

RESEARCH ARTICLE



An extension of grey relational analysis for intuitionistic and interval-valued intuitionistic fuzzy soft sets

OPEN ACCESS

Received: 27.08.2020

Accepted: 24.09.2020

Published: 21.10.2020

Editor: Dr. Natarajan Gajendran

Citation: Touqeer M, Saeed MN, Salamat N, Mustahsan M (2020) An extension of grey relational analysis for intuitionistic and interval-valued intuitionistic fuzzy soft sets. *Indian Journal of Science and Technology* 13(37): 3950-3959. <https://doi.org/10.17485/IJST/V13i37.1356>

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Funding: None

Competing Interests: None

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Published By Indian Society for Education and Environment ([iSee](https://www.isee.org/))

ISSN

Print: 0974-6846

Electronic: 0974-5645

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Abstract

Objectives: Neither any analytical (or numerical) nor any statistical approach is often helpful in these situations due to the reason that every person has his/her own choice. To cope with such situations usually we have to use fuzzy sets in combination with soft sets, which consist of predicates and approximate value sets as their images. **Material:** Choice values and comparison table techniques are two common decision-making techniques, which often don't result in same preference order or optimal choice. To overcome this kind of situation in decision-making problems, grey relational analysis method is used to get on a final decision. **Method:** Here we have used grey relational analysis method involving "intuitionistic fuzzy soft set" and "interval-valued intuitionistic fuzzy soft set" and "AND operation" to deal with such kind of problems. **Findings:** The proposed method is effective in seeking on an optimal choice in the case when common decision-making techniques fail to get on a final decision. **Novelty:** By using grey relational analysis, a suitable method to choose one object from different choices has been proposed. It overcome the greyness in decision-making problems for getting on a final decision when one gets too many options and finds it difficult to choose an optimal choice.

Keywords: Fuzzy soft set; intuitionistic fuzzy soft set; grey relational analysis; interval-valued intuitionistic fuzzy soft set

1 Introduction

Many problems of real life are not certain which cannot be solved used typical Mathematical rules involving methods based on precise reasoning. In ⁽¹⁾ has categorized the different nature of problems of real life as problems of "organized simplicity" and "disorganized complexity". The first type of the two involves analytical problems which can be solved used calculus while the second type of problems refer to the statistical approaches for dealing with physical problems at molecular level that involve numerous variables and randomness of high

degree. These problems are highly complementary to one another, under certain situations if one works then other fails. Majority of real life problems lie between these two which Weaver has named as the problems of organized complexity. Generally to deal with any type of problem, we have to construct a model based on reality aspects or some artificial objects. In construction of any model the factors affecting its usefulness are the credibility of the model, its complexity and uncertainty involved in it. Allowing more uncertainty will help in overcoming the complexity of the model and increasing its credibility. Therefore, the challenge was to develop techniques which can be used to estimate allowable uncertainty for such type of resulting models. The concept of fuzzy sets by⁽²⁾ in 1965 is considered as an evolution for dealing with uncertainty as his concept of fuzzy sets are the sets which do not have precise boundaries like the typical sets have.

Though the theory of fuzzy sets has served as the best tool for dealing with uncertainties but scarcity of criterion for modeling different linguistic uncertainties limits its use as is pointed out by⁽³⁾. To provide a rich platform for parameterizations by overcoming the deficiencies in the fuzzy set theory,⁽⁴⁾ introduced the idea of soft sets as a generalization of fuzzy sets. Fuzzy set theory in connection with soft sets have proved to be one of the most effective tool for dealing with uncertain situations some of which are discussed here.⁽⁵⁾ Propounded the perception of fuzzy soft sets and application of soft sets in a decision-making problems. Theoretic approach regarding “fuzzy soft set” offered by⁽⁶⁾.⁽⁷⁾ Deliberate reduct soft set’s notion and discussed soft set’s postulate. The abstraction of decision making through comparison table technique discussed by⁽⁸⁾. He made decision-making very useful by constructing comparison table technique.

In⁽⁹⁾ analyzed choice values and score values as evaluation bases to make a decision by discussing a counter example. After that⁽¹⁰⁾ introduced postulate of soft matrix and Uni-Int technique to make a decision for a problem. Un-Int technique facilitate decision maker to works on small number of attributes instead of larger number of attributes for soft set. He constructed Un-Int technique for “AND” and “OR” products. He also considers an example of 48 candidates and analyzed it with the help of Un-Int technique for “AND” product. To make a decision with uncertain problems^(11,12) presented “semantic” methodology by using “ontology” and properties of “intuitionistic fuzzy soft set”. In^(13,14) enhanced idea by presenting an adjustable approach for “level soft set” and “interval-valued soft set”.⁽¹⁵⁾ has given the conception of “interval-valued fuzzy soft set” and different applications.⁽¹⁶⁾ gave the conception of generalized fuzzy soft sets. Soft set’s algebra was presented by⁽¹⁷⁾. In⁽¹⁸⁾ introduced the conception of vague soft sets and its properties. By relating different parameters,⁽¹⁹⁾ introduced some advanced operations of soft set’s concept. In⁽²⁰⁾ gave an algorithm to overcome the problems of adding parameters and suboptimal choices.

Mostly decision making techniques involve “choice values” technique or “score values” technique for ranking of alternatives which often don’t result in same preference order. To overcome this kind of situation, grey relational analysis method by⁽²¹⁾ is used to get on a final decision. In⁽²²⁾ used grey relational projection, obtained by joining grey relational method and projection method. A combination of grey relational method and projection method is analyzed by⁽²³⁾ for ranking the alternatives/objects. To capture the uncertainty more effectively⁽²⁴⁾ introduced a technique that takes into consideration the left and right area of the three types of membership involved in three trapezoidal fuzzy numbers. In this Paper, we have developed an algorithm using grey relational analysis and “AND” operation for decision making problems. Moreover, the technique has been extended to “intuitionistic fuzzy soft sets” and “interval-valued intuitionistic fuzzy soft sets” by imposing different thresholds on different criterion using level soft sets.

In decision making problems, we often come across situations where we get different optimal choices while using different techniques, like choice value technique and score value technique. To overcome this greyness we use grey relational analysis to get on an optimal solution. Similar situation has been tackled in this article where we resolve the ambiguous situation of different choices based on choice value technique and score value technique. In our work we extend the idea of grey relational analysis by constructing an algorithms that is composed of grey relational analysis and AND operation for intuitionistic fuzzy soft set and interval-valued intuitionistic fuzzy soft set.

2 Preliminaries

Definition 2.1.⁽⁶⁾ For a universal set U and a parameters set E . Let $L(U)$ e a set of all fuzzy sets in U , then a pair (L, E) is called a fuzzy soft set over U , where L is a mapping as described below.

$$L : E \rightarrow L(U)$$

Definition 2.2.⁽⁶⁾ If we have two fuzzy soft sets (L, T_1) and (M, T_2) over a universe set U . Then we define (L, T_1) AND (M, T_2) is a fuzzy soft set denoted by $(L, T_1) \wedge (M, T_2)$ defined as $(L, T_1) \wedge (M, T_2) = (H, T_1 \times T_2)$, where $H(\alpha, \beta) = L(\alpha) \tilde{\cap} M(\beta), \forall \alpha \in T_1$ and $\forall \beta \in T_2$, $\tilde{\cap}$ is a operation “fuzzy intersection” of two fuzzy soft sets.

3 Choice value technique

The choice value⁽⁵⁾ of a participant/alternative $p_i \in P_t$ is c_{vi} , given by $c_{vi} = \sum_k p_{ik}$ where p_{ik} are the entries in the given table. We illustrate the idea by discussing an example. Suppose we have three participants and we want to select a participant by using choice values technique. Here p_{t3} is the best choice (Supplementary table 1).

4 Comparison table technique (or Score value technique)

A square table⁽²⁵⁾ where both the rows and columns involve alternatives/objects is called comparison table. Here each alternative/object is compared with every other alternative/object in the universal set U . For a comparison table involving n object $p_{t1}, p_{t2}, \dots, p_{tn}$, let c_{ik} = the count of attributes such that degree of membership grade of $p_{ti} \geq$ that of p_{tk} .

It can be observed that $c_{ik} \in \{0, 1, 2_{acw}, n\}$ and $c_{ik} = n$ if $i = k$. Thus c_{ik} indicates an integral number for which p_{ti} dominates p_{tk} for all $p_{tk} \in U$. In comparison table technique we use score of an alternative for their ranking process for which we need to calculate the row sum (r_i) and column sum (t_k) of each alternative computed as $r_i = \sum_{k=1}^n c_{ik}$ and $t_k = \sum_{i=1}^n c_{ik}$ respectively. Here r_i is the count of total attributes of U and t_i is the count of total attributes for which p_{tk} is dominated by all the members of U . Then the score j_i of an alternative/object p_{ti} is calculated as

$$j_i = r_i - t_i$$

5 Grey algorithm

Step I.

In first step we input the choice value sequence $\{c_{v1}, c_{v2}, \dots, c_{vn}\}$ and score value sequence $\{j_1, j_2, \dots, j_n\}$.

Step II.

“Grey relational generating”

$$c'_{vi} = \frac{c_{vi} - \min\{c_{vi}\}}{\max\{c_{vi}\} - \min\{c_{vi}\}}, j_{vi} = \frac{j_{vi} - \min\{j_{vi}\}}{\max\{j_{vi}\} - \min\{j_{vi}\}}, \text{ where } i = 1, 2, 3, \dots, n.$$

Step III.

In this step we reorder the sequence as $\{c'_{v1}, j'_1\}, \{c'_{v2}, j'_2\}, \dots, \{c'_{vn}, j'_n\}$

Step IV.

“Difference information”

$$c_{v\max} = \text{Max}\{c'_{vi}\}, j_{\max} = \text{Max}\{j'_i\}, \Delta c'_{vi} = |c_{v\max} - c'_{vi}|, \Delta j'_i = |j_{\max} - j'_i|$$

$$\Delta_{\max} = \text{Max}\{\Delta c'_{vi}, \Delta j'_i\}, \Delta_{\min} = \text{Min}\{\Delta c'_{vi}, \Delta j'_i\}, \text{ where } i=1, 2, \dots, n$$

Step V.

“Grey relational coefficient”

$$\gamma(c_v, c_{vi}) = \frac{\Delta_{\min} + \chi * \max}{\Delta c'_{vj} + \chi * \max} \text{ and } \gamma(j, j_i) = \frac{\Delta_{\min} + \chi * \max}{\Delta j_i + \chi * \max}$$

where χ is called “distinguishing coefficient” and $\chi \in [0, 1]$. Its principle is to amplify or shorten the amplitude of “grey relative coefficient”.

Step VI.

“Grey relational grade”

$\gamma(p_{ti}) = w_1 * \gamma(c_v, c_{vi}) + w_2 * \gamma(j, j_i)$, where w_1 and w_2 are weights of evaluation factor and $w_1 + w_2 = 1$.

Step VII.

“Decision making”.

p_{tk} is the optimal choice, where $p_{tk} = \max \gamma(p_{ti})$. If decision makers wish to select more than one participants then they will select the participants according to the maximum number of grey relational grade.

6 Grey relational analysis for intuitionistic fuzzy soft set (IFSS)

Definition 6.1. (10) For a universal set U and $T_1 \subset E$. where E be a set of parameters. Then (L, T_1) is known as IFSS over U where L is a mapping as follows

$$L : T_1 \rightarrow I.F(U)$$

Generally for $e \in T_1$, $L(e)$ is an intuitionistic fuzzy set of and is known as intuitionistic fuzzy value set of parameter. So clearly $L(e)$ can be written as an intuitionistic fuzzy set such that $L(e) = \{ \langle x, \mu_{L(e)}(x), \lambda_{L(e)}(x) \rangle \mid x \in U \}$, where $\mu_{L(e)}$ and $\lambda_{L(e)}$ are the membership and non-membership functions respectively.

Example 6.2. Let’s imagine a business organization needs to fill a vacant position. There are 10 participants who applied legally for vacant position. The organization have chosen two decision makers, one is from the panel of directors and second one is from the office of human development. They wish to select a participant to fill a vacant position. They separately judge the desired qualities that are required to fill a position by analyzing grey algorithm stand on “AND operation”.

Consider the set of participants $P_t = \{p_{t_1}, p_{t_2}, \dots, p_{t_{10}}\}$ which may be characterized by the set of parameters $E = \{x_1, x_2, x_3, x_4, x_5, x_6\}$. The parameters x_i where $i = 1, 2, \dots, 6$, signifies “experience”, “computer knowledge”, “training”, “young age”, “higher education” and “good health” respectively.

Step I.

First decision maker considered the set of parameters, $T_1 = \{x_1, x_3, x_4\}$ and second decision considered the set of parameters, $T_2 = \{x_2, x_3, x_4, x_6\}$, where $T_1, T_2 \subset E$

Step II.

In this step decision makers assign membership grades and non-membership grades to their desired parameters as in table given below. (Supplementary Table 2 and Table 3)

Step III.

Now we will find AND product $(L_{T_1} \wedge M_{T_2})$ of the fuzzy soft sets (L, T_1) and (M, T_2) .

Here we observe that if we perform the AND product of the above fuzzy soft sets then we will get $3 \times 4 = 12$ parameters of the form e_{ik} , where $e_{ik} = a_i \wedge b_k \forall i = 1, 2, 3$ and $k = 1, 2, 3, 4$, but here we need fuzzy soft set for the parameters $R = \{x_{12}, x_{32}, x_{33}, x_{44}, x_{46}\}$. So we will get the resultant of (L, T_1) and (M, T_2) say (K, R) after performing the AND operation as follows. (Supplementary Table 4)

Step IV.

We find Top-bottom level soft set with choice values, with corresponding parameters values are $x_{12} = [0.7, 0.2]$, $x_{32} = [0.7, 0.1]$, $x_{33} = [0.9, 0.1]$, $x_{44} = [0.6, 0.2]$, $x_{46} = [0.75, 0.2]$.(Supplementary Table 5)

Step V.

Top-bottom level soft set’s comparison table.(Supplementary Table 6)

Step VI.

Now we will calculate the score values $j_i = r_i - t_i$ where r_i denote the column sum and t_i denote the row sum of the above comparison table.(Supplementary Table 7)

According to choice values p_{t_7} and p_{t_9} are the optimal choices but score values shows that p_{t_9} is the best choice. To choose which answer is the best one, we use grey algorithm which have following steps.

Step I.

From the tables we write the choice value sequence $c_{vi} = \{1, 0, 1, 0, 1, 0, 4, 0, 4, 0\}$ and sore value sequence $j_i = \{17, -1, 3, -32, 17, -19, 35, -13, 36, -43\}$.

Step II.

“Grey relational generating”

We find the values through grey relational generating

$$c'_{vi} = \frac{c_{vi} - \min\{c_{vi}\}}{\max\{c_{vi}\} - \min\{c_{vi}\}}, j_{vi} = \frac{j_{vi} - \min\{j_{vi}\}}{\max\{j_{vi}\} - \min\{j_{vi}\}}, \text{ where } i = 1, 2, 3, \dots, 10 \text{ are}$$

$$c'_{vi} = \{0.25, 0, 0.25, 0, 0.25, 0, 1, 0, 1, 0\} \text{ and } j'_i = \{0.76, 0.53, 0.58, 0.14, 0.76, 0.30, 0.99, 0.38, 1, 0\}$$

Step III.

In this step we reorder the sequence as $\{c'_{v1}, j'_1\}, \{c'_{v1}, j'_1\}, \dots, \{c'_{vn}, j'_n\}$ and we get $\{c'_{v1}, j'_1\} = \{0.25, 0.76\}$, $\{c'_{v2}, j'_2\} = \{0, 0.53\}$, $\{c'_{v3}, j'_3\} = \{0.25, 0.58\}$, $\{c'_{v4}, j'_4\} = \{0, 0.14\}$, $\{c'_{v5}, j'_5\} = \{0.25, 0.76\}$, $\{c'_{v6}, j'_6\} = \{0, 0.30\}$, $\{c'_{v7}, j'_7\} = \{1, 0.99\}$, $\{c'_{v8}, j'_8\} = \{0, 0.38\}$, $\{c'_{v9}, j'_9\} = \{1, 1\}$, $\{c'_{v10}, j'_{10}\} = \{0, 0\}$

Step IV.

“Difference information”

To find $\Delta_{\max} = \text{Max}\{\Delta c'_{vi}, \Delta j'_i\}$ and $\Delta_{\min} = \text{Min}\{\Delta c'_{vi}, \Delta j'_i\}$ we have $c_{v_{\max} = \text{Max}\{c'_{vi}\} = 1}$ and $j_{\max} = \text{Max}\{j'_i\} = 1$. Calculated values are

$$\Delta c'_{vi} = \{0.75, 1, 0.75, 1, 0.75, 1, 0, 1, 0, 1\} \text{ and } \Delta j'_i = \{0.24, 0.47, 0.42, 0.86, 0.24, 0.7, 0.01, 0.62, 0, 1\}$$

So $\Delta_{\max} = 1$ and $\Delta_{\min} = 0$

Step V.

In this step we will find “grey relative coefficient” through

$$\gamma(c_v, c_{vi}) = \frac{\Delta_{\min} + \chi * \Delta_{\max}}{\Delta c'_{vi} + \chi * \Delta_{\max}} \text{ and } \gamma(j, j_i) = \frac{\Delta_{\min} + \chi * \Delta_{\max}}{\Delta j'_i + \chi * \Delta_{\max}}$$

Where $\chi \in [0, 1]$ is called “distinguishing” coefficient” and its aim is to amplify or shorten the amplitude of “grey relative coefficient”. Here $\chi = 0.5$. Calculated values are

$$\gamma(c_v, c_{vi}) = \{0.4, 0.33, 0.4, 0.33, 0.4, 0.33, 1, 0.33, 1, 0.33\}$$

$$\gamma(j, j_i) = \{0.68, 0.52, 0.54, 0.37, 0.68, 0.42, 0.98, 0.45, 1, 0.33\}.$$

Step VI.

In this step we find the grey relational grade through $\gamma(p_{t_i}) = w_1 * \gamma(c_v, c_{v_i}) + w_2 * \gamma(j, j_i)$, where w_1 and w_2 are weights of evaluation factor and $w_1 + w_2 = 1$ but in this the- sis $w_1 = w_2 = 0.5$. Calculated values are

$$\begin{aligned} \gamma(p_{t_1}) &= 0.54, \gamma(p_{t_2}) = 0.42, \gamma(p_{t_3}) = 0.47, \gamma(p_{t_4}) = 0.35, \gamma(p_{t_5}) = 0.54, \gamma(p_{t_6}) = 0.38, \gamma(p_{t_6}) = 0.38 \\ \gamma(p_{t_7}) &= 0.99, \gamma(p_{t_8}) = 0.39, \gamma(p_{t_9}) = 1, \gamma(p_{t_{10}}) = 0.33 \end{aligned}$$

Step VII.

“Decision making”

After analysis, we observe that p_{t_9} is optimal choice. If we select $\gamma(p_{t_i}) \geq 0.5$ then selected participants according to the maximum are $p_{t_9} = 1, p_{t_7} = 0.99, p_{t_5} = 0.54, p_{t_1} = 0.54$.

7 Bottom-bottom level soft set based decision-making

Bottom-bottom level soft set of $(L_{T_1} \wedge M_{T_2})$. (Supplementary Table 8)

Here is the case where all the choice values are zero but score values of $(L_{T_1} \wedge M_{T_2})$ shows that p_{t_9} is the est choice. to overcome this confusion of optimal decision in between choice values and score values we apply grey algorithm having following steps.

Step I.

From the tables we write the choice value sequence $c_{vi} = \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}$ and sore value sequence $j_i = \{17, -1, 3, -32, 17, -19, 35, -13, 36, -43\}$

Step II.

We compute “grey relational generating”

$$c'_{vi} = \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\} \text{ and } j_i = \{17, -1, 3, -32, 17, -19, 35, -13, 36, -43\}$$

Step III.

In this step we order the sequence as $\{c'_{v_1}, j'_1\} = \{0, 0.76\}, \{c'_{v_2}, j'_2\} = \{0, 0.53\}, \{c'_{v_3}, j'_3\} = \{0, 0.58\}, \{c'_{v_4}, j'_4\} = \{0, 0.14\}, \{c'_{v_5}, j'_5\} = \{0, 0.76\}, \{c'_{v_6}, j'_6\} = \{0, 0.30\}, \{c'_{v_7}, j'_7\} = \{0, 0.99\}, \{c'_{v_8}, j'_8\} = \{0, 0.38\}, \{c'_{v_9}, j'_9\} = \{0, 1\}, \{c'_{v_{10}}, j'_{10}\} = \{0, 0\}$

Step IV.

$$\Delta_{\max=1 \text{ and } \Delta_{\min=0}}$$

Step V.

$$\begin{aligned} \text{For } \chi = 0.5 \text{ “grey relative coefficient”. } \gamma(c_v, c_{v_x}) &= \{1, 1, 1, 1, 1, 1, 1, 1, 1, 1\} \\ \gamma(j, j_i) &= \{0.68, 0.52, 0.54, 0.37, 0.68, 0.42, 0.98, 0.45, 1, 0.33\} \end{aligned}$$

Step VI.

Calculated values of “grey relational grade”, for $w_1 = w_2 = 0.5$ $\gamma(p_{t_1}) = 0.84, \gamma(p_{t_2}) = 0.76, \gamma(p_{t_3}) = 0.77, \gamma(p_{t_4}) = 0.69, \gamma(p_{t_5}) = 0.84, \gamma(p_{t_6}) = 0.71, \gamma(p_{t_7}) = 0.99, \gamma(p_{t_8}) = 0.72, \gamma(p_{t_9}) = 1, \gamma(p_{t_{10}}) = 0.67$

Step VII.

According to grey relational analysis p_{t_9} is the optimal choice. If there are more than one seats then we select the candidate according to maximum numbers of grades for example if we take $\gamma(p_{t_i}) \geq 0.7$ then selected participants are $p_{t_9}, p_{t_7}, p_{t_1}, p_{t_5}, p_{t_3}, p_{t_2}, p_{t_8}, p_{t_6}$.

8 Grey relational analysis for interval-valued intuitionistic fuzzy soft set (IVIFSS)

Definition 8.1.⁽¹⁰⁾ “Consider U as universe set and E as a set of parameters. Interval- valued intuitionistic fuzzy set is presented by $E(U)$. A combination (L, E) is said to be soft set over U , where $L : E \rightarrow E(U)$.

For any $e \in T_1 \subset E$, IVIFSS $L(e)$ is presented as $L(e) = \{ \langle x, \mu_{L(e)}(x), \lambda_{L(e)}(x) \rangle \mid x \in U \}$ and $\mu_{L(e)}$ shows interval-valued membership degree that x have for the parameter e and $\lambda_{L(e)}$ shows interval-valued membership degree that x doesn't have for parameter e .

Example 8.2. Let’s imagine a business organization needs to fill a vacant position. There are 10 participants who applied legally for vacant position. The organization have chosen two decision makers, one is from the panel of directors and second one is from the office of human development. They wish to select a participant to fill a vacant position. They separately judge the desired qualities that are required to fill a position by using grey algorithm stand on AND operation for interval-valued intuitionistic fuzzy soft sets.

Consider the set of participants is $P_t = \{p_{t_1}, p_{t_2}, \dots, p_{t_{10}}\}$ which may be characterized by the set of parameters $E = \{x_1, x_2, x_3, x_4, x_5, x_6\}$. The parameters x_i where $i = 1, 2, \dots, 6$ signifies “experience”, “computer knowledge”, “training”, “young age”, “higher education” and “good health” respectively.

Step I.

First decision maker considered the set of parameters, $T_1 = \{x_1, x_3, x_4\}$ and second decision considered the set of parameters, $T_2 = \{x_2, x_3, x_4, x_6\}$, where $T_1, T_2 \subset E$

Step II.

In this step decision makers assign membership grades to their desired parameters as follows. (Supplementary Table 9 and Table 10)

Step III.

In this step we will find AND product $(L_{T_1} \wedge M_{T_2})$ of the fuzzy soft sets (L, T_1) and (M, T_2) Here we observe that if we perform the AND product of the above fuzzy soft sets then we will get $3 \times 4 = 12$ parameters in the form e_{ik} , where $e_{ik} = a_i \wedge b_k, \forall i = 1, 2, 3$ and $k = 1, 2, 3, 4$ but here we require fuzzy soft set of the form $R = \{x_{12}, x_{32}, x_{33}, x_{44}, x_{46}\}$. So we will get the resultant of (L, T_1) and (M, T_2) say (K, R) after performing the AND operation as follows. (Supplementary Table 11)

Step IV.

Optimistic-optimistic reduct intuitionistic fuzzy soft set of $(L_{T_1} \wedge M_{T_2})$. (Supplementary Table 12)

Step V.

Mid-level soft set of optimistic-optimistic reduct intuitionistic fuzzy soft set with choice values. For $x_{12} = [0.54, 0.26], x_{32} = [0.53, 0.25], x_{33} = [0.67, 0.18], x_{44} = [0.58, 0.28], x_{46} = [0.56, 0.23]$. (Supplementary Table 13)

Step VI.

Here we compute the comparison table of optimistic-optimistic reduct interval valued intuitionistic fuzzy soft set. (Supplementary Table 14)

Step VII.

Now we will calculate the score values $j_i = r_i - t_i$ where r_i denote the column sum and t_i denote the row sum as calculated in the supplementary table 15.

According to choice values there are four participants $p_{t_4}, p_{t_6}, p_{t_7}, p_{t_8}$ having same choice values but score values shows that p_{t_7} is the best choice. To choose which answer is best one we use grey algorithm that comprises of following steps.

Step I.

From the tables we write the choice value sequence $c_{v_i} = \{1, 2, 3, 4, 1, 4, 4, 4, 0, 0\}$ and score value sequence $j_i = \{-13, -8, 4, 18, -8, 17, 29, 20, -15, -34\}$.

Step II.

“Grey relational generating” $c'_{vi} = \frac{c_{vi} - \min\{c_{vi}\}}{\max\{c_{vi}\} - \min\{c_{vi}\}}, j'_{vi} = \frac{j_{vi} - \min\{j_{vi}\}}{\max\{j_{vi}\} - \min\{j_{vi}\}}$, where $i = 1, 2, 3, \dots, 10$ and generated values are $c'_{v_1} = \{0.25, 0.5, 0.75, 1, 0.25, 1, 1, 1, 0, 0\}$ and $j'_i = \{0.33, 0.41, 0.60, 0.82, 0.41, 0.81, 1, 0.86, 0.30, 0\}$

Step III.

In this step we reorder the sequence as $\{c'_{v_1}, j'_1\}, \dots, \{c'_{v_{10}}, j'_{10}\} \{c'_1, j'_1\} = \{0.25, 0.33\}, \{c'_{v_2}, j'_2\} = \{0.5, 0.41\}, \{c'_{v_3}, j'_3\} = \{0.75, 0.60\} \{c'_{v_4}, j'_4\} = \{1, 0.82\} \{c'_{v_5}, j'_5\} = \{0.25, 0.41\}, \{c'_{v_6}, j'_6\} = \{1, 0.81\} \{c'_{v_7}, j'_7\} = \{0.25, 0.33\}, \{c'_{v_8}, j'_8\} = \{0.5, 0.41\} \{c'_{v_9}, j'_9\} = \{0.75, 0.60\}, \{c'_{v_{10}}, j'_{10}\} = \{1, 0.82\}, \{c'_{v_1}, j'_1\} = \{0.25, 0.41\}, \{c'_{v_2}, j'_2\} = \{1, 0.81\} \{c'_{v_3}, j'_3\} = \{1, 1\}, \{c'_{v_4}, j'_4\} = \{1, 0.86\}, \{c'_{v_5}, j'_5\} = \{0, 0.30\}, \{c'_{v_6}, j'_6\} = \{0, 0\}$

Step IV.

“Difference information”

To find $\Delta_{max} = \text{Max}(\Delta c'_{v_i}, \Delta j'_i)$ and $\Delta_{min} = \text{Min}(\Delta c'_{v_i}, \Delta j'_i)$, we have $c_{v_{max}} = \text{Max}\{c'_{v_i}\} = 1$ and $j_{max} = \text{Max}\{j'_i\} = 1$. Now we find $\Delta c'_{v_i} = |c_{v_{max}} - c'_{v_i}|$ and $\Delta j'_i = |j_{max} - j'_i|$. Calculated values are $\Delta c'_v = \{0.75, 0.5, 0.25, 0, 0.75, 0, 0, 0, 1, 1\}$ and $\Delta j'_j = \{0.67, 0.59, 0.4, 0.18, 0.59, 0.19, 0, 0.14, 0.7, 1\}$, so $\Delta_{max}=1$ and $\Delta_{min}=0$

Step V.

In this step we will find “grey relative coefficient” through $\gamma(c_v, c_{v_i}) = \frac{\Delta_{min} + \chi * \Delta_{max}}{\Delta c'_{v_i} + \chi * \Delta_{max}}$ and $\gamma(j, j_i) = \frac{\Delta_{min} + \chi * \Delta_{max}}{\Delta j'_i + \chi * \Delta_{max}}$ where χ is

called “distinguishing coefficient” and its bourne is to amplify or shorten the amplitude of “grey relative coefficient”.

Here $\chi = 0.5$. Calculated values are $\gamma(c_v, c_{v_i}) = \{0.40, 0.5, 0.67, 1, 0.4, 1, 1, 1, 0.33, 0.33\}$ $\gamma(j, j_i) = \{0.43, 0.46, 0.56, 0.74, 0.46, 0.72, 1, 0.78, 0.42, 0.33\}$

Step VI.

In this step we find the grey relational grade through $\gamma(p_i) = w_1 * \gamma(c_v, c_{v_i}) + w_2 * \gamma(j, j_i)$, where w_1 and w_2 are weights of evaluation factor and $w_1 + w_2 = 1$ but in this thesis $w_1 = w_2 = 0.5$. Calculated values are $\gamma(p_{t_1}) = 0.42, \gamma(p_{t_2}) = 0.48, \gamma(p_{t_3}) = 0.62, \gamma(p_{t_4}) = 0.87, \gamma(p_{t_5}) = 0.43, \gamma(p_{t_6}) = 0.86, \gamma(p_{t_7}) = 1, \gamma(p_{t_8}) = 0.89, \gamma(p_{t_9}) = 0.38, \gamma(p_{t_{10}}) = 0.33$

Step VII.

“Decision making”

After analysis, we observe that p_{t_7} is optimal choice. If we select $\gamma(p_{t_i}) \geq 0.5$ then selected participants according to the maximum are $p_{t_7} = 1, p_{t_8} = 0.89, p_{t_4} = 0.87, p_{t_6} = 0.86, p_{t_3} = 0.62, p_{t_{10}} = 0.55$.

9 Pessimistic-pessimistic reduct IVIFSS based decision-making

Step I.

Tabular representation of $(L_{T_1} \wedge M_{T_2})$ for pessimistic-pessimistic reduct IVIFSS. (Supplementary Table 16)

Step II.

Mid level soft for the above pessimistic-pessimistic reduct interval-valued intuitionistic fuzzy soft set with choice values. For $x_{12} = [0.45, 0.36], x_{32} = [0.44, 0.34], x_{33} = [0.62, 0.24], x_{44} = [0.5, 0.33], x_{12} = [0.5, 0.29]$. (Supplementary Table 17)

Step III.

Comparison table of pessimistic-pessimistic reduct interval valued intuitionistic fuzzy soft set. (Supplementary Table 18)

Step IV.

Now we will calculate the score values $j_i = r_i - t_i$ where r_i denote the column sum and t_i denote the row sum as calculated in the table as follows. (Supplementary Table 19)

According to the choice values p_{t_6} but score values shows that p_{t_7} is the best choice. To overcome this confusion of optimal decision in between choice values and score values we apply grey algorithm having following steps.

Step I.

From the tables we write the choice value sequence $c_{v_i} = \{0, 0, 3, 4, 1, 5, 4, 4, 0, 0\}$ and score value sequence $j_i = \{-36, -15, 8, 26, -12, 22, 27, 26, -12, -34\}$.

Step II.

Values generated from “grey relational generating” are $c'_{v_i} = \{0, 0, 0.6, 0.8, 0.2, 1, 0.8, 0.8, 0, 0\}$ and $j'_i = \{0, 0.33, 0.7, 0.98, 0.38, 0.92, 1, 0.98, 0.38, 0.03\}$

Step III.

In this step we reorder the sequence.

$$\begin{cases} \{c'_{v_1}, j'_1\} = \{0, 0\}, \{c'_{v_2}, j'_2\} = \{0, 0.33\}, \{c'_{v_3}, j'_3\} = \{0.6, 0.7\}, \{c'_{v_4}, j'_4\} = \{0.8, 0.98\} \\ \{c'_{v_5}, j'_5\} = \{0.2, 0.38\}, \{c'_{v_6}, j'_6\} = \{1, 0.92\}, \{c'_{v_7}, j'_7\} = \{0.8, 1\}, \{c'_{v_8}, j'_8\} = \{0.8, 0.98\} \\ \{c'_{v_9}, j'_9\} = \{0, 0.38\}, \{c'_{v_{10}}, j'_{10}\} = \{0, 0.03\} \end{cases}$$

Step IV.

$$\Delta_{\max} = \text{Max}(\Delta c'_{v_i}, \Delta j'_i) = 1 \text{ and } \Delta_{\min} = \text{Min}(\Delta c'_{v_i}, \Delta j'_i) = 0$$

Step V.

For $\chi = 0.5$ “grey relative coefficient”.

$$\begin{aligned} \gamma(c_v, c_{v_i}) &= \{0.33, 0.33, 0.56, 0.71, 0.38, 1, 0.71, 0.71, 0.33, 0.33\} \\ \gamma(j, j_i) &= \{0.33, 0.43, 0.63, 0.96, 0.45, 0.86, 1, 0.96, 0.45, 0.34\} \end{aligned}$$

Step VI.

“Grey relational grade”, for $w_1 = w_2 = 0.5$

$$\begin{aligned} \gamma(p_{t_1}) &= 0.33, \gamma(p_{t_2}) = 0.38, \gamma(p_{t_3}) = 0.6, \gamma(p_{t_4}) = 0.84, \gamma(p_{t_5}) = 0.42 \\ \gamma(p_{t_6}) &= 0.93, \gamma(p_{t_7}) = 0.86, \gamma(p_{t_8}) = 0.84, \gamma(p_{t_9}) = 0.39, \gamma(p_{t_{10}}) = 0.34. \end{aligned}$$

Step VII.

According to grey relational analysis p_{t_6} is the optimal choice. If there are more than one seats then we select the candidate according to maximum numbers of grades for example if we take $\gamma(p_{t_i}) \geq 0.7$ then selected participants are $p_{t_6}, p_{t_7}, p_{t_8}, p_{t_4}$.

10 Conclusion

Here we have dealt with one of the ambiguous situations arising in solving a problem from the class of organized complexity by making use of grey relational analysis technique. By using IFSS and IVIFSS and level soft sets for imposing desired thresholds on different criterion, we arrived at different optimal choices by using Choice value technique and comparison table technique. To resolve the problem of preference order, grey relational analysis method with AND operation was used to get on a suitable selection.

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