\vec{w}_{m-1}) \quad (14)$$

Where

$$R_m(\vec{w}_{m-1}) = \frac{\partial w_{m-1}}{\partial t} - (1 + ia) \frac{\partial^2 w_{m-1}}{\partial x^2} - R w_{m-1} + (1 + ib) \sum_{k=0}^{m-1} \sum_{j=0}^k w_j w_{k-j} \bar{w}_{m-1-k} \quad (15)$$

So,

$$w_m = \chi_m w_{m-1} + h \int_0^t R_m(\vec{w}_{m-1}) dt + c, m \geq 1 \quad (16)$$

3.2 Homotopy perturbation method

Consider Equation (10) as cubic complex Ginzburg-Landau equation(cGL), to work out Equation (10) via homotopy perturbation method, we make the following homotopy

$$(1 - p) \left(\frac{\partial w}{\partial t} - \frac{\partial w_0}{\partial t} \right) + p \left(\frac{\partial w}{\partial t} - (1 + ai) \frac{\partial^2 w}{\partial x^2} - R w + (1 + ib) w^2 \bar{w} \right) = 0$$

or

$$\frac{\partial w}{\partial t} - \frac{\partial w_0}{\partial t} = p \left(-\frac{\partial w_0}{\partial t} + (1 + ai) \frac{\partial^2 w}{\partial x^2} + R w - (1 + ib) w^2 \bar{w} \right) \quad (17)$$

Suppose, the solution of Equation (10) is in the following form

$$w = w_0 + p w_1 + p^2 w_2 + \dots \quad (18)$$

by putting Equation (18) into Equation (17), and equating the coefficients of the parts with the same powers of p , we get

$$p^0 : \frac{\partial w_0}{\partial t} = \frac{\partial w_0}{\partial t}$$

$$p^1 : \frac{\partial w_1}{\partial t} = -\frac{\partial w_0}{\partial t} + (1 + ai) \frac{\partial^2 w_0}{\partial x^2} + R w_0 - (1 + ib) w_0^2 \bar{w}_0, w_1(x, 0) = 0$$

$$p^2 : \frac{\partial w_2}{\partial t} = (1 + ai) \frac{\partial^2 w_1}{\partial x^2} + R w_1 - (1 + ib) (2 w_0 \bar{w}_0 w_1 + \bar{w}_1 w_0^2), w_2(x, 0) = 0$$

$$p^j : \frac{\partial w_j}{\partial t} = (1 + ia) \frac{\partial^2 w_{j-1}}{\partial x^2} + R w_{j-1} - (1 + ib) \left(\sum_{i=0}^{j-1} \sum_{k=0}^j w_k w_{i-k} \bar{w}_{j-t-1} \right), w_j(x, 0) = 0$$

Obviously,

$$w_0(x, t) = w_0(x, t) = f(x) \quad (19)$$

and, by having these assumptions, we can write the following recursive relation

$$w_1(x, t) = \int_0^t \left(-\frac{\partial w_0}{\partial t} + (1 + ai) \frac{\partial^2 w_0}{\partial x^2} + R w_0 - (1 + ib) w_0^2 \bar{w}_0 \right) dt$$

$$w_j(x, t) = \int_0^t \left((1 + ai) \frac{\partial^2 w_{j-1}}{\partial x^2} + R w_{j-1} - (1 + ib) \left(\sum_{i=0}^{j-1} \sum_{k=0}^i w_k w_{i-k} \bar{w}_{j-t-1} \right) \right) dt, j \geq 2 \quad (20)$$

The approximate solution of Equation (10) can be obtained by setting $p = 1$, in Equation (18), that is,

$$w = \lim_{p \rightarrow 1} w_0 + p w_1 + p^2 w_2 + \dots = w_0 + w_1 + w_2 + \dots$$

3.3 Adomian decomposition method

Consider Equation (10) and operator $L_t = \frac{\partial}{\partial t}$, applying the inverse operator $L_t^{(-1)} = \int_0^t (.) dt$ to both sides of Equation (10), we have

$$w(x, t) = w(x, 0) + \int_0^t \left((1 + ia) \frac{\partial^2 w}{\partial x^2} + R w - (1 + ib) w^2 \bar{w} \right) dt$$

then

$$w(x, t) = f(x) + \int_0^t \left((1 + ia) \frac{\partial^2 w}{\partial x^2} + R w - (1 + ib) w^2 \bar{w} \right) dt \quad (21)$$

to solve Equation (10) by ADM, as usual the series solution $w = \sum_{n=0}^{\infty} w_n$ is considered. then, the components w_n can be determined recursively, now we consider $w^2 \bar{w} = \sum_{n=0}^{\infty} A_n$, where $A_n(w_0, w_1, \dots, w_n)$, $n \geq 0$ are Adomian's polynomials which are computed by using this method, so we get

$$\sum_{n=0}^{\infty} w_n = f(x) + \int_0^t \left[(1 + ia) \frac{\partial^2 \sum_{n=0}^{\infty} w_n}{\partial x^2} + R \sum_{n=0}^{\infty} w_n - (1 + ib) \sum_{n=0}^{\infty} A_n \right] dt$$

then, we obtain the following recursive relation:

$$w_0(x, t) = f(x)$$

$$w_{n+1} = \int_0^t \left((1 + ia) \frac{\partial^2 w_n}{\partial x^2} + R w_n - (1 + ib) A_n \right) dt, n \geq 0 \quad (22)$$

where,

$$A_0 = w_0^2 \bar{w}_0$$

$$A_1 = 2w_0 w_1 \bar{w}_0 + w_0^2 \bar{w}_1$$

$$A_2 = 2w_0 w_2 \bar{w}_0 + w_1^2 \bar{w}_0 + 2w_0 w_1 \bar{w}_1 + w_0^2 \bar{w}_2$$

$$A_3 = 2w_0 w_3 \bar{w}_0 + 2w_1 w_2 \bar{w}_0 + 2w_0 w_2 \bar{w}_1 + w_1^2 \bar{w}_1 + 2w_0 w_1 \bar{w}_2 + w_0^2 \bar{w}_3, \dots$$

4 Convergence of the HAM

In this section, we prove the convergence of the series solution obtained from the HAM to the exact solution of the Equation (10).

Theorem: If the series solution

$$w(x, t) = w_0(x, t) + w_1(x, t) + \dots$$

obtained from the HAM is convergent, it converges to the exact solution of the Equation (10).

proof: Let the series

$$\sum_{m=0}^{\infty} w_m(x, t)$$

be convergent. Then we consider

$$w(x, t) = \sum_{m=0}^{\infty} w_m(x, t)$$

In this case, we can write,

$$\lim_{m \rightarrow \infty} w_m(x, t) = 0 \quad (23)$$

So

$$\sum_{m=1}^n [w_m(x, t) - \chi_m w_{m-1}(x, t)] = w_n(x, t) \quad (24)$$

assuming convergence of series solution, we have:

$$\sum_{m=1}^{\infty} [w_m(x, t) - \chi_m w_{m-1}(x, t)] = \lim_{n \rightarrow \infty} w_n(x, t) = 0 \quad (25)$$

then,

$$\sum_{m=1}^{\infty} L[w_m(x, t) - \chi_m w_{m-1}(x, t)] = L(\sum_{m=1}^{\infty} (w_m(x, t) - \chi_m w_{m-1}(x, t))) = 0 \quad (26)$$

By applying the following statment

$$L[w_m(x, t) - \chi_m w_{m-1}] = hH(x, t)R_m(\bar{w}_{m-1}) \quad (27)$$

we get:

$$\sum_{m=1}^{\infty} L[w_m(x, t) - \chi_m w_{m-1}] = hH(x, t) \sum_{m=1}^{\infty} R_m(\bar{w}_{m-1}) \quad (28)$$

Moreover, since $h, H(x, t) \neq 0$,

$$\sum_{m=1}^{\infty} [R_m(\bar{w}_{m-1})] = 0 \quad (29)$$

According to Equation (15), it can be seen:

$$\sum_{m=1}^{\infty} [R_m(\bar{w}_{m-1})] = \sum_{m=1}^{\infty} \frac{\partial w_{m-1}}{\partial t} - (1 + ia) \sum_{m=1}^{\infty} \frac{\partial^2 w_{m-1}}{\partial x^2} - R \sum_{m=1}^{\infty} w_{m-1} + (1 + ib) \sum_{m=1}^{\infty} \sum_{k=0}^{m-1} \sum_{j=0}^k w_j w_{k-j} \bar{w}_{m-1-k}$$

Therefore,

$$\sum_{m=1}^{\infty} [R_m(\bar{w}_{m-1})] = \sum_{m=1}^{\infty} \frac{\partial w_{m-1}}{\partial t} - (1 + ia) \sum_{m=1}^{\infty} \frac{\partial^2 w_{m-1}}{\partial x^2} - R \sum_{m=1}^{\infty} w_{m-1} + (1 + ib) \sum_{k=0}^{\infty} \sum_{m=k+1}^{\infty} \sum_{j=0}^k w_j w_{k-j} \bar{w}_{m-k-1}$$

then

$$\sum_{m=1}^{\infty} [R_m(\bar{w}_{m-1})] = \sum_{m=1}^{\infty} \frac{\partial w_{m-1}}{\partial t} - (1 + ia) \sum_{m=1}^{\infty} \frac{\partial^2 w_{m-1}}{\partial x^2} - R \sum_{m=1}^{\infty} w_{m-1} + (1 + ib) \sum_{j=0}^{\infty} \sum_{k=1}^{\infty} \sum_{m=1}^{\infty} w_j w_{k-j} \bar{w}_{m-1}$$

so, we obtain

$$\sum_{m=1}^{\infty} [R_m(\bar{w}_{m-1})] = \sum_{m=1}^{\infty} \frac{\partial w_{m-1}}{\partial t} - (1 + ia) \sum_{m=1}^{\infty} \frac{\partial^2 w_{m-1}}{\partial x^2} - R \sum_{m=1}^{\infty} w_{m-1} + (1 + ib) \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \sum_{m=1}^{\infty} w_j w_k \bar{w}_{m-1}$$

then, it is found that

$$\sum_{m=1}^{\infty} [R_m(\bar{w}_{m-1})] = \frac{\partial}{\partial t} - 0 - (1 + ia) \frac{\partial^2 \sum_{m=0}^{\infty} w_m}{\partial x^2} - R \sum_{m=0}^{\infty} w_m + (1 + ib) \sum_{j=0}^{\infty} w_j \sum_{k=0}^{\infty} w_k \sum_{m=0}^{\infty} w \quad (30)$$

From Equation (29) and Equation (30) and the relation

$$\sum_{m=0}^{\infty} \bar{w} = \sum_{m=0}^{\infty} w_m$$

we conclude that

$$w(x, t) = \sum_{m=0}^{\infty} w_m(x, t)$$

is the exact solution of Equation (10).

5 Sample Examples

In this section, we solve three cubic complex Ginzburg-Laundau equations via the HAM and the results are compared with the ADM and the HPM. Also, the region of convergence are shown in the HAM by plotting the h -curves. The programs have been provided and the figures have been plotted by Matlab package.

Example 1: Consider the following cGL equation:

$$w_t = (1 - 3/2i)w_{xx} + 2w - (1 + i/2)|w|^2w, w(x, 0) = e^{ix}, i^2 = -1$$

We solve the equation by the HAM, using Equation (16), we get:

$$w_0(x, t) = e^{ix}$$

$$w_1(x, t) = -hti e^{xi}$$

$$w_2(x, t) = - (hte^{xi}(2hi + ht + 2i)) / 2$$

$$w_3(x, i) = - (hte^{xt} (-h^2t^2i + 6h^2t + 6h^2i + 6ht + 12hi + 6i)) / 6, \dots$$

When $h = -1$, we can write

$$w_0(x, i) = e^k$$

$$w_1(x, t) = ite^k$$

$$w_2(x, t) = - (t^2e^{xi}) / 2 = \frac{(it)^2}{2!}e^h$$

$$w_3(x, t) = \frac{-(it^3e^{ix})}{6} = \frac{(it)^3}{3!}e^i, \dots,$$

so, we can easily see

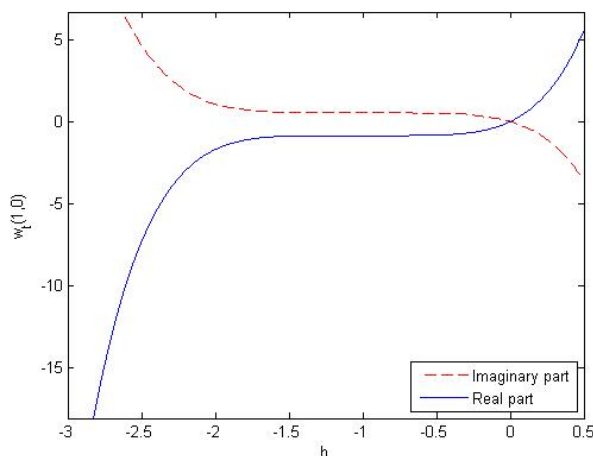
$$\begin{aligned} w(x, t) &= w_0(x, t) + w_1(x, t) + w_2(x, t) + w_3(x, t) + \dots = \\ &e^{ix} + ite^{ix} + \frac{(it)^2}{2!}e^{ix} + \frac{(it)^3}{3!}e^{ix} + \dots = e^{i(x+t)} \end{aligned}$$

that is the exact solution of the equation.

Table 1 shows the errors of HAM at $x = -2$ with different t when $n = 12$ and $h = -1$

Table 1. The errors of the HAM at $x = -2$ when $n = 12$ and $h = -1$

t	0.2	0.4	0.6	0.8	1
Error	2.2204e-016	9.9093e-016	2.0959e-013	8.8161e-012	1.6024e-010


Fig 1. The h -curve of 5-approximation ($n = 5$) of example1 when $x = 1$ and $t = 0$
Table 2. The errors of the HAM at the point (2,1)

n	Approximation at by HAM ($h = -1$)	Error
2	-1.117370845099253e+000 +3.850187686569845e-002i	1.635717717580491e-001
4	-9.831607254844369e-001 +1.457470757412923e-001i	8.251233356899829e-003
6	-9.901602223238910e-001 +1.410162723439193e-001i	1.972131260047780e-004
8	-9.899901272698717e-001 +1.411213931802091e-001i	2.744500998182562e-006
10	-9.899925183709056e-001 +1.411199958130925e-001i	2.497871950008141e-008
12	-9.899924964598675e-001 +1.411200081367663e-001i	1.602361278494555e-010
14	-9.899924966011185e-001 +1.411200080595068e-001i	7.635667678970700e-013
16	-9.899924966004430e-001 +1.411200080598685e-001i	2.769027319221402e-015

Figure 1 shows, the region of convergence of the example 1 at the point (1,0), which is $-1.5 < h < 0$

Table 2 shows the convergence of the HAM at the point (2,1) and $n = 2, 4, 6, 8, 10, 12, 14, 16$ and error is calculated by $|\sum_{i=0}^n w_i - w|$

Now, we solve the equation by HPM to compare the results. By using Equation (19) and Equation (20), we obtain

$$w_0(x, t) = e^x$$

$$w_1(x, t) = ti e^{ix}$$

$$w_2(x, t) = -(t^2 e^{ix})/2 = \frac{(it)^2}{2!} e^{ix}$$

$$w_3(x, t) = - (t^3 e^{xi} i) / 6 = \frac{(it)^3}{3!} e^{ix}, \dots$$

so, we get

$$w(x, t) = w_0(x, t) + w_1(x, t) + w_2(x, t) + w_3(x, t) + \dots = e^{ix} + ite^{ix} + \frac{(it)^2}{2!} e^{ix} + \frac{(it)^3}{3!} e^{ix} + \dots = e^{i(x+t)}$$

which is the same results of HAM when $(h = -1)$. The results of the ADM by using Equation (22) are as follows:

$$w_0(x, t) = e^{ix}$$

$$w_1(x, t) = ti e^{ix}$$

$$w_2(x, t) = - (t^2 e^{ix}) / 2 = \frac{(it)^2}{2!} e^{ix}$$

$$w_3(x, t) = - (t^3 e^{xi} i) / 6 = \frac{(it)^3}{3!} e^{ix}, \dots$$

therefore, we can write

$$w(x, t) = w_0(x, t) + w_1(x, t) + w_2(x, t) + w_3(x, t) + \dots = e^{ix} + ite^{ix} + \frac{(it)^2}{2!} e^{ix} + \frac{(it)^3}{3!} e^{ix} + \dots = e^{i(x+t)}$$

which is the same results of HPM and HAM when $(h = -1)$. It can be seen that all three methods are able to produce the similar results, via the HAM, we can avoid of difficulties in calculation of Adomian's polynomials specially for larger values of n , and computation of powers of p by the HPM.

Example 2: Consider the following cGL:

$$w_t = (1 - 9i)w_{xx} + 10/9w - (1 - i)|w|^2w, w(x, 0) = e^{-ix/3}, i^2 = -1$$

By using Equation (16), namely HAM, it can be seen:

$$w_0(x, t) = e^{-ix/3}$$

$$w_1(x, t) = -2ht \left(1/e^{(ix)/3} \right) i$$

$$w_2(x, t) = -2ht \left(1/e^{(ix)/3} \right) (hi + ht + i)$$

$$w_3(x, t) = - \left(4ht \left(1/e^{(xx)/3} \right) \left(-h^2t^2i + 3h^2t + (3h^2i)/2 + 3ht + 3hi + (3i)/2 \right) \right) / 3, \dots$$

When $(h = -1)$, we can see

$$w_0(x, t) = e^{-ix/3}$$

$$w_1(x, t) = 2t \left(1/e^{(ix)/3} \right) i = 2ti e^{-ix/3}$$

$$w_2(x, t) = -2t^2 \left(1/e^{ix/3} \right) = \frac{(2ti)^2 e^{-ix/3}}{2!}$$

$$w_3(x, t) = - \left(4t^3 \left(1/e^{ix/3} \right) i \right) / 3 = \frac{(2ti)^3 e^{-ix/3}}{3!}, \dots$$

so,

$$w(x, t) = w_0(x, t) + w_1(x, t) + w_2(x, t) + w_3(x, t) + \dots = e^{-ix/3} + 2ti e^{-ix/3} + \frac{(2ti)^2}{2!} e^{-ix/3} + \frac{(2ti)^3}{3!} e^{-ix/3} + \dots = e^{2ti - ix/3}$$

which is the accurate solution of the equation.

Figure 2 presents the convergence region of the example 2 at the point $(-1, 0)$.

Table 3 shows the errors of HAM ($h=-1$) at $x=1$ with different values of t , when $n=18$.

Table 3. The errors of HAM ($h=-1$) at $x=1$, when $n=18$

t	0.2	0.4	0.6	0.8	1
Error	1.1189e-016	3.1402e-016	1.5701e-016	6.1733e-014	4.2904e-012

Figure 3 shows the real part of the approximate solution for $n=6$ and the real part of exact solution, respectively. Also, **Figure 4** compares the imaginary part of the approximate solution for $n=6$ and the exact solution respectively.

Table 4 shows the convergence of this method at the point $(1, 0.6)$ and for $n=3, 6, 9, 12, 15, 18$ and the error is $|\sum_{i=0}^n w_i - w|$

Table 5 shows the errors, at the point $(1, 0.6)$, for different values of h .

Applying HPM, namely **Equation (19)** and **Equation (20)**, we get

$$w_0(x, t) = e^{-ix/3}$$

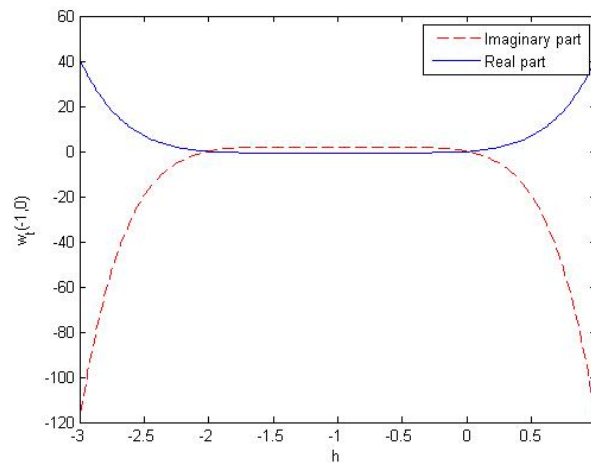


Fig 2. The -curve of 6-approximation ($n=2$) of example 2 when $x=-1$ and $t=0$

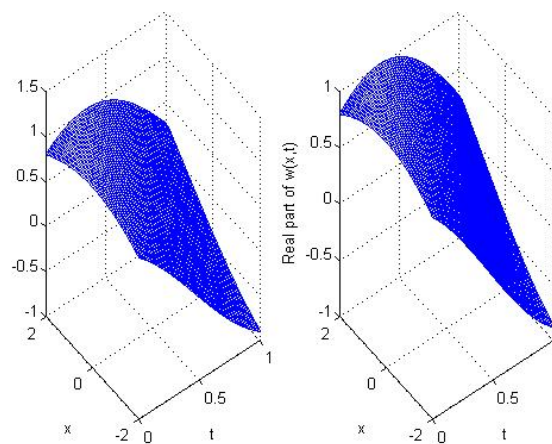


Fig 3. Real parts of the approximate(left) and the exact solutions of example 2 when $n=6$ and $h=-1$

Table 4. The errors of the HAM at the point $(1, 0.6)$.

n	Approximation at $(1, 0.6)$ via HAM ($h=-1$)	Error
3	$5.629895084462173e-001 + 7.701862199361181e-001i$	$8.476063171568474e-002$
6	$6.474995723928192e-001 + 7.628681672182660e-001i$	$7.047727373065257e-004$
9	$6.473723785312548e-001 + 7.621748949527030e-001i$	$1.697855943744531e-006$
12	$6.473707228589358e-001 + 7.621752712533816e-001i$	$1.712756669817200e-009$
15	$6.473707232781010e-001 + 7.621752729140687e-001i$	$8.816695142989886e-013$
18	$6.473707232789525e-001 + 7.621752729138399e-001i$	$1.570092458683775e-016$

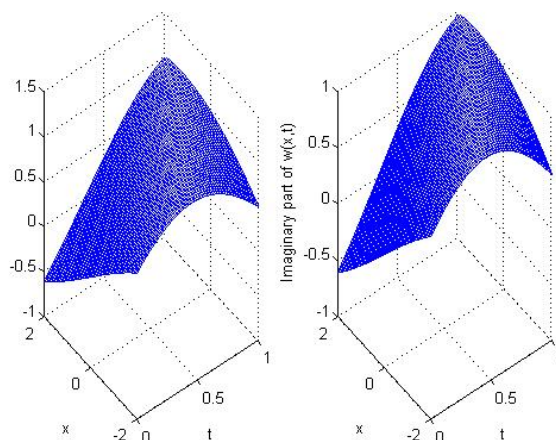


Fig 4. Imaginary parts of the approximate(left) and the exact solutions of example 2 when $n=6$ and $h=-1$

Table 5. The errors of the HAM, at the point $(1, 0.6)$

h	-1.25	-1	-0.75	-0.5
3	2.8584e-001	8.4761e-002	1.3122e-001	2.8949e-001
6	3.0911e-002	7.0477e-004	8.1473e-003	6.7683e-002
9	2.3592e-003	1.6979e-006	4.0043e-004	1.4635e-002
12	1.4666e-004	1.7128e-009	1.7133e-005	3.0039e-003
15	7.9268e-006	8.8167e-013	6.6626e-007	5.9337e-004

$$w_1(x, t) = 2t \left(1/e^{(ix)/3} i \right) = 2ti e^{-ix/3}$$

$$w_2(x, t) = -2t^2 \left(1/e^{ix/3} \right) = \frac{(2ti)^2 e^{-ix/3}}{2!}$$

$$w_3(x, t) = - \left(4t^3 \left(1/e^{ix/3} \right) i \right) / 3 = \frac{(2ti)^3 e^{-ix/3}}{3!}, \dots$$

So, we have

$$w(x, t) = w_0(x, t) + w_1(x, t) + w_2(x, t) + w_3(x, t) + \dots = e^{-ix/3} + 2ti e^{-ix/3} + \frac{(2ti)^2}{2!} e^{-ix/3} + \frac{(2ti)^3}{3!} e^{-ix/3} + \dots = e^{2ti - ix/3}$$

which is the same results of HAM when $h=-1$. Using Equation (22), the results of the ADM are as follows:

$$w_0(x, t) = e^{-ix/3}$$

$$w_1(x, t) = 2t \left(1/e^{(ix)/3} i \right) = 2ti e^{-ix/3}$$

$$w_2(x, t) = -2t^2 \left(1/e^{ix/3} \right) = \frac{(2ti)^2 e^{-ix/3}}{2!}$$

$$w_3(x, t) = - \left(4t^3 \left(1/e^{ix/3} \right) i \right) / 3 = \frac{(2ti)^3 e^{-ix/3}}{3!}, \dots$$

therefore, we can write

$$w(x, t) = w_0(x, t) + w_1(x, t) + w_2(x, t) + w_3(x, t) + \dots = e^{-ix/3} + 2ti e^{-ix/3} + \frac{(2ti)^2}{2!} e^{-ix/3} + \frac{(2ti)^3}{3!} e^{-ix/3} + \dots = e^{2ti-ix/3}$$

Example 3: consider the following PDE:

$$iu_t + u_{xx} + 2|u|^2 u - u + iu = 0, u(x, 0) = e^{ix} 30$$

Applying the HAM for this equation, we get

$$u_0(x, t) = e^{ix}$$

$$u_1(x, t) = hte^{ix}$$

$$u_2(x, t) = (hte^{ix}(2h + 2 + ht(1 - 4i))) / 2$$

$$u_3(x, t) = (hte^{ix}(h^2 t^2(1 - 20i) + h^2 t(6 - 24i) + 6h^2 + ht(6 - 24i) + 12h + 6)) / 6$$

$$u_4(x, t) = (hte^{ix}(h^3 t^3(-72i - 47) + h^3 t^2(12 - 240i) + h^3 t(36 - 144i) + 24h^3 + h^2 t^2(12 - 240i) + h^2 t(72 - 288i) + 72h^2 + ht(36 - 144i) + 72h + 24)) / 24$$

$$u_5(x, t) = (hte^{ix}(h^4 t^4(-232i - 559) + h^4 t^3(-1440i - 940) + h^4 t^2(120 - 2400i) + h^4 t(240 - 960i) + 120h^4 + h^3 t^3(-1440i - 940) + h^3 t^2(240 - 4800i) + h^3 t(720 - 2880i) + 480h^3 + h^2 t^2(120 - 2400i) + h^2 t(720 - 2880i) + 720h^2 + ht(240 - 960i) + 480h + 120)) / 120$$

Setting $h=-1$, we obtain

$$u_0(x, t) = e^{ix}$$

$$u_1(x, t) = -te^{ix}$$

$$u_2(x, t) = t^2 e^{xi} (1/2 - 2i)$$

$$u_3(x, t) = t^3 e^{xi} ((10i)/3 - 1/6)$$

$$u_4(x, t) = t^4 e^{xi} (-3i - 47/24)$$

$$u_5(x, t) = t^5 e^{xi} ((29i)/15 + 559/120)$$

so with the help of these computations, we have

$$u(x, t) = u_0(x, t) + u_1(x, t) + u_2(x, t) + u_3(x, t) + \dots = e^{ix} - t e^{ix} + t^2 e^{xi} (1/2 - 2i) + t^3 e^{xi} ((10i)/3 - 1/6) +$$

$$t^4 e^{xi} (-3i - 47/24) + t^5 e^{xi} ((29i)/15 + 559/120) + \dots = \sum_{n=0}^{\infty} \frac{(-t)^n}{n!} e^{ix} + i e^{ix} (-2t^2 + 10/3 t^3 - 3t^4 + 29/15 t^5 - \dots) + e^{ix} (-2t^4 + 4t^5 - \dots)$$

Which is the same series solution that has been calculated by HPM and ADM⁽²⁹⁾. According to the h-curve at any point and by changing h. it is possible to obtain other approximation solution. Also, Figure 5 shows the region of convergence of the example 3, when x=1 and t=0.

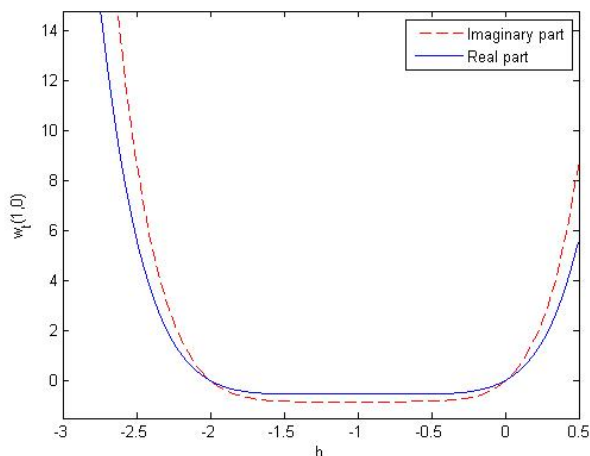


Fig 5. The h-curve of 6-approximation (n=6) of example 3 when x=1 and t=0.

6 Conclusion

This paper used, the homotopy analysis method, homotopy perturbation method, and Adomian decomposition method to solve the cubic complex Ginzburg-Landau equation and a theorem of convergence of the HAM was proved. Also, three examples were solved and the h-curves of the examples were drawn and some numerical results were presented to show the importance and applicability of the HAM, likewise one can observe that all three methods produce the similar results, however the HAM provides a situation that one can avoid of difficulties in calculation of Adomian's polynomials specially for larger values of n , and computation of powers of p by the HPM, also if necessary, by different values of h , region of convergence can be controlled via the HAM. Furthermore, the ADM and HPM are a specific case of the HAM when $h=-1$. Consequently, the HAM can be used to work out the cGL as a reliable and valid scheme.

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