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Green design and product stewardship approach for two-warehouse inventory model

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Abstract

Background/Objectives: To trim down the recycling cost of any manufactured goods with the help of green design and product stewardship. **Methods/Statistical analysis:** For the planned EPQ (economic production quantity model) model, all costs are calculated to find total cost and this total cost is optimized with the help of the Hessian matrix. Sensitivity analysis is also carried w.r.t. different parameters, to illustrate the impact of these parameters on the proposed model. The convexity of the total cost function is also checked with the help of mathematical software Mathematica 9.0. **Findings:** Major finding of the proposed model are as follows: (i) Increase in the number of recycles results in the reduction of the total cost. (ii) Product stewardship parameter has a negative effect on total cost as the PS increases from 1 to 4 units, total cost decreases from 5926.00 to 5918.96 units (see Table 9) (similar findings can be written for numeric example 1 after correcting it). (iii) Green design costs have a positive effect on total cost, as the green design cost increases from 3 to 6 units, total cost also increases from 5918.49 to 5920.37 units (see Table 10). (iv) increase in the number of recycles results in the reduction of the total cost, as the number of recycles increases from 20 to 50 units total cost decreases from 5922.87 to 5919.12 units (see Table 11). **Novelty/Applications:** The Study of the effects of recycling by this green design and product stewardship approach makes the proposed model distinctive from the existing methods. The proposed model applies to eco-friendly manufacturing items with green design and product stewardship.

Keywords: Green design; product stewardship; production model; own warehouse (OW); rented warehouse (RW); shortage; deterioration

1 Introduction

In the current inventory modeling, green design and product stewardship are quite emerging issues. To resolve environmental stumbling blocks, inventory modeling can play a significant role in terms of green design and product stewardship. Manufacturer designs products considering it to be a profitable task, but most of the time environment is ignored deliberately or unintentionally. Through green design and product stewardship, a manufacturer can maintain both profit as well as an environmental concern. Frosch and Gallopoulos[1] decided strategies for the manufacturer to reduce environmental impact. To resolve issues of manufacturer, Glantschnig[2] proposed green design and an introduction of issues and challenges related to manufacturing green design. Due to demand of product stewardship, Shapiro and White[3] gave a right start to product stewardship through the life cycle and design of a product. Bellmann and Khare[4] resolved some economic issues in recycling end of life vehicles. To reduce all impact on environment, government should also play a role in safety check on manufacturing process adopted by the manufacturer. In this field, Das[5] revealed the responsibilities of government and manufacture to green concern or environmental issues. Lewis[6] proposed product stewardship in a new pattern and sustainability of it in the Australian packaging industry. A practical approach of putting product stewardship into practice, a commercial moss harvest in North West Oregon studied by Pack and Christy[7]. Burger[8] recommended environmental strategies to integrate ecological evolution, remediation, restoration, natural resources damage assessment, and long term stewardship on contaminated lands. Chung and Wee[9] investigated a green component life cycle value on design and reverse manufacturing in a semi-closed supply chain. Natural resources are also used in the manufacturing, and they should be utilized in a smart and economical way, so that Guang et al.[10] proposed a green supply chain based on utilization of natural resources. Christina et al.[11] scrutinized green operation and moderating role of environmental management capability of suppliers manufacturing firm performance. Zhou et al.[12] explored an integrated approach for optimizing green production strategies. Mallidis et al.[13] proposed a design and planning for the green global supply chain under periodic review replenishment policies. Domingo and Rio[14] revealed the linking use stage life cycle inventories with the product design model of usage. Zhu and He[15] approached green product design in the supply chain under competition. Some of the researchers may approach in a different way to resolve environmental issues like Balakrishnan and Suresh[16]. They studied the conceptual framework of multimodal logistics to improve logistics performance and descriptive statistics of multimodal of rail and road transportation. They also analyzed a case study of Ford India's finished vehicle distribution after using a multimodal network. In this same chain. Yadav et al.[17] analyze a sustainable approach for a green supply chain for warehouse with environmental collaboration by using genetic algorithm. To extend yadav et al.[17] model, Khalafi and Zarei[18] proposed a multi-periodic and multiproduct production model considering the environmental issue for the single objective green routing problem with the help of fuzzy theory and mathematical programming. Huo et al.[19] proposed a lean approach by which economic, social and environmental performance of a supply chain can be improve while any green process can improve only environmental performance of supply chain. Recently Du et al.[20] studied strategies for green design in a sustainable supply chain management.

Initially, most of the inventory models were based on the assumption that every manufacturer has its warehouse with unlimited capacity but in a realistic world, it is not possible. Every warehouse has limited capacity due to some real reasons. When manufacturer starts production, he accumulates all well in his warehouse. But when manufacturer got attractive price discount on raw material for bulk purchase or maybe the order cost is too higher than manufacturer have to use RW for the storage purpose. Inventory managers usually attracted to hold more quantity of items that can be stored in an owned warehouse. Rented ware house (RW) may also useful due to its good condition, like low-risk factors, low deterioration rate, and better storage facility, etc. But the manufacturer has to pay some rent or cost which is much greater than own ware house (OW). So that manufacturer uses the RW but tries to utilize the inventory of RW first and then OW inventory. In reality, various factors was induced by the decision-maker to order more items. Recently warehouse situation generally arises when the acquisition is higher than maintaining an RW which have better preservation technology. Hartely[21] was first who proposed a two warehouse inventory system. Sarma[22] generalized Hartely[21] to cover the transportation cost from RW to OW. Dave[23] proposed an economic quantity model with two levels of storage. Kar et al.[24] presented a deterministic inventory model with two levels of storage, a linear trend in demand, and a fixed time horizon. Zohu[25] approaches a multi-warehouse inventory model for deteriorating items with time-varying demand and storage. Yang et al.[26] investigated a two warehouse partial backlogging inventory model for deteriorating items under inflation. To survive in high demand, Singh et al.[27] again presented a two warehouse partial backlogging inventory model for the perishable product having exponential demand. Demand depends on several factors like time etc. to reveal this factor, Lee et al.[28] proposed a two warehouse production model for deteriorating items with time-dependent demand. Singh et al.[29] evaluated an inventory model for deteriorating items with shortages and stock-dependent demand under inflation for two shops under one management. Hariga[30] worked on an inventory model for the multi-warehouse under a fixed and flexible space leasing contract. Sett et al.[31] discussed a two warehouse inventory model with increasing demand and a time-dependent deteriorating rate. Singh et al.[32] proposed an optimal ordering policy for deteriorating items with power form stock dependent demand under two warehouse storage facilities. Kumar et al.[33] developed a two warehouse inventory model for deteriorating items with three component demand rates and a time-proportional backlogging rate and fuzzy environment. Recently, some more researchers like Singh et al.[34] and Khurana[35] also presented different approaches for different models.

In this study, a green design and product stewardship approach is used in the model for deteriorating items with shortages. Product stewardship is a predominant factor in green design. It is assumed that the demand rate is a function of price and time. The model is developed with a two-warehouse storage facility. To illustrate the utility of the model, two numerical examples are expounded; convexity and sensitivity analysis is also illuminating the constructive path for the proposed model.

The proposed study is further dealt in the following way: In section 2 deals with research gap analysis regarding the utility of the proposed model, in section 3, notations, and assumptions are provided which is used for the development of the proposed model. In section 4, the problem's definition is presented in the form of a flow chart. In section 5, a mathematical model is derived. In sections 6 and 7, an algorithm to solve the mathematical model and numerical examples are shown respectively.

2 Research gap

In the existing literature, different kinds of production inventory models are introduced and studied yet a lot of work had been done by the researcher in the field of green production. In a two warehouses inventory model green design and product stewardship is not used yet. In the proposed paper production depends on demand and still manufacturer needs a rented ware house due to sudden fluctuation in market. To resolve these fluctuation manufacturer should store goods in rented ware house. In the proposed work green design and product stewardship are applicable in the form of cost, which may be responsible to make a product green. This cost may increase the total cost, but it can reduce the recycling cost of inventory and it may reduce the salvage of the system. This concept become the main focal point of this study and that is achieved in it. To show the research gap the previous reports are tabulated as follows (Table 1).

Table 1. Comparison of parameters considered in inventory modeling reported earlier.

References	Parameters				
	Two warehouse	Recycling	Product Stewardship	Green design	Environmental Management capabilities
1	No	No	No	No	No
2	No	No	No	Yes	No
3	No	Yes	No	No	Yes
4	No	No	No	No	Yes
5	No	No	Yes	No	Yes
6	No	No	Yes	No	Yes
7	No	No	Yes	No	Yes
8	No	Yes	No	Yes	No
9	No	No	No	Yes	Yes
10	Yes	No	No	No	No
11	No	Yes	No	No	Yes
12	No	No	No	Yes	No
13	No	No	No	Yes	Yes
14	No	No	No	No	Yes
15	No	No	No	No	Yes
16	No	No	No	No	Yes
17	No	No	No	Yes	No
18	Yes	No	No	No	Yes
19	No	No	No	No	Yes
20	No	No	No	Yes	Yes
21	Yes	No	No	No	No
22	Yes	No	No	No	No
23	Yes	No	No	No	No
24	Yes	No	No	No	No
25	No	Yes	No	Yes	No
26	Yes	No	No	No	No
27	Yes	No	No	No	No
28	Yes	No	No	No	No
29	Yes	No	No	No	No
30	Yes	No	No	No	No
31	Yes	No	No	No	No
32	Yes	No	No	No	No

Continued on next page

Table 1 continued

33	Yes	No	No	No	No
34	Yes	No	No	No	No
35	Yes	No	No	No	No
Proposed Model	Yes	Yes	Yes	Yes	Yes

3 Notations and assumptions

3.1 Notation

1. P (t) Production rate is demand dependent (KD(t)).
2. D(t) Demand rate which is (a-bp)t (a, b>0), p is the selling price.
3. $\alpha(t)$ Deterioration rate of inventory items in OW is time-dependent is $\theta_1 t$..
4. $\beta(t)$Deterioration rate of inventory items in RW is time-dependent is i$\theta_2 t$
5. $I_1(t)$ Inventory level in OW at time t, t$\in [t_0, t_1]$.
6. $I_2(t)$ Inventory level in RW at time t, t$\in [t_1, t_2]$.
7. $I_3(t)$ Inventory level in RW at time t, t$\in [t_2, t_3]$.
8. $I_4(t)$ Inventory level in OW at time t, t$\in [t_3, t_4]$.
9. $I_5(t)$ Inventory level in OW at time t, t$\in [t_1, t_3]$.
10. $I_6(t)$ Inventory level in OW at time t, t$\in [t_4, t_5]$.
11. $I_7(t)$ Inventory level in OW at time t, t$\in [t_5, t_6]$.
12. c_1 Setup cost per production run.
13. w Storage capacity of OW.
14. c_2 Cost of deteriorated items.
15. D_i Quantity of deteriorated items during the production cycle.
16. H_i Quantity of stored items during the production cycle in OW and RW.
17. C_{OW} Carrying cost per inventory unit per unit time in OW.
18. C_{RW} Carrying cost per inventory unit per unit time in RW.
19. G$_d$ Green design life cycle cost.
20. G$_{dv}$ Fixed product life cycle design cost ratio for green design.
21. N Number of life cycles before a product is disposed of or recycles.
22. RW Rented ware house.
23. OW Own ware house.
24. R$_{DV}$ Variable product life design cost ratio for green design.
25. R$_j$ Reliability of the function.
26. PS Product stewardship.
27. TC Total cost.

3.2 Assumption

1. Demand is price and time dependent in this model. D(t)=(a-bp)t, where a,b are constant, p is selling price of the commodity and t is time. (for example Zhou[25] and Singh et. al[27])
2. The OW has a limited capacity of w units and the RW has unlimited capacity.
3. The inventory cost of RW is greater than OW.
4. Inventory decreased due to demand and deterioration only.
5. Production rate depends on time and rate of production is KD(t), where K is constant.
6. The deteriorating rate depends on time and deteriorating units cannot be replaced or repaired. Deterioration rate for the model is θt, where θ is deterioration rate and t in time.(for example sett et. al[31])
7. The RW is located near the OW so the transportation cost is negligible.
8. Deterioration rate of RW is lower than the OW.
9. The lead time is considered as negligible.
10. The shortage is allowed and completely backlogged.

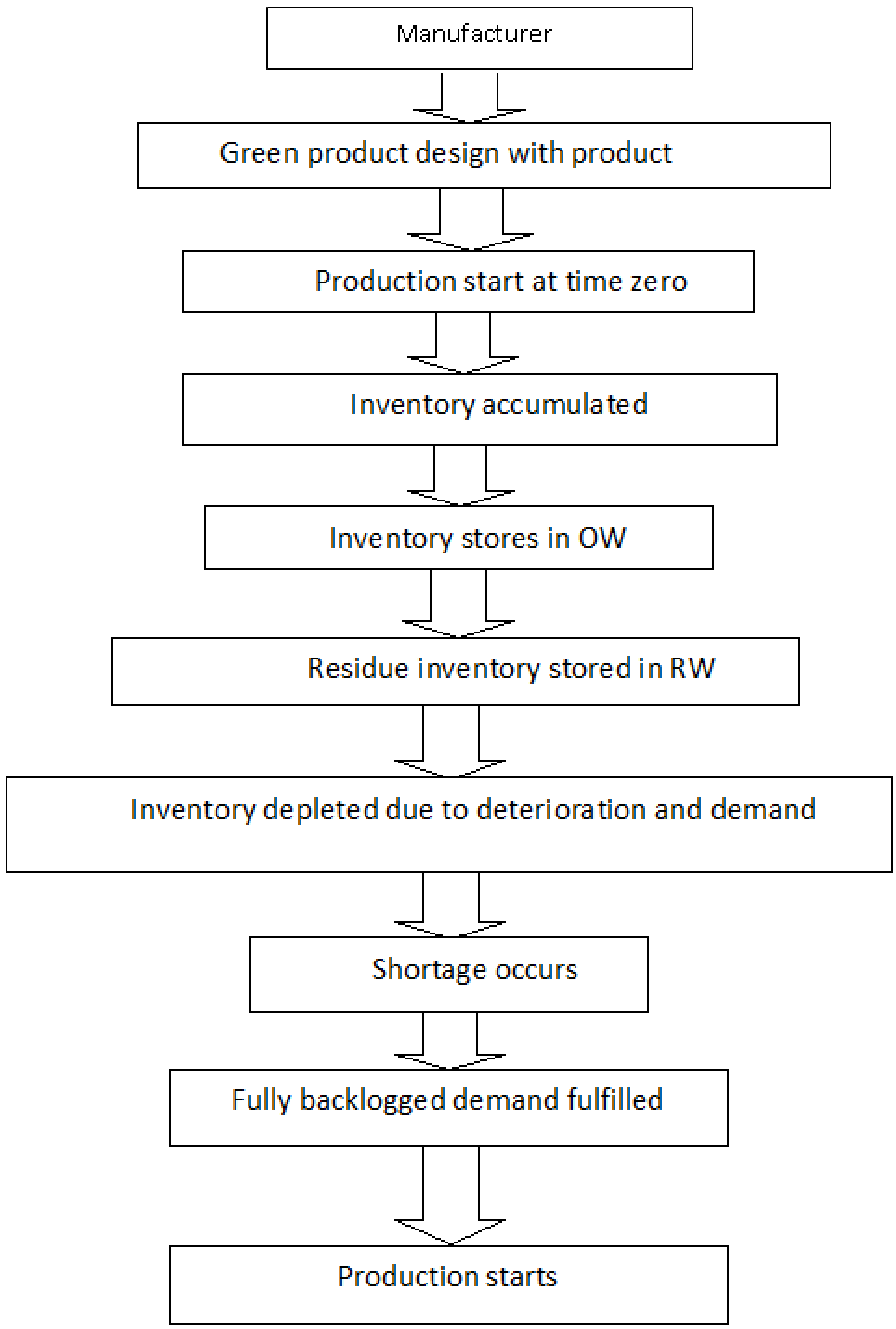

Fig 1. Flow chart of the production model

4 Mathematical model

In the proposed mathematical model, price and time-dependent demand are considered for different inventory levels. Here the production starts at zero time. Figure 1 shows the different inventory levels during the production cycle when two different warehouses are used OW and RW.

Throughout the time interval, 0 to t_1 inventory level levitate at the rate of $P-D(t)$ for manufactured own warehouse, where P is the rate of production, and D(t) is the demand rate. This situation solved with the help of differential equation

$$\frac{dI_1(t)}{dt}+\alpha(t)I_1(t)=P-D(t) \tag{1}$$

With the boundary conditions $0\leq t\leq t_1, \quad I_1(0)=0$

Throughout the time interval, t_1 to t_2 inventory level levitate at the rate of $P-D(t)$ for the rented warehouse, where P is the rate of production, and D(t) is the demand rate. This situation solved with the help of differential equation

$$\frac{dI_2(t)}{dt}+\beta(t)I_2(t)=P-D(t) \tag{2}$$

With the boundary conditions $t_1\leq t\leq t_2, \quad I_2(t_1)=0$

Throughout the time interval, t_2 to t_3 inventory level diminished at the rate of $D(t)$ for the rented warehouse, where D(t) is the demand rate. This situation is solved with the help of differential equation

$$\frac{dI_3(t)}{dt}+\beta(t)I_3(t)=-D(t) \tag{3}$$

With the boundary conditions $t_2\leq t\leq t_3, \quad I_3(t_3)=0$

Throughout the time interval, t_3 to t_4 inventory level diminished at the rate of $D(t)$ for manufactured own warehouse, where D(t) is the demand rate. This situation is solved with the help of differential equation

$$\frac{dI_4(t)}{dt}+\alpha(t)I_4(t)=-D(t) \tag{4}$$

With the boundary conditions $t_3\leq t\leq t_4, \quad I_4(t_4)=0$

Throughout the time interval, t_1 to t_3 inventory level diminished at the rate of $\alpha(t)$ from manufactured own warehouse, where $\alpha(t)$ is the deterioration rate of own warehouse. In time interval t_1 to t_3 inventory stored in the manufactured own warehouse deteriorate. This deteriorated inventory is calculated with the help of differential equations given below, where 'w' is the capacity of manufactured own warehouse.

$$\frac{dI_5(t)}{dt}+\alpha(t)I_5(t)=0 \tag{5}$$

With the boundary conditions $t_1\leq t\leq t_3, \quad I_5(t_1)=w$

Throughout the time interval, t_4 to t_5 shortage occurs at the rate of $D(t)$, where D(t) is the demand rate. This I_6(t) inventory is calculated with the help of differential equation

$$\frac{dI_6(t)}{dt}=-D(t) \tag{6}$$

With the boundary conditions $t_4\leq t\leq t_5, \quad I_6(t_4)=0$

Throughout the time interval t_5 to t_6 production starts again and inventory level increases at the rate $P-D(t)$, where P and D(t) are the rates of production the rate of demand respectively. The levitated inventory is calculated with the help of differential equation

$$\frac{dI_7(t)}{dt}+\alpha(t)I_4(t)=P-D(t) \tag{7}$$

With the boundary conditions $t_5\leq t\leq t_6, \quad I_7(t_6)=0$

On solving above differential equation (1), subject to the boundary condition

$$I_1(t) = \{(1-k)(a-bp)\left[\frac{t^2}{2} - \frac{3\theta_1 t^4}{8} - \frac{\theta_1^2 t^6}{16}\right]\} \tag{8}$$

On solving above differential equation (2), subject to the boundary condition

$$I_2(t) = \{(1-k)(a-bp)\left(1 - \frac{\theta_2^2 t_1^4}{2}\right)\left[\left(\frac{t^2}{2} - \frac{\theta_2 t^4}{8}\right) - \left(\frac{\theta_2 t^4}{8} + \frac{t_1^2}{2}\right)\right]\} \tag{9}$$

On solving above differential equation (3), subject to the boundary condition

$$I_3(t) = \{(a-bp)\left[\left(\frac{\theta_2 t^2}{2} - 1\right)\left(\frac{\theta_2 t_3^4}{8} + \frac{t_3^2}{2}\right) - \left(\frac{\theta_2 t^4}{4} - \frac{t^2}{2} - \frac{\theta_2 t^4}{8} + \frac{\theta_2^2 t^6}{8}\right)\right]\} \tag{10}$$

On solving above differential equation (4), subject to the boundary condition

$$I_4(t) = \{(a-bp)\left[\left(\frac{\theta_1 t^2}{2} - 1\right)\left(\frac{\theta_1 t_4^4}{8} + \frac{t_4^2}{2}\right) - \left(\frac{\theta_1 t^4}{4} - \frac{t^2}{2} - \frac{\theta_1 t^4}{8} + \frac{\theta_1^2 t^6}{8}\right)\right]\} \tag{11}$$

On solving above differential equation (5), subject to the boundary condition

$$I_5(t) = w\left(1 - \frac{\theta_1 t^2}{2}\right)\left(1 + \frac{\theta_1 t_1^2}{2}\right) \tag{12}$$

On solving above differential equation (6), subject to the boundary condition

$$I_6(t) = (a-bp)\left[\left(\frac{t_4^2}{2} - \frac{t^2}{2}\right)\right] \tag{13}$$

On solving above differential equation (7), subject to the boundary condition

$$I_7(t) = \{(1-k)(a-bp)\left(1 - \frac{\theta_1 t^2}{2}\right)\left[\left(\left(\frac{t^2}{2} - \frac{\theta_1 t^4}{8}\right)\right) - \left(\frac{\theta_1 t_6^4}{8} + \frac{t_6^2}{2}\right)\right]\} \tag{14}$$

The different inventory levels obtained as follows:

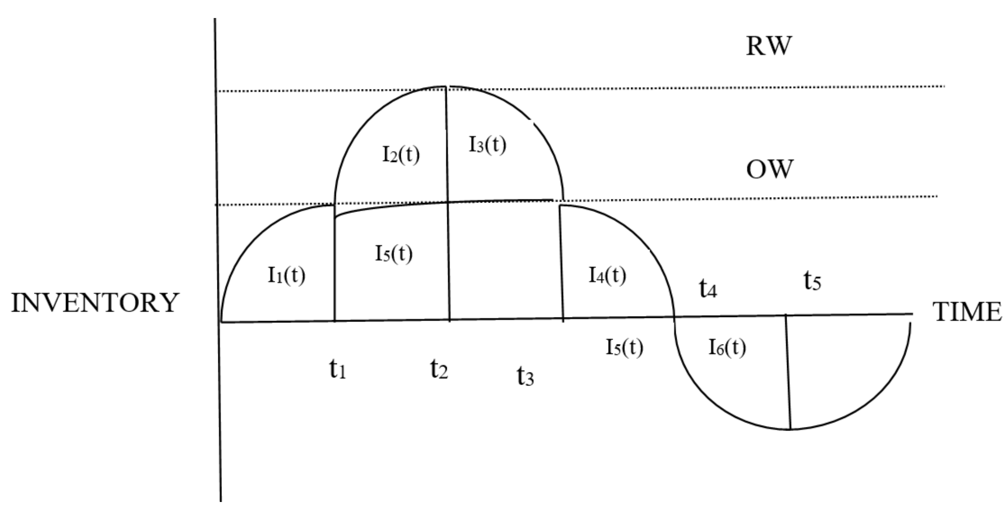

Fig 2. Inventory level

To find out the relation between t_2, t_3, and t_5 the equation of continuity is solved

$$I_2(t_2) = I_3(t_2)\ \left\{(1-k)(a-bp)\left(1-\frac{\theta_2^2 t_1^4}{2}\right)\left[\left(\frac{t_2^2}{2}-\frac{\theta_2 t_2^4}{8}\right)-\left(\frac{\theta_2 t_1^4}{8}+\frac{t_1^2}{2}\right)\right]\right\} = \left\{(a-bp)\left[\left(\frac{\theta_2 t_2^2}{2}-1\right)\left(\frac{\theta_2 t_3^4}{8}+\frac{t_3^2}{2}\right)-\left(\frac{\theta_2 t_2^4}{4}-\frac{t_2^2}{2}-\frac{\theta_2 t_2^4}{8}+\frac{\theta_2^2 t_2 s^6}{8}\right)\right]\right\} \tag{15}$$

$$I_4(t_3) = I_5(t_3)\ \left(\frac{t_4^2}{2}\right)+\left(\frac{\theta_2 t_4^4}{8}\right)+\frac{w\left(1-\frac{\theta_1 t_1^2}{2}\right)}{(a-bp)}-\left(\frac{t_3^2}{2}\right)+\left(\frac{\theta_2 t_3^4}{8}\right)=0 \tag{16}$$

$$I_7(t_5) = -I_6(t_5)\left\{(1-k)(a-bp)\left(1-\frac{\theta_1 t^2}{2}\right)\left[\left(\left(\frac{t_5^2}{2}-\frac{\theta_1 t_5^4}{8}\right)\right)-\left(\frac{\theta_1 t_6^4}{8}+\frac{t_6^2}{2}\right)\right]\right\} = -(a-bp)\left(\frac{t_4^2}{2}-\frac{t_5^2}{2}\right) \tag{17}$$

Inventory stored at the manufacturer own warehouse is calculated by integrating different inventory level at a different time period

$$I_{OW}(t) = \int_0^{t_1} I_1(t)\,dt + \int_{t_3}^{t_4} I_4(t)\,dt + \int_{t_1}^{t_3} I_5(t)\,dt + \int_{t_4}^{t_5} I_6(t)\,dt + \int_{t_5}^{t_6} I_7(t)\,dt$$

$$\begin{aligned} I_{OW}(t) &= (a-bp)(1-k)\left[\frac{t_1^2}{2}+\frac{3\theta_1 t_1^4}{8}-\frac{\theta_1^2 t_1^6}{16}\right]+ \\ &(a-bp)\ \left[\left(\frac{t_4^2}{2}+\frac{\theta_1 t_4^2}{2}\right)\left(\frac{\theta_1 t_4^3}{6}-\frac{\theta_1 t_3^3}{6}-t_4+t_3\right)-\left(\frac{\theta_1 t_4^5}{20}-\frac{\theta_1 t_3^5}{20}+\frac{\theta_1^2 t_4^7}{56}-\frac{\theta_1^2 t_3^7}{56}-\frac{t_4^3}{6}+\frac{t_3^3}{6}-\frac{\theta_2 t_4^5}{40}+\frac{\theta_2 t_3^5}{40}\right)\right]- \\ &\left[w\left(1+\frac{\theta_1 t_1^2}{2}\right)\left(t_3-t_1-\frac{\theta_1 t_3^3}{6}+\frac{\theta_1 t_1^3}{6}\right)\right]+\ (1-k)(a-bp) \\ &\left[\frac{t_6^3}{6}+\frac{t_5^3}{6}-\frac{3\theta_1 t_6^5}{40}+\frac{\theta_1 t_5^5}{40}-\frac{\theta_1^2 t_6^7}{112}+\frac{\theta_1^2 t_5^7}{112}-\left(\frac{t_6^2}{2}+\frac{\theta_1 t_6^4}{8}\right)\left(t_6-t_5-\frac{\theta_1 t_6^3}{6}+\frac{\theta_1 t_5^3}{6}\right)\right]+(a-bp)\left[\frac{t_4^2}{2}(t_5-t_4)-\frac{t_5^3}{6}+\frac{t_4^3}{6}\right] \end{aligned} \tag{18}$$

The inventory stored at the rented warehouse when the capacity of own warehouse is filled can be calculated as given below:

$$I_{RW}(t) = \int_{t_1}^{t_2} I_2(t)\,dt + \int_{t_2}^{t_3} I_3(t)\,dt$$

$$\begin{aligned} I_{RW}(t) &= (1-k)(a-bp)\left[\left(\frac{t_2^3}{6}-\frac{t_1^3}{6}+\frac{3\theta_2 t_2^5}{40}-\frac{3\theta_2 t_1^5}{40}-\frac{\theta_2^2 t_2^7}{112}+\frac{\theta_2^2 t_1^7}{112}\right)-\left(\frac{\theta_2 t_1^4}{8}+\frac{t_1^2}{2}\right)\left(t_2-t_1-\ \frac{\theta_2 t_2^3}{6}-\frac{\theta_2 t_1^3}{6}\right)\right]+ \\ &(a-bp)\left[\left(\frac{\theta_2 t_3^4}{2}+\frac{t_3^3}{8}\right)\left(\frac{\theta_2 t_3^3}{6}-\frac{\theta_2 t_2^3}{6}-t_3+t_2\right)-\left(\frac{\theta_2 t_3^5}{20}-\frac{\theta_2 t_2^5}{20}+\frac{{\theta_2}^2 t_3^5}{56}-\frac{\theta_2^2 t_2^5}{56}-\frac{t_3^3}{6}+\frac{t_2^3}{6}-\ \frac{\theta_2 t_3^5}{40}+\frac{\theta_2 t_2^5}{40}\right)\right] \end{aligned} \tag{19}$$

The number of deteriorating stocks in both own and rented warehouse during the production cycle:

$$D_i = \int_{t_0}^{t_1} \alpha(t) I_1(t)\,dt + \int_{t_1}^{t_3} \alpha(t) I_5(t)\,dt + \int_{t_1}^{t_2} \beta(t) I_2(t)\,dt + \int_{t_2}^{t_3} \beta(t) I_3(t)\,dt + \int_{t_3}^{t_4} \alpha(t) I_4(t)\,dt \; = \\ (1-\mathrm{k})(\mathrm{a}-\mathrm{bp})\theta_1\left\{\frac{\mathrm{t}_1^4}{6}-\frac{3\theta_1\mathrm{t}_1^8}{64}+\frac{\theta_1^2\mathrm{t}_1^8}{112}\right\}+(1-\mathrm{k})(\mathrm{a}-\mathrm{bp}) \\ \left[\left(\frac{\mathrm{t}_2^3}{8}+\frac{\mathrm{t}_1^3}{8}-\frac{\theta_1\mathrm{t}_2^5}{40}+\frac{\theta_1\mathrm{t}_1^5}{40}-\frac{\theta_1^2\mathrm{t}_2^8}{128}+\frac{\theta_1^2\mathrm{t}_1^8}{128}-\left(\frac{t_1^2}{2}+\frac{\theta_1 t_1^4}{8}\right)\left(t_2-t_1-\frac{\theta_1 t_2^3}{6}+\frac{\theta_1 t_1^3}{6}\right)\right]+ \\ w\theta_1\left(1+\frac{\theta_1 t_1^2}{2}\right)\left(\frac{\theta_1 t_1^4}{8}-\frac{\theta_1 t_3^4}{8}+\frac{t_3^2}{2}-\frac{t_1^2}{2}\right)+(a-bp)\theta_2\left\{\left(\frac{\theta_2 t_3^4}{8}-\frac{\theta_2\mathrm{t}_2^4}{8}+\frac{\mathrm{t}_3^2}{2}-\frac{\mathrm{t}_2^2}{2}\right)\right. \\ \left.\left(\frac{\theta_2\mathrm{t}_3^4}{4}+\frac{\mathrm{t}_3^2}{2}\right)-\left(\frac{\theta_2\mathrm{t}_3^6}{24}-\frac{\theta_2\mathrm{t}_2^6}{24}+\frac{\theta_2\mathrm{t}_2^6}{48}-\frac{\theta_2\mathrm{t}_3^6}{48}-\frac{\theta_2\mathrm{t}_2^8}{64}+\frac{\theta_2\mathrm{t}_3^8}{64}+\frac{\mathrm{t}_2^4}{8}-\frac{\mathrm{t}_3^4}{8}\right)\right\}+ \\ (\mathrm{a}-bp)\theta_1\left\{\left(\frac{\theta_1\mathrm{t}_4^4}{8}-\frac{\theta_1\mathrm{t}_3^4}{8}+\frac{\mathrm{t}_3^2}{2}-\frac{\mathrm{t}_4^2}{2}\right)\left(\frac{\theta_1\mathrm{t}_4^4}{4}+\frac{\mathrm{t}_4^2}{2}\right)-\left(\frac{\theta_1\mathrm{t}_4^6}{24}-\frac{\theta_1\mathrm{t}_3^6}{24}+\frac{\theta_1\mathrm{t}_3^6}{48}-\frac{\theta_1\mathrm{t}_4^6}{48}-\frac{\theta_1^2\mathrm{t}_1^8}{64}+\frac{\theta_1^2\mathrm{t}_4^8}{64}+\frac{\mathrm{t}_3^4}{8}-\frac{\mathrm{t}_4^4}{8}\right)\right\}+ \\ (1-\mathrm{k})(\mathrm{a}-\mathrm{bp})\left[\left(\frac{\mathrm{t}_6^3}{8}+\frac{\mathrm{t}_5^3}{8}-\frac{\theta_1\mathrm{t}_6^5}{40}+\frac{\theta_1\mathrm{t}_5^5}{40}-\frac{\theta_1^2\mathrm{t}_6^8}{128}+\frac{\theta_1^2\mathrm{t}_5^8}{128}-\left(\frac{\mathrm{t}_6^2}{2}+\frac{\theta_1\mathrm{t}_6^4}{8}\right)\left(\mathrm{t}_6-\mathrm{t}_5-\frac{\theta_1\mathrm{t}_6^3}{6}+\frac{\theta_1\mathrm{t}_5^3}{6}\right)\right]\tag{20}$$

Total holding cost for rented and own warehouse:

$$H_i = \int_{t_0}^{t_1} \alpha(t) I_1(t)\,dt + \int_{t_1}^{t_3} \alpha(t) I_5(t)\,dt + \int_{t_1}^{t_2} \beta(t) I_2(t)\,dt + \int_{t_2}^{t_3} \beta(t) I_3(t)\,dt + \int_{t_3}^{t_4} \alpha(t) I_4(t)\,dt \\ =(1-\mathrm{k})(\mathrm{a}-\mathrm{bp})\theta_1\left\{\frac{\mathrm{t}_1^4}{6}-\frac{3\theta_1\mathrm{t}_1^8}{64}+\frac{\theta_1^2\mathrm{t}_1^8}{112}\right\}+(1-\mathrm{k})(\mathrm{a}-\mathrm{bp})\left[\left(\frac{\mathrm{t}_2^3}{8}+\frac{\mathrm{t}_1^3}{8}-\frac{\theta_1\mathrm{t}_2^5}{40}+\frac{\theta_1\mathrm{t}_1^5}{40}-\frac{\theta_1^2\mathrm{t}_2^8}{128}+\right.\right. \\ \left.\left.\frac{\theta_1^2 t_1^8}{128}-\left(\frac{t_1^2}{2}+\frac{\theta_1 t_1^4}{8}\right)\left(t_2-t_1-\frac{\theta_1 t_2^3}{6}+\frac{\theta_1 t_1^3}{6}\right)\right]+w\theta_1\left(1+\frac{\theta_1 t_1^2}{2}\right)\left(\frac{\theta_1 t_1^4}{8}-\frac{\theta_1 t_3^4}{8}+\frac{t_3^2}{2}-\frac{t_1^2}{2}\right)+ \\ (a-\mathrm{bp})\theta_2\left\{\left(\frac{\theta_2 t_3^4}{8}-\frac{\theta_2 t_2^4}{8}+\frac{t_3^2}{2}-\frac{t_2^2}{2}\right)\left(\frac{\theta_2 t_3^4}{4}+\frac{t_3^2}{2}\right)-\left(\frac{\theta_2 t_3^6}{24}-\frac{\theta_2 t_2^6}{24}+\frac{\theta_2 t_2^6}{48}-\frac{\theta_2 t_3^6}{48}-\frac{\theta_2 t_2^8}{64}+\frac{\theta_2 t_3^8}{64}+\frac{t_2^4}{8}-\frac{t_3^4}{8}\right)\right\}+ \\ (a-bp)\theta_1\left\{\left(\frac{\theta_1 t_4^4}{8}-\frac{\theta_1 t_3^4}{8}+\frac{t_3^2}{2}-\frac{t_4^2}{2}\right)\left(\frac{\theta_1 t_4^4}{4}+\frac{t_4^2}{2}\right)-\left(\frac{\theta_1 t_4^6}{24}-\frac{\theta_1 t_3^6}{24}+\frac{\theta_1 t_3^6}{48}-\frac{\theta_1 t_4^6}{48}-\frac{\theta_1^2 t_1^8}{64}+\frac{\theta_1^2 t_4^8}{64}+\frac{t_3^4}{8}-\frac{t_4^4}{8}\right)\right\}+ \\ (1-k)(a-bp)\left[\left(\frac{t_6^3}{8}+\frac{t_5^3}{8}-\frac{\theta_1 t_6^5}{40}+\frac{\theta_1 t_5^5}{40}-\frac{\theta_1^2 t_6^8}{128}+\frac{\theta_1^2 t_5^8}{128}-\left(\frac{t_6^2}{2}+\frac{\theta_1 t_6^4}{8}\right)\left(t_6-t_5-\frac{\theta_1 t_6^3}{6}+\frac{\theta_1 t_5^3}{6}\right)\right]\tag{21}$$

Shortage cost: When a shortage occurs in any production system manufacturer lose some sale. This cost is called a shortage cost.

$$=-C_s\int_{t_4}^{t_5} I_6(t)dt \;=\; C_s(a-bp)\left\{\left(\frac{t_5^3}{6}-\frac{t_4^3}{6}-\frac{t_4^2}{2}(t_5-t_4)\right)\right\}\tag{22}$$

Green design cost: When a green design product is manufactured there is some extra expenditure that occurs in this process, that expenditure is called green design cost.

$$G(N)=\frac{G_D}{PS}\left\{\frac{G_{DF}}{N}+N.R_{DV}.\prod_{j=1}^{2} R_j\right\}\tag{23}$$

TC=Setup cost+ Holding cost+ Deterioration cost+ Shortage cost+ Green design cost

$$
\begin{aligned}
TC &= A+(1-k)(a-bp)\theta_1\left\{\frac{t_1^4}{6}-\frac{3\theta_1 t_1^8}{64}+\frac{\theta_1^2 t_1^8}{112}\right\}+(1-k)(a-bp)\left[\left(\frac{t_2^3}{8}+\frac{t_1^3}{8}-\frac{\theta_1 t_2^5}{40}+\frac{\theta_1 t_1^5}{40}-\frac{\theta_1^2 t_2^8}{128}+\right.\right.\\
&\left.\frac{\theta_1^2 t_1^8}{128}-\left(\frac{t_1^2}{2}+\frac{\theta_1 t_1^4}{8}\right)\left(t_2-t_1-\frac{\theta_1 t_2^3}{6}+\frac{\theta_1 t_1^3}{6}\right)\right]+w\theta_1\left(1+\frac{\theta_1 t_1^2}{2}\right)\left(\frac{\theta_1 t_1^4}{8}-\frac{\theta_1 t_3^4}{8}+\frac{t_3^2}{2}-\frac{t_1^2}{2}\right)+(a-\\
&bp)\theta_2\left\{\left(\frac{\theta_2 t_3^4}{8}-\frac{\theta_2 t_2^4}{8}+\frac{t_3^2}{2}-\frac{t_2^2}{2}\right)\left(\frac{\theta_2 t_3^4}{4}+\frac{t_3^2}{2}\right)-\left(\frac{\theta_2 t_3^6}{24}-\frac{\theta_2 t_2^6}{24}+\frac{\theta_2 t_2^6}{48}-\frac{\theta_2 t_3^6}{48}-\frac{\theta_2 t_2^8}{64}+\frac{\theta_2 t_3^8}{64}+\frac{t_2^4}{8}-\frac{t_3^4}{8}\right)\right\}+\\
&(a-bp)\theta_1\left\{\left(\frac{\theta_1 t_4^4}{8}-\frac{\theta_1 t_3^4}{8}+\frac{t_3^2}{2}-\frac{t_4^2}{2}\right)\left(\frac{\theta_1 t_4^4}{4}+\frac{t_4^2}{2}\right)-\left(\frac{\theta_1 t_4^6}{24}-\frac{\theta_1 t_3^6}{24}+\frac{\theta_1 t_3^6}{48}-\frac{\theta_1 t_4^6}{48}-\frac{\theta_1^2 t_1^8}{64}+\frac{\theta_1^2 t_4^8}{64}+\frac{t_3^4}{8}-\right.\right.\\
&\left.\left.\frac{t_4^4}{8}\right)\right\}+(1-k)(a-bp)\left[\left(\frac{t_6^3}{8}+\frac{t_5^3}{8}-\frac{\theta_1 t_6^5}{40}+\frac{\theta_1 t_5^5}{40}-\frac{\theta_1^2 t_6^8}{128}+\frac{\theta_1^2 t_5^8}{128}-\left(\frac{t_6^2}{2}+\frac{\theta_1 t_6^4}{8}\right)\left(t_6-t_5-\frac{\theta_1 t_6^3}{6}+\frac{\theta_1 t_5^3}{6}\right)\right]\right.\\
&\left(\frac{t_1^2}{2}+\frac{\theta_1 t_1^4}{8}\right)\left(t_2-t_1-\frac{\theta_1 t_2^3}{6}+\frac{\theta_1 t_1^3}{6}\right)\Bigg]+w\theta_1\left(1+\frac{\theta_1 t_1^2}{2}\right)\left(\frac{\theta_1 t_1^4}{8}-\frac{\theta_1 t_3^4}{8}+\frac{t_3^2}{2}-\frac{t_1^2}{2}\right)+(a-bp)\theta_2\left\{\left(\frac{\theta_2 t_3^4}{8}-\right.\right.\\
&\frac{\theta_2 t_2^4}{8}+\frac{t_3^2}{2}-\frac{t_2^2}{2}\Bigg)\left(\frac{\theta_2 t_3^4}{4}+\frac{t_3^2}{2}\right)-\left(\frac{\theta_2 t_3^6}{24}-\frac{\theta_2 t_2^6}{24}+\frac{\theta_2 t_2^6}{48}-\frac{\theta_2 t_3^6}{48}-\frac{\theta_2 t_2^8}{64}+\frac{\theta_2 t_3^8}{64}+\frac{t_2^4}{8}-\frac{t_3^4}{8}\right)\Bigg\}+(a-\\
&bp)\theta_1\left\{\left(\frac{\theta_1 t_4^4}{8}-\frac{\theta_1 t_3^4}{8}+\frac{t_3^2}{2}-\frac{t_4^2}{2}\right)\left(\frac{\theta_1 t_4^4}{4}+\frac{t_4^2}{2}\right)-\left(\frac{\theta_1 t_4^6}{24}-\frac{\theta_1 t_3^6}{24}+\frac{\theta_1 t_3^6}{48}-\frac{\theta_1 t_4^6}{48}-\frac{\theta_1^2 t_1^8}{64}+\frac{\theta_1^2 t_4^8}{64}+\frac{t_3^4}{8}-\frac{t_4^4}{8}\right)\right\}+(1-\\
&k)(a-bp)\left[\left(\frac{t_6^3}{8}+\frac{t_5^3}{8}-\frac{\theta_1 t_6^5}{40}+\frac{\theta_1 t_5^5}{40}-\frac{\theta_1^2 t_6^8}{128}+\frac{\theta_1^2 t_5^8}{128}-\left(\frac{t_6^2}{2}+\frac{\theta_1 t_6^4}{8}\right)\left(t_6-t_5-\frac{\theta_1 t_6^3}{6}+\frac{\theta_1 t_5^3}{6}\right)\right]\right.\quad +C_s(a-\\
&bp)\left\{\left(\frac{t_5^3}{6}-\frac{t_4^3}{6}-\frac{t_4^2}{2}(t_5-t_4)\right)\right\}+\frac{G_D}{PS}\left\{\frac{G_{DF}}{N}+N.R_{DV}\cdot\prod_{j=1}^{2}R_j\right\}
\end{aligned}
\tag{24}
$$

5 Algorithm to solve the mathematical model

According to the proposed model, there are three independent variables in the total cost equationt_1, t_2andt_5. To optimize the total cost equation and to find the value of all the independent parameters, the following steps are pursued.

Step. 1

Calculate the first-order partial derivatives w.r.t all the independent variable $\frac{\partial f}{\partial t_1}, \frac{\partial f}{\partial t_2}$ and $\frac{\partial f}{\partial t_3}$

Step 2.

Equate the first-order partial derivatives to zero and solve for the value of t_1, t_2, t_3

Step 3.

Now, calculate the second-order partial derivative w.r.t. all the independent variables like $\frac{\partial^2 f}{\partial t_1^2}, \frac{\partial^2 f}{\partial t_1 \partial t_2}, \frac{\partial^2 f}{\partial t_1 \partial t_3}, \frac{\partial^2 f}{\partial t_2 \partial t_1}$ etc.

Step 4.

Now, form a Hessian matrix as follows

$$
\begin{pmatrix}
\frac{\partial^2 f}{\partial {t_1}^2} & \frac{\partial^2 f}{\partial t_1 \partial t_2} & \frac{\partial^2 f}{\partial t_1 \partial t_3}\\
\frac{\partial^2 f}{\partial t_2 \partial t_1} & \frac{\partial^2 f}{\partial {t_2}^2} & \frac{\partial^2 f}{\partial t_2 \partial t_3}\\
\frac{\partial^2 f}{\partial t_3 \partial t_1} & \frac{\partial^2 f}{\partial t_3 \partial t_2} & \frac{\partial^2 f}{\partial {t_3}^2}
\end{pmatrix}
$$

Step 5.

Find H_1, H_2, and H_3, where, H_1, H_2, and H_3 denote the first principal minor, second principal minor, and third principal minor respectively. If det $(H_1)>0$, det$(H_2)>0$, and det$(H_3)>0$, then the matrix is a positive definite matrix, and f is called convex function.

6 Numerical analysis

Two numerical examples are solved to show the reliability of the model. Random data is used to solve the following numerical.

6.1 Numerical example 1

Following parameters are used to demonstrate the numerical:

θ_2=\$0.5; θ_1=\$0.9; w=\$0.3; a=44; b=0.01; p=\$100; c_s=\$0.01; D_i=\$0.001; k=0.50; A=\$6000; c_{rw}=\$0.13; c_{ow}=\$0.8; G_d=\$5; G_{dv}=\$80; N=70; R_{DV}=\$0.1; r_1=0.1; r_2=0.9; PS=\$4;

By using the above parameters, we minimize the total cost function with green design and product stewardship, get the following values:

TC=\$5974.68; t_1=0.78214; t_2=1.22029; t_5=1.38124;

Total cost and other parameter value without green design and product stewardship:

TC=\$5972.68; t_1=0.78214; t_2=1.22029; t_5=1.38124;

6.2 Sensitivity analysis for numerical 1

Table 2. Sensitivity for parameter 'a'

a	t_1	t_2	t_5	TC
44	0.78214	1.22029	1.38124	5974.68
45	0.78163	1.22025	1.38136	5974.26
46	0.78121	1.22028	1.38144	5973.62
47	0.78075	1.22024	1.38155	5972.98

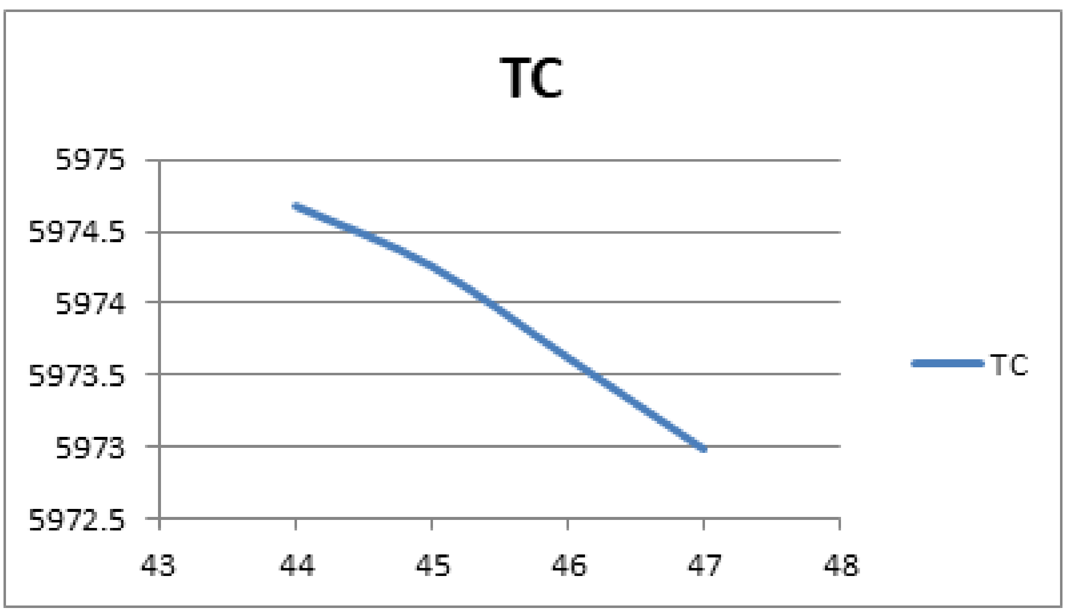

Fig 3. Sensitivity analysis between total cost and parameter 'a'

Table 3. Sensitivity for the parameter 'b'

b	t_1	t_2	t_5	TC
0.008	0.782051	1.22029	1.38126	5974.77
0.009	0.782083	1.22029	1.38126	5974.83
0.010	0.782140	1.22029	1.38126	5974.89
0.011	0.782150	1.22029	1.38126	5975.15

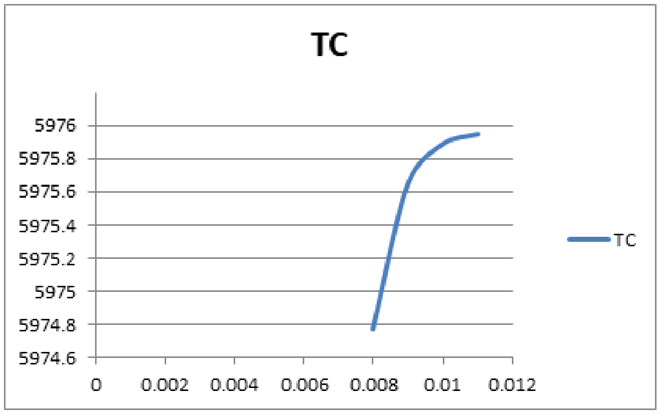

Fig 4. Sensitivity analysis between total cost and parameter 'b'

Table 4. Sensitivity for the parameter 'PS'

PS	t_1	t_2	t_5	TC
4	0.78214	1.22029	1.38124	5974.68
5	0.78214	1.22029	1.38124	5974.45
6	0.78214	1.22029	1.38124	5975.05
7	0.78214	1.22029	1.38124	5975.94

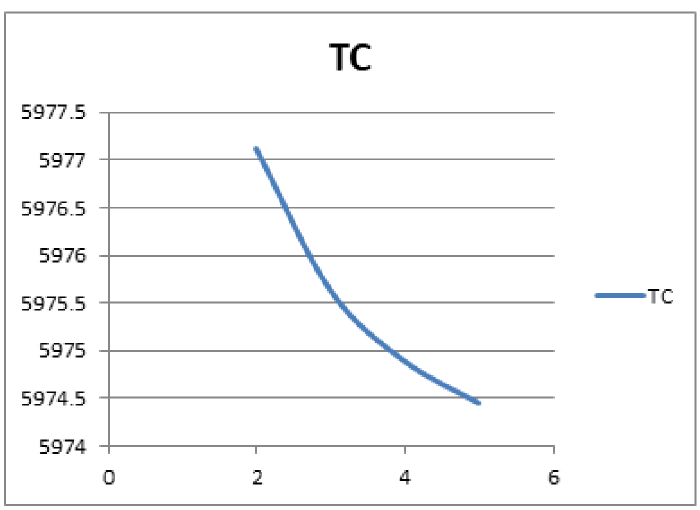

Fig 5. Sensitivity analysis between total cost and parameter 'PS'

Table 5. Sensitivity for the parameter ’G_d’

G_d	t_1	t_2	t_5	TC
5	0.78214	1.22029	1.38124	5974.68
6	0.78214	1.22029	1.38124	5975.34
7	0.78214	1.22029	1.38124	5975.78
8	0.78214	1.22029	1.38124	5976.22

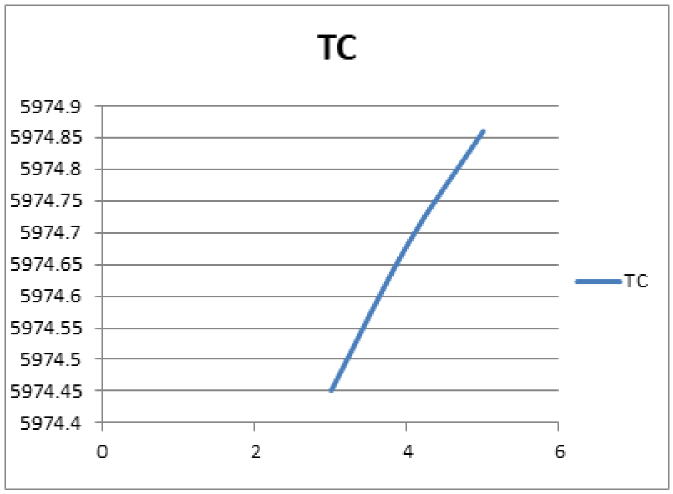

Fig 6. Sensitivity analysis between total cost and parameter ’G_d’

Table 6. Sensitivity for the parameter ’N’

N	t_1	t_2	t_5	TC
60	0.78214	1.22029	1.38124	5975.02
65	0.78214	1.22029	1.38124	5974.95
70	0.78214	1.22029	1.38124	5974.89
75	0.78214	1.22029	1.38124	5974.82

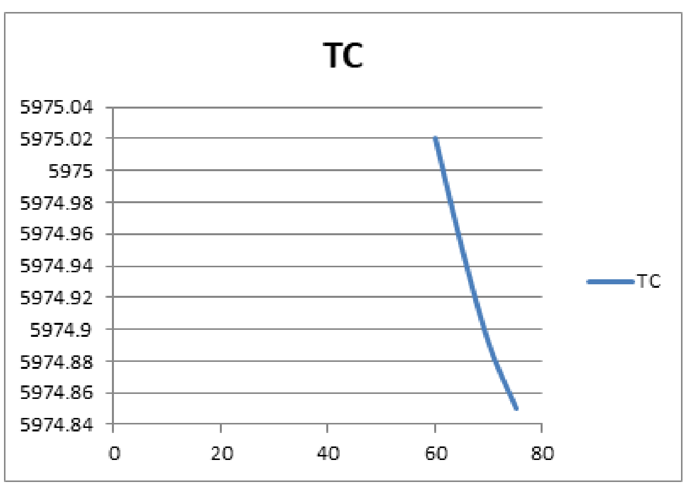

Fig 7. Sensitivity analysis between total cost and parameter ’N’

6.3 Observation for the above sensitivity analysis

1. Firstly, when the value of 'a' increases then there is a decrease in the value of TC, but t_1, t_2 and t_5 becomes constant regularly. In any real market situation when demand increases then total cost decreases. Parameter 'a' is a factor of the demand function. Hence when the value of 'a' increases the value of demand function also increases, which may reduce total cost.
2. In the next step if we increase the value of 'b' then TC increases, but t_1, t_2 and t_5 will remain constant. Parameter 'b' is a negative impact on demand function. On increasing the value of 'b' demand decreases and if demand decrease then the total cost goes up.
3. On increasing the value of parameter 'PS' then TC increases regularly but, the value of t_5, t_1 and t_2 remains constant. Product stewardship is a concept which helps in making a product green. In the proposed research green design and product stewardship collaborated, by increasing the value of product stewardship, the total cost increases.
4. When the 'G_d' increases then TC increases on the other hand t_1, t_2, and t_5 remains constant. 'G_d' is a green design life cycle cost if 'G_d' increases then it also increases the value of total cost.
5. On increasing the number or recycling the product 'N' then the value of t_1, t_2, and t_5 remains constant, but the value of TC continuously reduces. It is the main objective of the paper that recycling of a commodity will reduce the total cost.

6.4 Convexity of TC

The Figures 8, 9 and 10 show the convexity of TC function w.r.t different independent variables.

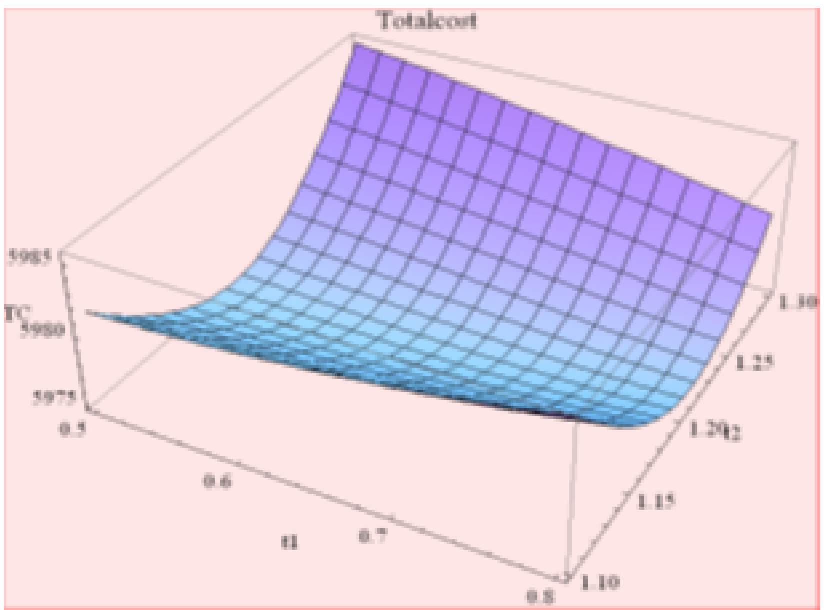

Fig 8. TC w.r.t t_1 and t_2

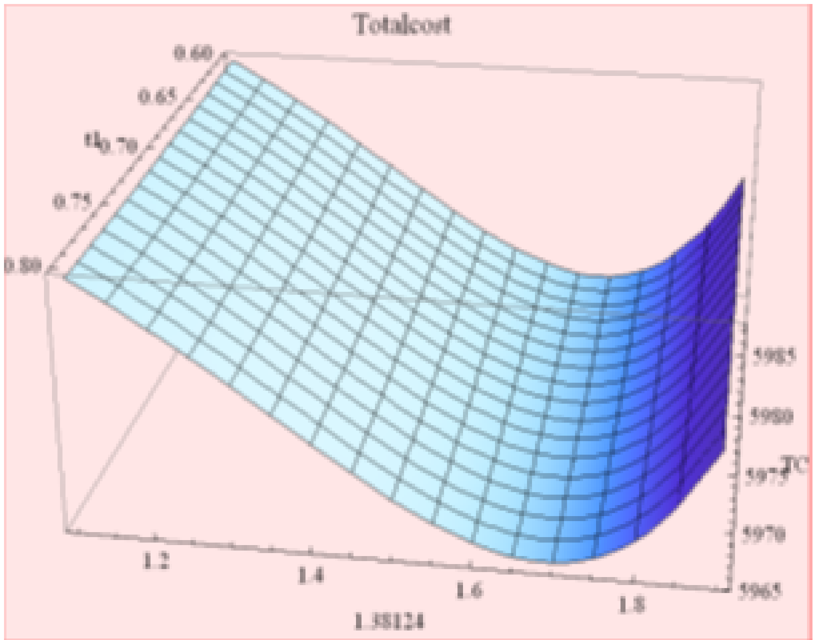

Fig 9. TC w.r.t t_1 and t_5

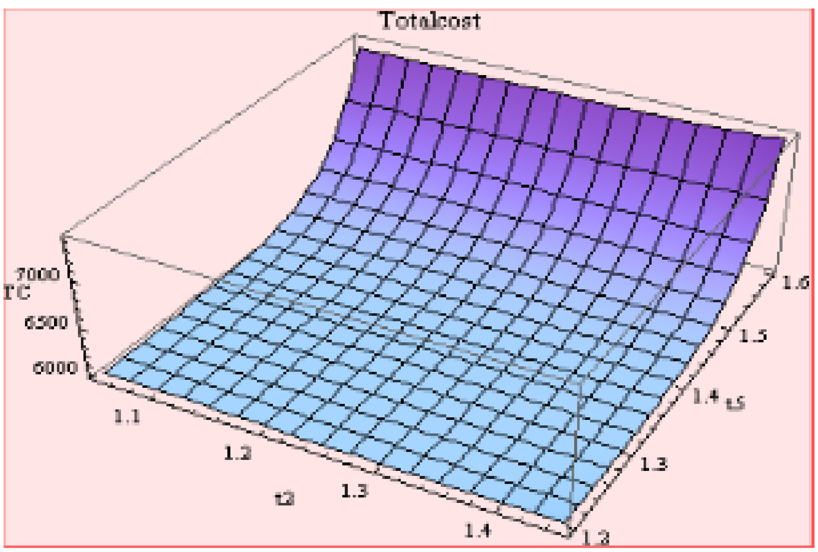

Fig 10. TCw.r.t t_2 and t_5

6.5 Numerical example 2

Following parameters are used to demonstrate the numerical:

θ_2=\$0.1; θ_1=\$0.5; w=\$0.1; a=40; b=0.001; p=\$90; c_s=\$0.01; D_i=\$0.004; k=0.40; A=\$6000; c_{rw}=\$0.09; c_{ow}=\$0.6; G_d=\$5; G_{dv}=\$75; N=40; R_{DV}=\$0.01; r_1=0.01; r_2=0.7; PS=\$3;

By using above parameters, we minimize the total cost function and get the following values:

TC=\$5919.74; t_1=0.7.334; t_2=0.831332; t_5=2.09251;

6.6 Sensitivity analysis for above numerical

Table 7. Sensitivity for the parameter 'a'

a	t_1	t_2	t_5	TC
20	0.712615	0.83245	2.09258	5961.51
30	0.708811	0.835243	2.09256	5940.60
40	0.70334	0.831332	2.09251	5919.74
50	0.70225	0.832285	2.09252	5898.85

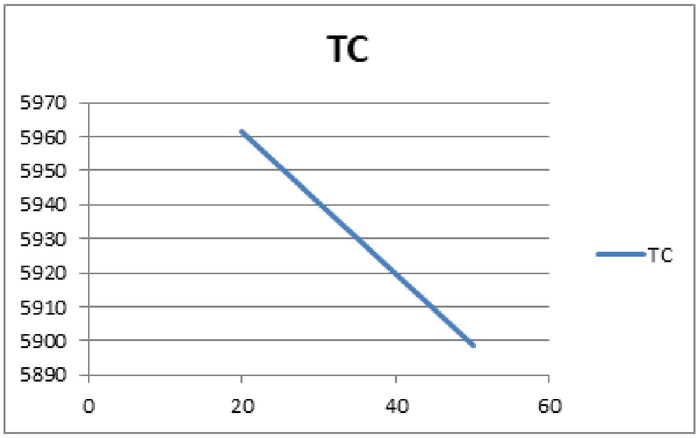

Fig 11. Sensitivity analysis between total cost and parameter 'a'

Table 8. Sensitivity for the parameter 'b'

b	t_1	t_2	t_5	TC
0.0005	0.703323	0.831330	2.09251	5919.65
0.0010	0.70334	0.831332	2.09251	5919.74
0.0015	0.703342	0.8331329	2.09251	5919.84
0.0020	0.703358	0.831338	2.09251	5919.93

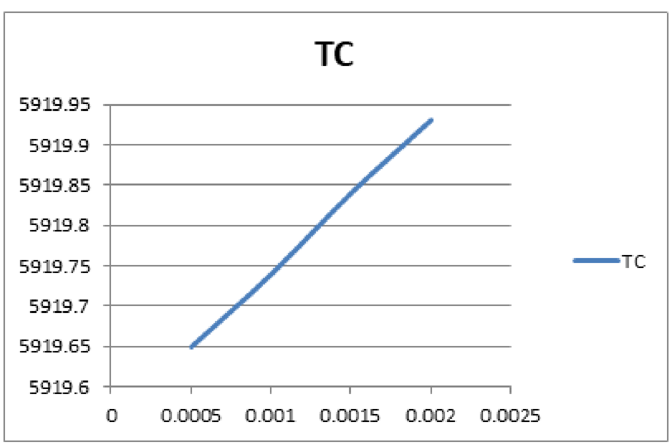

Fig 12. Sensitivity analysis between total cost and parameter 'b'

Table 9. Sensitivity for the parameter 'PS'

PS	t_1	t_2	t_5	TC
1	0.70334	0.831332	2.09251	5926.00
2	0.703334	0.831332	2.09251	5921.31
3	0.703334	0.831332	2.09251	5919.74
4	0.7.3334	0.831332	2.09251	5918.96

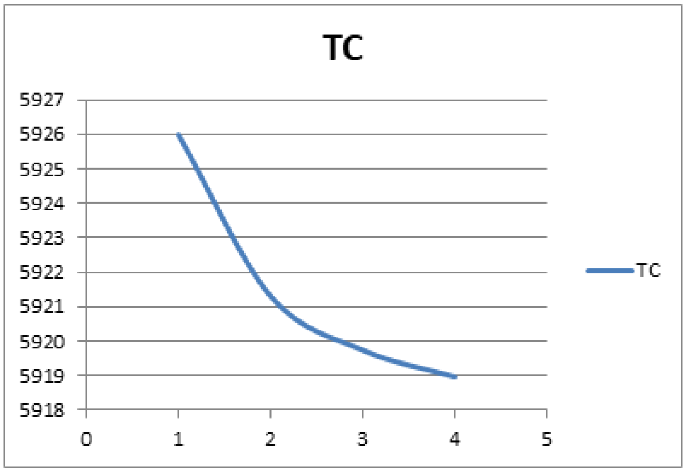

Fig 13. Sensitivity analysis between total cost and parameter 'PS'

Table 10. Sensitivity for the parameter 'G_d'

G_d	t_1	t_2	t_5	TC
3	0.7.3334	0.831332	2.09251	5918.49
4	0.703334	0.831332	2.09251	5919.12
5	0.703334	0.831332	2.09251	5919.74
6	0.7.3334	0.831332	2.09251	5920.37

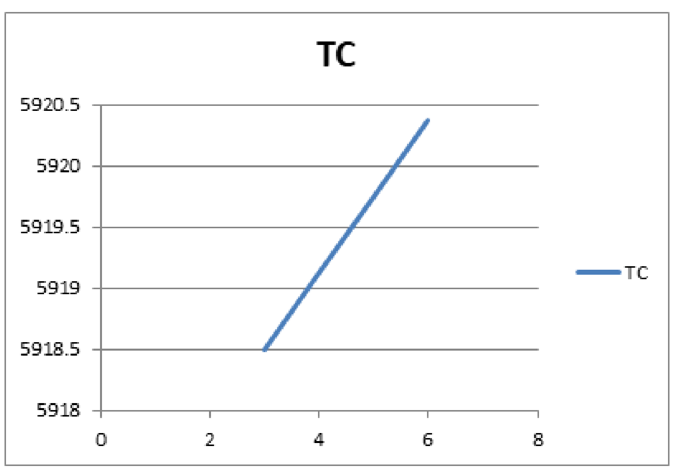

Fig 14. Sensitivity analysis between total cost and parameter 'G_d'

Table 11. Sensitivity for the parameter 'N'

N	t1	t2	t5	TC
20	0.703334	0.831332	2.09251	5922.87
30	0.703334	0.831332	2.09251	5920.78
40	0.703334	0.831332	2.09251	5919.74
50	0.73334	0.831332	2.09251	5919.12

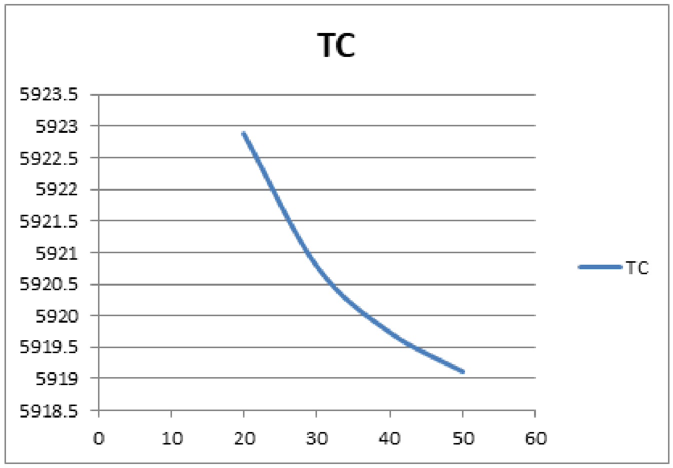

Fig 15. Sensitivity analysis between total cost and parameter 'N'

6.7 Observation for the above sensitivity analysis:

1. Firstly, when the value of 'a' increases then there is decrease in the value of TC and t_1, but on the other side the values of t_2 fluctuate regularly. The value of t_5 increases first and after that decreases. On increasing the value of parameter 'a' demand function is also increasing. In any situation when demand goes up, it will decrease the total cost.
2. In the next step if the value of 'b' increased then TC increases, also increase, and t_1, t_5 andt_2 will remain constant. On increasing the value of parameter 'b' demand function decreases and when demand decrease then total cost increase.
3. On increasing the value of parameter 'PS'(product stewardship), TC decreased. On the other hand t_1, t_5 and t_2 become constant. Product stewardship has a sharp effect on green product design. If the parameter of product stewardship increases then total cost decreased.
4. When 'G_d' increase then TC increased, but the value oft_2, $t_{5,}$ and t_1 will become constant. When green design cycle cost increases then green product design cost also increases, it will increase the total cost also..
5. n increasing the number of recycling the product 'N' then the value of t1$_,$ t2$_{, a}$nd t5remains constant, but the value of TC continuously reduces.

6.8 Convexity of TC

In the following Figures 16, 17 and 18 shows the convexity of TC function w.r.t independent variables.

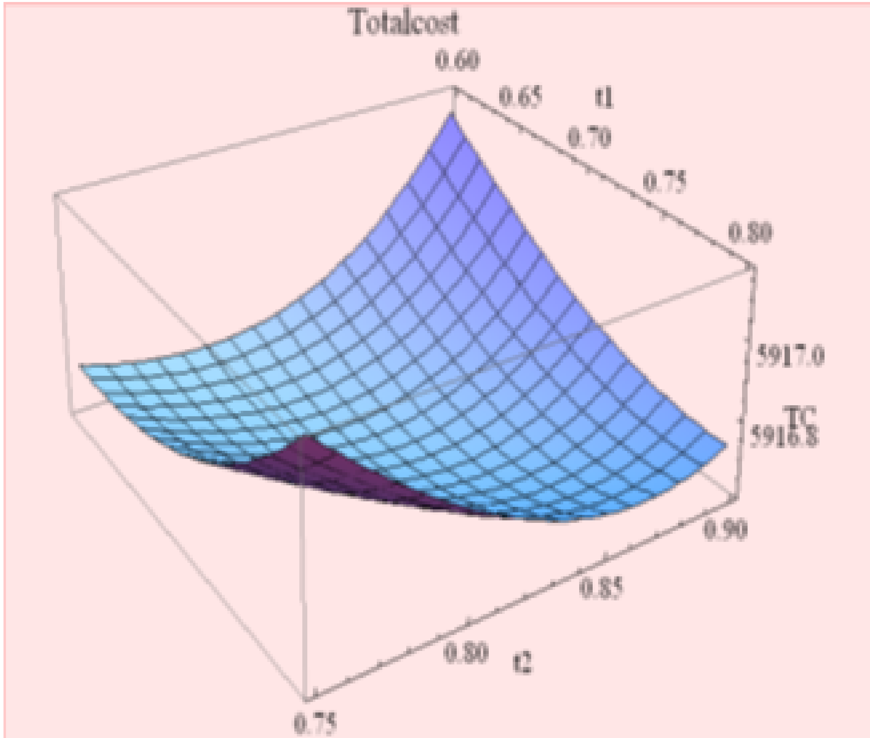

Fig 16. TCw.r.t t_1 and t_2

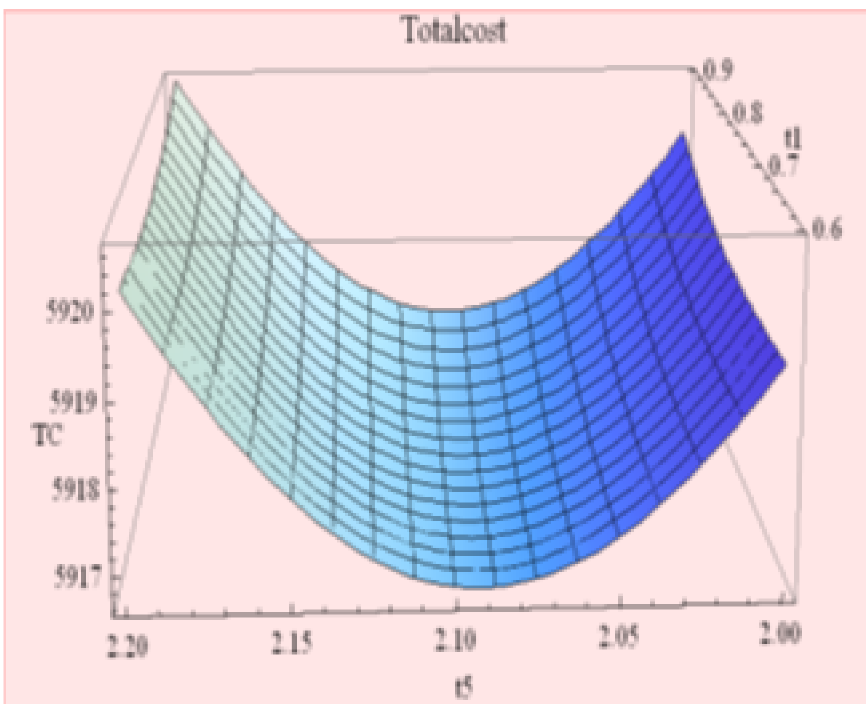

Fig 17. TCw.r.t t_1 and t_5

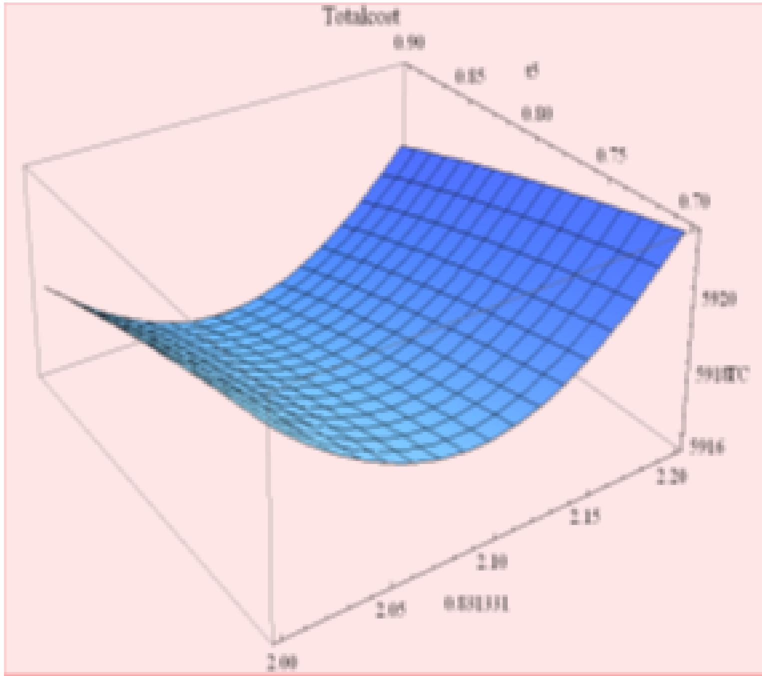

Fig 18. TC w.r.t t_2 and t_5

7 Conclusion

This study has collaborated on the concepts of green design, product stewardship, and two warehouses with the recycling of the used items. Numerical examples and sensitivity analysis illustrate that number of recycling of the items had a reverse effect on the total cost. i.e., an increase in the number of recycles results in the reduction of the total cost. Green design and product stewardship help the manufacturers to decrease the recycling cost. This study may be very useful for the manufacturer to deal with inventory management as well as environmental issues. Future research can be unfolded as integral research model by which of inflation, trade credit policy, different demand patterns, etc. can be included.

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