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An extension of grey relational analysis for interval-valued fuzzy soft sets

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Abstract

Background: Neither any analytical (nor numerical) nor any statistical approach is often helpful in a situation where every person has his/her own choice. To cope with such situations usually we have to use fuzzy sets in combination with soft sets, which consist of predicates and approximate value sets as their images. **Material**: Choice values and comparison table techniques are two common decision-making techniques, which often don't result in same preference order or optimal choice. To overcome this kind of situation in decision-making problems, grey relational analysis method is used to get on a final decision. **Method**: Here we have used grey relational analysis method involving "interval-valued fuzzy soft sets" and "AND operation" to deal with such kind of problems. Our approach is also a correction to the method used by Kong et al.[1], where the Uni-Int technique is incorrectly used in case of soft sets instead of fuzzy soft sets. **Result:** The proposed method is effective in seeking an optimal choice in the case when common decision-making techniques fail to get on a final decision. **Conclusion**: By using grey relational analysis, a suitable method to choose one object from different choices has been proposed. It overcomes the grayness in decision-making problems for getting on a final decision when one gets too many options and finds it difficult to choose an optimal choice.

Keywords: Fuzzy soft set; Grey relational analysis; Un-Int; interval-valued fuzzy soft set

1 Introduction

Many problems of real life cannot be solved used typical Mathematical rules involving methods based on precise reasoning. Weaver[2] has categorized the different nature of problems of real life as problems of "organized simplicity" and "disorganized complexity". The first type of the two involves analytical problems which can be solved used calculus while the second type of problems refer to the statistical approaches for dealing with physical problems at molecular level that involve numerous variables and randomness of high degree. These problems are highly complementary to one another, under certain situations if one works then other fails. Majority of real life problems lie between these two which Weaver has named as the problems of organized complexity. The introduction of computer technology during world war II helped to deal many problems resembling the problems of organized complexity but it was realized that there are certain limits in dealing with complexity, which are unable to be overcome by either any computer technology or human abilities. Generally to deal with any type of problem, we have to construct a model based on reality aspects or some artificial objects. In construction of any model the factors affecting its usefulness are the credibility of the model, its complexity and uncertainty involved in it. Allowing more uncertainty will help in overcoming the complexity of the model and increasing its credibility. Therefore, the challenge was to develop techniques which can be used to estimate allowable uncertainty for such type of resulting models. The concept of fuzzy sets by Zadeh [3] in 1965 is considered as an evolution for dealing with uncertainty as his concept of fuzzy sets are the sets which do not have price boundaries like the typical sets have.

Though the theory of fuzzy sets has served as the best tool for dealing with uncertainties but scarcity of criterion for modeling different linguistic uncertainties limits its use as is pointed out by Molodtsov [4]. To provide a rich platform for parameterizations by overcoming the deficiencies in the fuzzy set theory, Molodtsov [5] introduced the idea of soft sets as a generalization of fuzzy sets. Fuzzy set theory in connection with soft sets have proved to be one of the most effective tool for dealing with uncertain situations some of which are discussed here. Maji et al. [6] propounded the perception of fuzzy soft sets and application of soft sets in a decision-making problems. Theoretic approach regarding "fuzzy soft set" offered by Roy et al. [7]. Maji et al. [8] deliberate reduct soft set's notion and discussed soft set's postulate. The abstraction of decision-making through comparison table technique discussed by Roy et al. He made decision-making very useful by constructing comparison table technique. Kong et al. [1] analyzed choice values and score values as evaluation bases to make a decision by discussing a counter example. After that Cagman et al. [9] introduced postulate of soft matrix and Uni-Int technique to make a decision for a problem. Un-Int technique facilitate decision maker to works on small number of attributes instead of larger number of attributes for soft set. He constructed Un-int technique for "AND", "OR", (AND), (OR) products. He also considered an example of 48 candidate and analyzed it with the help of Un-int technique for "AND" product. To make a decision with uncertain problems Jiang et al. [10,11] presented "semantic" methodology by using "ontology" and properties of "intuitionistic fuzzy soft set". Feng et al. [12,13] enhanced idea by presenting an adjustable approach for "level soft set" and "interval-valued soft set". Yang et al. [14] has given the conception of "interval-valued fuzzy soft set" "AND operation" and different applications. Majumdar et al. [15] gave the conception of generalized fuzzy soft sets. The soft set's algebra was presented by Zhan et al. [16]. Xu et al. [17] introduced the conception of vague soft sets and its properties. By relating different parameters, Ali et al. [18] introduced some advanced operations of soft set's concept. Kong et al. [19] gave an algorithm to overcome the problems of adding parameters and suboptimal choices.

Mostly decision-making techniques involve "choice values" technique or "score values" technique for ranking of alternatives which often don't result in same preference order. To overcome this kind of situation, grey relational analysis method is used to get on a final decision. Here we have used grey relational analysis method involving "interval-valued fuzzy soft sets" and "AND operation" to deal with such kind of problems. Our approach is also a correction to the method used by Kong et al. [1], where the Uni-Int technique is incorrectly used in case of soft sets instead of fuzzy soft sets. Moreover, the technique has been extended to "intuitionistic fuzzy soft sets" and "interval-valued intuitionistic fuzzy soft sets" by imposing different thresholds on different criterion using level soft sets.

2 Preliminaries

Definition 2.1. [7] For a universal set U and a parameters set E. Let (L, E) be a set of all fuzzy in U then a combination (L, E) is known as fuzzy soft set over , where L is a mapping as described below.

$$\mathrm{L} : \mathrm{E} \to \mathrm{L(U)}$$

Definition 2.2. [7] If we have two fuzzy soft sets $(\mathrm{L},\mathrm{T}_1)$ and $(\mathrm{M},\mathrm{T}_2)$ over a universe set U. Then we define $(\mathrm{L},\mathrm{T}_1)$ AND $(\mathrm{M},\mathrm{T}_2)$ is a fuzzy soft set denoted by $(L,T_1) \wedge (M,T_2)$ defined as $(L,T_1) \wedge (M,T_2) = (H,\ T_1\ X\ T_2)$ where $\mathrm{H}(\alpha,\beta) = \mathrm{L}(\alpha) \widetilde{\cap} \mathrm{M}(\beta), \forall\alpha \in \mathrm{T}_1$ and $\forall\beta \in \mathrm{T}_2$, $\widetilde{\cap}$ is a operation "fuzzy intersection" of two fuzzy soft sets.

3 Choice value technique

The choice value [6] of a participant/alternative $p_{t_i} \in P_t$ is c_{v_i}, given by $c_{v_i} = \Sigma_k\, p_{ik}$where p_{ik} are the entries in the given table. We illustrate the idea by discussing an example.

Suppose we have three participants, and we want to select a participant by using choice values technique. Here p_{t3} is the best choice.

P_t	x_1	x_3	x_5	Choice values
p_{t1}	0.1	0.2	0.3	c_{v1} = 0.6
p_{t2}	0.2	0.3	0.4	c_{v2} = 0.9
p_{t3}	0.8	0.7	0.5	c_{v3} = 2.0

4 Comparison table technique (or Score value technique)

A square table[7] where both the rows and columns involve alternatives/objects is called comparison table. Here each alternative/object is compared with every other alternative/object in the universal set U. For a comparison table involving n object p_{t1} , p_{t2} ,...p_{tn} , let c_{ik} = the count of attributes such that degree of membership grade of $p_{ti} \geq$ that of p_{tk} .

It can be observed that $c_{ik} \in \{0, 1, 2, ...n\}$ and c_{ik} = n if i=k. Thus c_{ik} indicates an integral number for which p_ti dominates p_tk for all $p_{tk} \in$ U. In comparison table technique we use score of an alternative for their ranking process for which we need to calculate the row sum (r_i) and column sum (t_k) of each alternative computed as $r_i = \sum_{k=1}^{n} c_{ik}$ and $t_k = \sum_{k=1}^{n} c_{ik}$ respectively. Here r_i is the count of total attributes of U and t_i is the count of total attributes for which p_{tk} is dominated by all the members of U. Then the score j_i of an alternative/object p_{ti} is calculated as

$$j_i = r_i - t_i$$

5 Grey Relational Analysis algorithm

Step 1 .

In first step we input the choice value sequence {c_{v1} , c_{v2} ,..., c_{vn} } and score value sequence {j_1 , j_2 ,..., j_n }.

Step 2.

"Grey relational generating"

$$c'_{vi} = \frac{c_{vi} - \min\{c_{vi}\}}{\max\{c_{vj}\} - \min\{c_{vi}\}},, \quad j'_{vi} = \frac{j_{vi} - \min\{j_{vi}\}}{\max\{j_{vi}\} - \min\{j_{vi}\}} \text{where} 1 = 1,2,3,4,\ldots,n$$

Step 3.

In this step we reorder the sequence as

$$\left\{c'_{v1},\ j'_1\right\}, \left\{c'_{v2},\ j'_2\right\}, \ldots, \left\{c'_m,\ j'_n\right\}$$

Step 4.

"Difference information"

$$c_{v_{\max}} = \text{Max}\left\{c'_{vi}\right\}, j_{\max} = \text{Max}\left\{\Delta j'_i\right\}, \Delta c'_{vi} = \left|c_{v_{\max}} - c'_{vi}\right|, \Delta j'_i = \left|j_{\max} - j'_i\right|$$

$$\Delta_{\max} = \text{Max}\left\{\Delta c'_{\text{v}}, \Delta j'_i\right\}, \Delta\min = \text{Min}\left\{\Delta c'_{\text{v}}, \Delta j'_i\right\}, \text{ where } i = 1,2,\ldots,n$$

Step 5.

"Grey relational coefficient"

$$\gamma(c_v, c_{vi}) = \frac{\Delta_{\min + \chi^*{}_{\Delta}\max}}{\Delta c'_{ij} + \chi^*_{\Delta}\max} \text{ and } \gamma(j, j_i) = \frac{\Delta_{\min + \chi * \Delta_{\max}}}{\Delta j'_i + \chi *_{\Delta_{\max}}}$$

Where χ is called "distinguishing coefficient" and $\chi \in [0,1]$. Its principle is to amplify or shorten the amplitude of "grey relative coefficient".

Step 6.

"Grey relational grade"

$$\gamma(p_{t_i}) = w_1 * \gamma\left(c_v, c_{v_j}\right) + w_2 * \gamma(j, j_i)$$

Where w1 and w2 are weights of evaluation factor and $w_1 + w_2 = 1$

Step 7.

"Decision making".

p_{tk} is the optimal choice, where $p_{t_k} = \max\ \gamma(p_{t_r})$. If decision makers wish to select more than one participants then they will select the participants according to the maximum number of grey relational grade.

6 Grey relational analysis for interval-valued fuzzy soft sets (IVFSS)

Definition 6.1. [20] For a universal set $\cup$ and parameter set $E, T_1 \subset E$, a combination (L, T_1) is known as IVFSS over $\cup$, where F is a mapping such that

$$L : T_1 \rightarrow P(U)$$

An IVFSS is a parameterized family of interval-valued fuzzy subsets of $U, \forall s \in T_1, L(s)$ is referred as the interval fuzzy value set of parameter s. Clearly $L(s)$ is written as $L(s) = \{(x, L(s^-), L(s^+)) : X \in U\}$ where $L(s^+)$ and $L(s^-)$ be the upper and lower membership's degrees of x to $L(s)$respectively.

Example 6.2. Let's imagine a business organization needs to fill a vacant position. There are 10 participants who applied legally for vacant position. The organization have chosen two decision makers, one is from the panel of directors and second one is from the office of human development. They wish to select a participant to fill a vacant position. They separately judge the desired qualities that are required to fill a vacant position by using grey algorithm based on "AND" operation.

Consider the set of participants $P_t = \{p_{t_1}, p_{t_2} \ldots, p_{t_10}\}$ which may be characterized by the set of parameters $E = \{x_1, x_2, x_3, x_4, x_5, x_6\}$. The parameters $x_i$where i = 1,2, ... 6, signifies "experience", "computer knowledge","training","young age","higher education" "good health" respectively.

Step 1.

First decision maker considered the set of parameters, $T_1 = \{x_1, x_3, x_4\}$ and second decision considered the set of parameters, $T_2 = \{x_2, x_3, x_4, x_6\}$, where $T_1, T_2 \subset E$.

Step 2.

In this step decision makers assign membership grades to their desired parameters as in the table given below.

Table 1. (L, T_1)

P_t	x_1	x_3	x_4
p_{t1}	[0.3,0.5]	[0.48,0.5]	[0.3,0.8]
p_{t2}	[0.4,0.6]	[0.54,0.58]	[0.5,0.54]
p_{t3}	[0.64,0.68]	[0.72,0.76]	[0.43,0.47]
p_{t4}	[0.62,0.68]	[0.51,0.55]	[0.78,0.82]
p_{t5}	[0.88,0.92]	[0.71,0.77]	[0.39,0.41]
p_{t6}	[0.78,0.82]	[0.84,0.88]	[0.63,0.67]
p_{t7}	[0.69,0.72]	[0.71,0.78]	[0.58,0.62]
p_{t8}	[0.52,0.58]	[0.61,0.69]	[0.78,0.82]
p_{t9}	[0.49,0.55]	[0.68,0.7]	[0.48,0.52]
p_{t10}	[0.78,0.82]	[0.41,0.47]	[0.32,0.38]

Table 2. (M, T_2)

P_t	x_2	x_3	x_4	x_6
p_{t1}	[0.1,0.3]	[0.48,0.5]	[0.31,0.81]	[0.58,0.62]
p_{t2}	[0.45,0.55]	[0.54,0.58]	[0.52,0.56]	[0.32,0.34]
p_{t3}	[0.5,0.7]	[0.72,0.76]	[0.45,0.48]	[0.69,0.71]
p_{t4}	[0.68,0.72]	[0.51,0.55]	[0.79,0.83]	[0.87,0.93]
p_{t5}	[0.31,0.35]	[0.71,0.77]	[0.39,0.41]	[0.63,0.68]
p_{t6}	[0.45,0.55]	[0.84,0.88]	[0.64,0.68]	[0.93,0.98]
p_{t7}	[0.88,0.92]	[0.71,0.78]	[0.58,0.63]	[0.78,0.82]
p_{t8}	[0.58,0.62]	[0.61,0.69]	[0.78,0.82]	[0.87,0.93]
p_{t9}	[0.43,0.45]	[0.68,0.7]	[0.48,0.52]	[0.52,0.58]
p_{t10}	[0.47,0.51]	[0.41,0.47]	[0.33,0.39]	[0.3,0.36]

Step 3.

Now we will find AND product $(L_{T_1} \wedge M_{T_2})$ of the fuzzy soft sets $(L, T_1)$and(M, T_2). Here we observe that if we perform the AND product of the above fuzzy soft sets then we will get 3×4=12 parameters of the form e_{ik} , where $e_{ik} = a_i \wedge b_k \forall i =$

$1,2,3$ and $k = 1,2,3,4$, but here we need fuzzy soft set for the parameters $R = \{x_{12}, x_{32}, x_{33}, x_{44}, x_{46}\}$. So we will get the resultant of $(L,\ T_1)$and$(M,\ T_2)$say $(K,\ R)$ after performing the AND operation as follows.

Table 3. $(L_{T_1} \wedge M_{T_2})$

P_t	x_{12}	x_{32}	x_{33}	x_{44}	x_{46}
p_{t1}	[0.1,0.3]	[0.1,0.3]	[0.48,0.5]	[0.3,0.8]	[0.3,0.62]
p_{t2}	[0.4,0.55]	[.45,0.55]	[0.54,0.58]	[0.5,0.54]	[0.32,0.34]
p_{t3}	[0.5,0.68]	[0.5,0.7]	[0.72,0.76]	[0.43,0.47]	[0.43,0.47]
p_{t4}	[0.62,0.68]	[0.51,0.55]	[0.51,0.55]	[0.78,0.82]	[0.78,0.82]
p_{t5}	[0.31,0.35]	[0.31,0.35]	[0.71,0.77]	[0.39,0.41]	[0.39,0.41]
p_{t6}	[0.45,0.55]	[0.45,0.55]	[0.84,0.88]	[0.63,0.67]	[0.63,0.67]
p_{t7}	[0.69,0.72]	[0.71,0.78]	[0.71,0.78]	[0.58,0.62]	[0.58,0.62]
p_{t8}	[0.52,0.58]	[0.58,0.62]	[0.61,0.69]	[0.78,0.82]	[0.78,0.82]
p_{t9}	[0.43,0.45]	[0.43,0.45]	[0.68,0.7]	[0.48,0.52]	[0.48,0.52]
p_{t10}	[0.47,0.51]	[0.41,0.47]	[0.41,0.47]	[0.32,0.38]	[0.3,0.36]

Step 4.

We find the optimistic reduct fuzzy soft set for the table.

Table 4. Optimistic reduct FSS of the table 9

P_t	x_{12}	x_{32}	x_{33}	x_{44}	x_{46}	Choice value
p_{t1}	0.3	0.3	0.5	0.8	0.62	c_{v1} =2.52
p_{t2}	0.5	0.55	0.58	0.54	0.34	c_{v2} =2.56
p_{t3}	0.68	0.7	0.76	0.47	0.47	c_{v3} =3.08
p_{t4}	0.68	0.55	0.55	0.82	0.82	c_{v4} =3.42
p_{t5}	0.35	0.35	0.77	0.41	0.41	c_{v5} =2.29
p_{t6}	0.55	0.55	0.88	0.67	0.67	c_{v6} =3.32
p_{t7}	0.72	0.78	0.78	0.62	0.62	c_{v7} =3.52
p_{t8}	0.58	0.62	0.69	0.82	0.82	c_{v8} =3.53
p_{t9}	0.45	0.45	0.7	0.52	0.52	c_{v9} =2.64
	0.51	0.47	0.47	0.38	0.36	c_{v10} =2.19

Step 5.

After computing "AND" product we will compose the comparison table of optimistic reduct FSS.

Table 5. Comparison table of optimistic reduct FSS

P_t	p_{t1}	p_{t2}	p_{t3}	p_{t4}	p_{t5}	p_{t6}	p_{t7}	p_{t8}	p_{t9}	p_{t10}
p_{t1}	5	2	2	0	2	1	2	0	2	3
p_{t2}	3	5	1	2	3	2	0	0	3	4
p_{t3}	3	4	5	3	4	2	0	3	3	5
p_{t4}	5	4	3	5	4	4	2	3	4	5
p_{t5}	3	2	1	1	5	0	0	1	1	3
p_{t6}	4	5	3	2	5	5	3	1	5	5
p_{t7}	4	5	5	3	5	2	5	3	5	5
p_{t8}	5	5	2	4	4	4	2	5	4	5
p_{t9}	3	2	2	1	4	0	0	1	5	3
p_{t10}	2	1	0	0	2	0	0	0	2	5

Step 6.

Now we will calculate the score values $j_i = r_i - t_i$ where r_i denote the column sum and t_i denote the row sum as calculated in table 3.6 as follows.

Table 6. Score values table

P_t	Row sum	Column sum	Score value
p_{t1}	19	37	j_1=-18
p_{t2}	23	35	j_2=-12
p_{t3}	32	24	j_3= 8
p_{t4}	39	21	j_4= 18
p_{t5}	17	38	j_5=-21
p_{t6}	38	20	j_6= 18
p_{t7}	42	14	j_7= 28
p_{t8}	40	17	j_8= 23
p_{t9}	21	34	j_9=-13
p_{t10}	12	43	j_{10}=-31

According to choice values p_{t8} is the optimal choice but score values shows that p_{t7} is the best choice. To choose which answer is the best one we use Grey algorithm which have following steps.

Step 1.

From the tables we write the choice value sequence $c_{v_i} = \{2.52,\ 2.56,\ 3.08,\ 3.42,\ 2.29,\ 3.32,\ 3.52,\ 3.53,\ 2.64,\ 2.19\}$ and sore value sequence $j_i = \{-18,\ -12,\ 8,\ 18,\ -21,\ 18,\ 28,\ 23,\ -13,\ -31\}$.

Step 2.

"Grey relational generating",

We find the values through grey relational generating

$$c'_{vi} = \frac{c_{vi} - \min\{c_{vi}\}}{\max\{c_{vi}\} - \min\{c_{vi}\}}, \quad j'_{vi} = \frac{j_{vj} - \min\{j_{vj}\}}{\max\{j_{vj}\} - \min\{j_{vj}\}}, \text{ where } i = 1,2,3,\ldots,10 \textit{ are}$$

$c'_{vi} = \{0.23, 0.28, 0.66, 0.92, 0.07, 0.84, 0.99, 1, 0.34, 0\}$ and $j'_i = \{0.22, 0.32, 0.66, 0.83, 0.17, 0.83, 1, 0.92, 0.30, 0\}$

Step 3.

In this step we reorder the sequence as $\left\{c'_{v1},\ j'_1\right\},\ \left\{c'_{v1},\ j'_1\right\},\ldots,\left\{c'_{vn},\ j'_n\right\}$ and we get $\left\{c'_{v1},\ j'_1\right\} = \{0.23, 0.22\}$, $\left\{c'_{v2},\ j'_2\right\} = \{0.28, 0.32\}$, $\left\{c'_{v3},\ j'_3\right\} = \{0.66, 0.66\}$, $\left\{c'_{v4},\ j'_4\right\} = \{0.92, 0.83\}$, $\left\{c'_{v5},\ j'_5\right\} = \{0.07, 0.17\}$, $\left\{c'_{v6},\ j'_6\right\} = \{0.84, 0.83\}$, $\left\{c'_{v7},\ j'_7\right\} = \{0.99, 1\}$, $\left\{c'_{v8},\ j'_8\right\} = \{1, 0.92\}$, $\left\{c'_{v9},\ j'_9\right\} = \{0.34, 0.30\}$, $\left\{c'_{v10},\ j'_{10}\right\} = \{0, 0\}$.

Step 4.

"Difference information"

To find $\Delta_{\max=\mathrm{Max}\left\{\Delta c'_v, \Delta j'_i\right\} \text{ and } \Delta\min=\mathrm{Min}\left\{\Delta c'_v, \Delta j'_i\right\} \text{ we have } c_{v\max=\mathrm{Max}\left\{c'_{vi}\right\}=1}}$ and $j_{\max} = \mathrm{Max}\{\Delta j'_i\} = 1$. Calculated values are $\Delta c'_{vi}\ \{0.77, 0.72, 0.34, 0.08, 0.93, 0.16, 0.01, 0, 0.66, 1\}$ and $\Delta j'_i = \{0.78, 0.68, 0.34, 0.17, 0.83, 0.17, 0, 0.08, 0.7, 1\}$ So $\Delta_{\max} = 1$ and $\Delta\min = 0$

Step 5.

In this step we will find "grey relative coefficient" through

$$\gamma(c_v, c_{vi}) = \frac{\Delta_{\min + \chi * \Delta\max}}{\Delta C'_{vi} + \chi *_{\Delta_{\max}} \text{ and } \gamma(j, j_i) = \frac{\Delta_{\min + \chi * \Delta\max}}{\Delta j'_i + \chi *_{\Delta\max}}}$$

Where χ is called "distinguishing coefficient" and $\chi \in [0,1]$. Its principle is to amplify or shorten the amplitude of "grey relative coefficient". Here we took $\chi = 0.52$. Calculated values are $\gamma(c_v,\ c_{v_i}) = \{0.40,\ 0.42,\ 0.60,\ 0.87,\ 0.36,\ 0.76,\ 0.98,\ 1,\ 0.44,\ 0.84\}$

$\gamma(j,\ j_i) = \{0.40,\ 0.43,\ 0.60,\ 0.75,\ 0.38,\ 0.75,\ 1,\ 0.87,\ 0.43,\ 0.34\}$

Step 6.

In this step we find the "grey relational grade" through

$\gamma(p_{t_i}) = w_1 * \ \gamma(c_v,\ c_{v_i}) + w_2 * \ \gamma(j,\ j_i)$, where w_1 and w_2 are weights of evaluation factor and $w_1 + w_2 = 1$ but in this thesis w_1=w_2=0.5. Calculated values are $\gamma(p_{t_1}) = 0.4,\ \gamma(p_{t_2}) = 0.42,\ \gamma(p_{t_3}) = 0.6,\ \gamma(p_{t_4}) = 0.81,\ \gamma(p_{ts}) = 0.37, \gamma(p_{t_6}) = 0.76,\ \gamma(p_{t_7}) = 0.99,\ \gamma(p_{t_8}) = 0.94,\ \gamma(p_{t_9}) = 0.44,\ \gamma(p_{t_{10}}) = 0.59$.

Step 7.

"Decision making"

After analysis, we observe that p_{t7} is optimal choice. If we select $\gamma(u_i) \geq 0.5$ then selected participants according to the maximum are $p_{t7} = 0.99,\ p_{ts} = 0.94,\ p_{t_4} = 0.81,\ p_{t6} = 0.76,\ p_{t3} = 0.61,\ p_{t10} = 0.59$.

7 Neutral reduct fuzzy soft set based decision-making

Step 1.

"Neutral redut" fuzzy soft set with choice values.

Table 7. Neutral reduct fuzzy soft set

P_t	x_{12}	x_{32}	x_{33}	x_{44}	x_{46}	Choice value
p_{t1}	0.2	0.2	0.49	0.55	0.47	c_{v1} =1.91
p_{t2}	0.48	0.5	0.56	0.52	0.33	c_{v2} =2.39
p_{t3}	0.59	0.6	0.74	0.45	0.45	c_{v3} =2.83
p_{t4}	0.65	0.53	0.53	0.8	0.8	c_{v4} =3.31
p_{t5}	0.33	0.33	0.74	0.4	0.4	c_{v5} =2.2
p_{t6}	0.5	0.48	0.86	0.65	0.65	c_{v6} =3.14
p_{t7}	0.7	0.74	0.74	0.6	0.6	c_{v7} =3.38
p_{t8}	0.55	0.6	0.65	0.8	0.8	c_{v8} =3.4
p_{t9}	0.44	0.44	0.69	0.5	0.5	c_{v9} =2.57
p_{t10}	0.49	0.44	0.44	0.35	0.33	c_{v10} =2.05

Step 2.

Neutral reduct fuzzy soft set's comparison table.

Table 8. Comparison table of neutral reduct fuzzy soft set

P_t	p_{t1}	p_{t2}	p_{t3}	p_{t4}	p_{t5}	p_{t6}	p_{t7}	p_{t8}	p_{t9}	p_{t10}
p_{t1}	5	2	2	0	2	0	0	0	1	3
p_{t2}	3	5	1	1	3	1	0	0	3	4
p_{t3}	3	4	5	2	5	2	1	3	3	5
p_{t4}	5	4	3	5	4	4	2	3	4	5
p_{t5}	3	2	1	1	5	0	1	1	1	3
p_{t6}	5	4	3	1	5	5	3	1	5	5
p_{t7}	5	5	5	3	5	2	5	3	5	5
p_{t8}	5	5	3	4	4	4	2	5	4	5
p_{t9}	4	2	2	1	4	0	0	1	5	4
p_{t10}	2	2	0	2	0	0	0	0	2	5

Step3.

Now we will calculate the score values $j_i = r_i - t_i$ where r_i denote the column sum and t_i denote the row sum as calculated in the table as follows.

Table 9. Score values table

P_t	Row sum	Column sum	Score value
p_{t1}	15	40	j_1= -25
p_{t2}	21	35	j_2= -14
p_{t3}	33	25	j_3= 8
p_{t4}	40	20	j_4= 20
p_{t5}	18	37	j_5= -19
p_{t6}	37	18	j_6= 19
p_{t7}	43	14	j_7= 29
p_{t8}	41	17	j_8= 24
p_{t9}	23	33	j_9= -10
p_{t10}	13	44	j_{10}= -31

According to choice values p_{t8} is the optimal choice but score values shows that p_{t7} is the best choice. To choose which answer is the best one we use grey algorithm which have following steps.

Step 1.

From the tables we write the choice value sequence $c_{vi}\,\{1.91,\ 2.39,\ 2.83,\ 3.31,\ 2.2,\ 3.14,\ 3.38,\ 3.4,\ 2.57,\ 2.05\}$ and score value sequence $j_i = \{-25,\ -14,\ 8,\ 20,\ -19,\ 19,\ 29,\ 24,\ -10,\ -31\}$.

Step 2.

Values generated through "grey relational generating" are $c'_{vi} = \{0,\ 0.32,\ 0.62,\ 0.94,\ 0.19,\ 0.83,\ 0.99,\ 1,\ 0.44,\ 0.09\}$ and $j'_i = \{0.1,\ 0.28,\ 0.65,\ 0.83,\ 0.2,\ 0.83,\ 1,\ 0.92\ ,\ 0.35,\ 0\}$.

Step 3.

In this step we reorder the sequence $\left\{c'_{v1},\ j'_1\right\} = \{0.0, 0.1\}$, $\left\{c'_{v2},\ j'_2\right\} = \{0.32, 0.28\}$, $\left\{c'_{v3},\ j'_3\right\} = \{0.62,\ \ 0.65\}$, $\left\{c'_{v4},\ j'_4\right\} = \{0.94, 0.85\}$, $\left\{c'_{v5},\ j'_5\right\} = \{0.19, 0.2\}$, $\left\{c'_{v6},\ j'_6\right\} = \{0.83,\ \ 0.83\}\left\{c'_{v7},\ j'_7\right\} = \{0.99, 1\}$, $\left\{c'_{v8},\ j'_8\right\} = \{1, 0.92\}$, $\left\{c'_{v9},\ j'_9\right\} = \{0.44,\ 0.35\}$, $\left\{c'_{v10},\ j'_{10}\right\} = \{0.09,\ 0\}$.

Step 4 .

$$\Delta_{\max} = \mathrm{Max}\left\{\Delta c'_{vi}, \Delta j'_i\right\} = 1 \text{ and } \Delta \min = \mathrm{Min}\left\{\Delta c'_{vi}, \Delta j'_i\right\} = 0$$

Step 5.

For $\chi = 0.52$"grey relative coefficient" $\gamma(c_v,\ c_{vi}) = \{0.34,\ 0.43,\ 0.58,\ 0.9,\ 0.39,\ 0.75,\ 0.98,\ 1,\ 0.48,\ 0.36\}\gamma(j,\ j_i) = \{0.37,\ 0.42,\ 0.6,\ 0.78,\ 0.39,\ 0.75,\ 1,\ 0.87,\ 0.44,\ 0.34\}$

Step 6.

"grey relational grade" for w_1=w_2=0.5

$\gamma(p_{t_1}) = 0.36,\ \gamma(p_{t_2}) = 0.42,\ \gamma\left(p_{t_3}\right) = 0.59,\ \gamma(p_{t_4}) = 0.84,\ \gamma(p_{ts}) = 0.39,\ \gamma(p_{ts}) = 0.39, \gamma\left(p_{t_6}\right) = 0.75,\ \gamma(p_{t7}) = 0.99,\ \gamma\left(p_{t_8}\right) = 0.94,\ \gamma\left(p_{t_9}\right) = 0.46,\ \gamma\left(p_{t_{10}}\right) = 0.35.$

Step 7.

According to grey relational analysis p_{t7} is the optimal choice. If there are more than one seats then we select the participants according to maximum numbers of grades for example if we take $\gamma(p_t i) \geq 0.5$ then selected participants are p_{t7}, p_{t6}, p_{t4}, p_{t8}, p_{t3}.

8 Mid-level soft set based decision-making

Mid-level soft set of neutral reduct fuzzy soft set. Thresholds for parameters $\{x_{12}, x_{32}, x_{33}, x_{44}, x_{46}\}$ are {0.49, 0.49, 0.64, 0.56, 0.53} respectively. Mid-level soft set's tabular representation with choice values.

Table 10. Mid level soft set

P_t	x_{12}	x_{32}	x_{33}	x_{44}	x_{46}	Choice value
p_{t1}	0	0	0	0	0	$c_{v1}=0$
p_{t2}	0	1	0	0	0	$c_{v2}=1$
p_{t3}	1	1	1	0	0	$c_{v3}=3$
p_{t4}	1	1	0	1	1	$c_{v4}=4$
p_{t5}	0	0	1	0	0	$c_{v5}=1$
p_{t6}	1	0	1	1	1	$c_{v6}=4$
p_{t7}	1	1	1	1	1	$c_{v7}=5$
p_{t8}	1	1	1	1	1	$c_{v8}=5$
p_{t9}	0	0	1	0	0	$c_{v9}=1$
p_{t10}	1	0	0	0	0	$c_{v10}=1$

Here choice values shows that p_{t7} and p_{t8} are the best choices but score value from table 3.9 of neutral reduct fuzzy soft set shows that p_{t7} is the optimal decision. To overcome this confusion we use grey algorithm.

Step 1.

From the tables we write the choice value sequence $c_{vi} = \{0,\ 1,\ 3,\ 4,\ 1,\ 4,\ 5,\ 5,\ 1,\ 1\}$ and score value sequence $j_i = \{-25,\ -14,\ 8,\ 20,\ -19,\ 19,\ 29,\ 24,\ -10,\ -31\}$.

Step 2.

We compute "grey relational generating" $c'_{vi} = \{0,\ 0.2,\ 0.6,\ 0.8,\ 0.2,\ 0.8,\ 1,\ 1,\ 0.2,\ 0.2\}$ and $j'_i = \{0.1,\ 0.28,\ 0.65,\ 0.83,\ 0.2,\ 0.83,\ 1,\ 0.92,\ 0.35,\ 0\}$.

Step 3.

In this step we reorder the sequence $\left\{c'_{v1},\ j'_1\right\} = \{0.0, 0.1\}$, $\left\{c'_{v2},\ j'_2\right\} = \{0.2, 0.28\}$, $\left\{c'_{v3},\ j'_3\right\} = \{0.6, 0.65\}$, $\left\{c'_{v4},\ j'_4\right\} = \{0.8, 0.85\}$, $\left\{c'_{v5},\ j'_5\right\} = \{0.2, 0.2\}$, $\left\{c'_{v6},\ j'_6\right\} = \{0.8, 0.83\}$, $\left\{c'_{v7},\ j'_7\right\} = \{1, 1\}$, $\left\{c'_{v8},\ j'_8\right\} = \{1, 0.92\}$, $\left\{c'_{v9},\ j'_9\right\} = \{0.2, 0.35\}$, $\left\{c'_{v10},\ j'_{10}\right\} = \{0.2, 0\}$.

Step 4.

$$\Delta_{\max} = \mathrm{Max}\left\{\Delta c'_{\mathrm{v}}, \Delta j'_i\right\} = 1 \text{ and } \Delta \min = \mathrm{Min}\left\{\Delta c'_{\mathrm{vi}}, \Delta j'_i\right\} = 0$$

Step 5.

For $\chi = 0.52$ generated values of "grey relative coefficient" are $\gamma(c_v,\ c_{vi}) = \{0.34,\ 0.39,\ 0.56,\ 0.72,\ 0.39,\ 0.72,\ 1,\ 1,\ 0.39,\ 0.39\}$

$\gamma(j,\ j_i) = \{0.37,\ 0.42,\ 0.6,\ 0.78,\ 0.39,\ 0.75,\ 1,\ 0.87,\ 0.44,\ 0.34\}$

Step 6.

"grey relational grade", for $w_1 = w_2 = 0.5$

$\gamma(p_{t_1}) = 0.36,\ \gamma(p_{t_2}) = 0.40,\ \gamma(p_{t_3}) = 0.58,\ \gamma(p_{t_4}) = 0.75,\ \gamma(p_{t_5}) = 0.39,\ \gamma(p_{t6}) = 0.74,\ \gamma(p_{t7}) = 1,\ \gamma(p_{t8}) = 0.94,\ \gamma(p_{t9}) = 0.42,\ \gamma(p_{t10}) = 0.36$.

Step 7.

According to grey relational analysis p_{t7} is the optimal choice. If there are more than one seats then we select the candidate according to maximum numbers of grades for example if we take $\gamma(p_{t_i}) \geq 0.54$ then selected participants are p_{t7}, p_{t8}, p_{t4}, p_{t6}, p_{t3}.

9 Pessimistic reduct fuzzy soft set based decision-making

Step 1.

Pessimistic reduct FSS of $(L_{T_1} \wedge M_{T_2})$ with choice values.

Table 11. Pessimistic reduct fuzzy soft set of $(L_{T_1} \wedge M_{T_2})$

P_t	x_{12}	x_{32}	x_{33}	x_{44}	x_{46}	choice value
p_{t1}	0.1	0.1	0.48	0.3	0.3	c_{v1} =1.28
p_{t2}	0.4	0.45	0.54	0.5	0.32	c_{v2} =2.21
p_{t3}	0.5	0.5	0.72	0.43	0.43	c_{v3} =2.58
p_{t4}	0.62	0.51	0.51	0.78	0.78	c_{v4} =3.20
p_{t5}	0.31	0.31	0.71	0.39	0.39	c_{v5} =2.11
p_{t6}	0.45	0.45	0.84	0.63	0.63	c_{v6} =3.00
p_{t7}	0.69	0.71	0.71	0.58	0.58	c_{v7} =3.27
p_{t8}	0.52	0.58	0.61	0.78	0.78	c_{v8} =3.27
p_{t9}	0.43	0.43	0.68	0.48	0.48	c_{v9} =2.50
p_{t10}	0.47	0.41	0.41	0.32	0.3	c_{v10} =1.91

Step 2.

Table of comparison for pessimistic reduct fuzzy soft set presented as follows.

Table 12. Comparison table of pessimistic reduct fuzzy soft set

p_t	p_{t1}	p_{t2}	p_{t3}	p_{t4}	p_{t5}	p_{t6}	p_{t7}	p_{t8}	p_{t9}	p_{t10}
p_{t1}	5	0	0	0	0	0	0	0	0	2
p_{t2}	5	5	1	1	3	1	0	0	2	4
p_{t3}	5	4	5	1	5	2	1	1	3	5
p_{t4}	5	4	4	5	4	4	2	3	4	5
p_{t5}	5	2	0	1	5	0	1	1	1	3
p_{t6}	5	5	3	1	5	5	3	1	5	4
p_{t7}	5	5	5	5	5	2	5	3	5	5
p_{t8}	5	5	4	4	4	4	2	5	4	5
p_{t9}	5	3	2	1	4	0	0	1	5	4
p_{t10}	4	1	0	0	2	1	0	0	1	5

Step 3.

Tabular representation of Score values presented in table 3.13.

Table 13. Score values table

P_t	Row sum	Column sum	Score value
p_{t1}	7	49	j_1=-42
p_{t2}	22	34	j_2=-12
p_{t3}	32	24	j_3= 8
p_{t4}	40	19	j_4= 21
p_{t5}	19	37	j_5=-18
p_{t6}	37	19	j_6= 18
p_{t7}	45	14	j_7= 31
p_{t8}	42	15	j_8= 27
p_{t9}	25	30	j_9=-5
p_{t10}	14	42	j_{10}=-28

Here choice values shows that p_{t7} and p_{t8} are the best choices but score value table of neutral reduct fuzzy soft set shows that p_{t7} is the optimal decision. To overcome this confusion we use grey algorithm.

Step 1.

From the tables we write the choice value sequence $c_{vi} = \{1.28,\ 2.21,\ 2.58,\ 3.2,\ 2.11,\ 3,\ 3.27,\ 3.27,\ 2.5,\ 1.91\}$and sore value sequence $j_i = \{-42,\ -12,\ 8,\ 21,\ -18,\ 18,\ 31,\ 27,\ -5,\ -28\}$.

Step 2.

we compute "grey relational generating" $c'_{vi} = \{0,\ 0.47,\ 0.65,\ 0.96,\ 0.42,\ 0.86,\ 1,\ 1,\ 0.61,\ 0.32\}$ and $j'_i = \{0,\ 0.41,\ 0.68,\ 0.86,\ 0.33,\ 0.82,\ 1,\ 0.94,\ 0.51,\ 0.19\}$

Step 3.

In this step we reorder the sequence.

$\left\{c'_{v1},\ j'_1\right\} = \{0,0\}$, $\left\{c'_{v2},\ j'_2\right\} = \{0.47,0.41\}$, $\left\{c'_{v3},\ j'_3\right\} = \{0.65,0.68\}$, $\left\{c'_{v4},\ j'_4\right\} = \{0.96,0.86\}$, $\left\{c'_{v5},\ j'_5\right\} = \{0.42,0.33\}$, $\left\{c'_{v6},\ j'_6\right\} = \{0.86,0.82\}$, $\left\{c'_{v7},\ j'_7\right\} = \{1,1\}$, $\left\{c_{v8},\ j'_8\right\} = \{1,0.94\}$, $\left\{c'_{v9},\ j'_9\right\} = \{0.61,0.51\}$, $\left\{c'_{v10}, j'_{10}\right\} = \{0.32,0.19\}$.

Step 4.

$$\Delta_{\max = \mathrm{Max}\left\{\Delta c'_{vi}, \Delta j'_i\right\} = 1 \text{ and } \Delta \min = \mathrm{Min}\left\{\Delta c'_{vi}, \Delta j'_i\right\} = 0}$$

Step 5.

For $\chi = 0.52$ values of "grey relative coefficient" are

$\gamma(c_v,\quad c_{vi}) = \{0.34,\ 0.50,\ 0.60,\ 0.93,\ 0.47,\ 0.79,\ 1,\ 1,\ 0.57,\ 0.43\}$

$\gamma(j,\ j_i) = \{0.34,\ 0.47,\ 0.62,\ 0.79,\ 0.44,\ 0.74,\ 1,\ 0.90,\ 0.51,\ 0.39\}$

Step 6.

"Grey relational grade", for $w_1 = w_2 = 0.5$

$\gamma(p_{t_1}) = 0.34,\ \gamma(p_{t_2}) = 0.48,\ \gamma\left(p_{t_3}\right) = 0.61,\ \gamma(p_{t_4}) = 0.86,\ \gamma(p_{ts}) = 0.46,\ \gamma\left(p_{t_6}\right) = 0.76,\ \gamma(p_{t_7}) = 1,\ \gamma\left(p_{t_8}\right) = 95,\ \gamma(p_{t9}) = 0.54,\ \gamma\left(p_{t_{10}}\right) = 0.41.$

Step 7.

According to grey relational analysis p_{t7} is the optimal choice. If there are more than one seats then we select the candidate according to maximum numbers of grades for example if we take $\gamma\left(p_{t_i}\right) \geq 0.55$ then selected participants are $p_{t7},\ p_{t8},\ p_{t4},\ p_{t6}, p_{t3}.$

10 Conclusion

Here we have dealt with one of the ambiguous situations arising in solving a problem from the class of organized complexity by making use of grey relational analysis technique. By using IVFSS and level soft sets for imposing desired thresholds on different criterion, we arrived at different optimal choices by using Choice value technique and comparison table technique. To resolve the problem of preference order, grey relational analysis method was used to get on a suitable selection. The approach used here is a correction to the method used by Kong et al.[19], where the Uni-Int technique was incorrectly used in case of soft sets instead of fuzzy soft sets.

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