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RESEARCH ARTICLE

Some results in multiplicative metric space using absorbing mappings

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Abstract

Objective/Aim: To generate three common fixed point results for four self mappings in complete multiplicative metric space (MMS). **Method:** The method involves applying of point wise absorbing mappings with different combinations such as complete subspace, reciprocally continuous and compatible mappings and semi-compatible mappings. **Findings:** All the results are supported by the provision of valid examples. **Novelty/Improvement:** The concept of reciprocally continuity along with semi compatible mappings is used which is weaker than the concept of continuity and compatibility.

Keywords: Fixed point; absorbing maps; compatible mappings; semi-compatible mappings; reciprocally continuous mappings

1 Introduction

The extraction of fixed point theorems has been fascinating area to the researchers due to its remarkable applications in many areas of mathematics and other allied subjects. The notion of multiplicative distance is initiated in multiplicative calculus [(1)]. Subsequently, the multiplicative metric space (MMS) has been introduced [(2)]. Proved fixed point theorem in multiplicative metric space using weak commuting mappings [(3)]. Introduced the notion of Absorbing mappings in fuzzy metric space and prove common fixed point theorems [(4)]. The notion of semi-compatible mappings is introduced in d-topological space [(5)]. The notion of reciprocally continuous mappings became instrumental in proving some common fixed point theorems in metric space [(6)]. Recently some results in multiplicative space are seen in [(7)] and [(8)] using the concept of semi compatible mappings.

In this study, absorbing mappings notion is initiated in MMS to generate some common fixed point theorems under different conditions.

2 Preliminaries

Before establishing our Theorems we present some definitions and results that they are needed.

Definition 2.1. [(1)]**:** For a non empty set X ,a MMS $(X,\ d^{*})$ is defined as function $d^{*}: X\times X\to(0,\infty)$ holding the conditions:

MMS (i) $d^{*}(\alpha,\beta)\geq 1\ for\ all\ \alpha,\beta\in X$ and $d^{*}(\alpha,\beta)=1\iff\alpha=\beta$

MMS (ii) $d^*(\alpha,\beta) = d^*(\beta,\alpha)$ for all $\alpha,\beta \in X$

MMS (iii) $d^*(\alpha,\beta) \leq d^*(\alpha,\gamma).d^*(\gamma,\beta)$ for all $\alpha,\beta,\gamma \in X$.

Definition 2.2. [(2)]**:** In a MMS a sequence $\{\alpha_\eta\}$ converges to α in X if $d^*(\alpha_\eta,\alpha) < \varepsilon$ for each $\in > 1$ and for all $\eta \geq \eta_0$ and $\eta_0 \in N$.

Definition 2.3. [(2)]**: M**ultiplicative Cauchy sequence in MMS is one which holds $d^*(\alpha_\eta,\ \beta_m) < \in$ for all $m,\eta > N$ and for all $\in > 1$.

Definition 2.4. [(2)]**:** A complete MMS is one in which every Cauchy sequence is converges in it.

Definition 2.5 : In a MMS having two maps A and S then S said to be A-absorbing if $d^*(A\alpha,AS\alpha) \leq d^{*R}(A\alpha,S\alpha)$ for some real number R>0 and for all $\alpha \in X$.

Example 2.5 .1 : Let $X = (0,\infty)$.Define $d^*: X\times X \to (0,\infty)$ by $d^*(\alpha,\beta) = e^{|\alpha-\beta|}$ then (X,d^*) is multiplicative metric space. Defined $A,S: X\times X \to (0,\infty)$ as

$$A(\alpha) = \begin{cases} \frac{1}{2} \text{ for } \alpha \neq \frac{1}{2} \\ 0 \text{ for } \alpha = \frac{1}{2} \end{cases} \quad and\ S(\alpha) = \frac{1}{2} \text{ for all } \alpha \in \mathrm{X}$$

Then A is S-absorbing for $R \geq 1$.

Definition 2.6: A condition $d^*(Ax,ASx) \leq d^{*R}(Ax,Sx)$ for some $R > 0$ and for given xX holds then the map S in a MMS is said to be point wise A-absorbing.

Example 2.6.1:*Let* $X = [0,10]$. Define $d^*: X\times X \to R^+$ by $d^* = e^{|\alpha-\beta|}$ then (X,d^*) is MMS. The mappings $A(\alpha) = 1\ \ and\ \ S(\alpha) = \frac{3\alpha}{2\alpha+1}$ for all $\alpha \in X$ then A is point wise S-absorbing.

Definition 2.7: We define the pair (S,A) in MMS as compatible or asymptotically commuting if for some $t \in X$, $\lim_{\eta\to\infty} d^*(SA\alpha_\eta, AS\alpha_\eta) = 1$ whenever $\{\alpha_\eta\}$ is a sequence in X such that $\lim_{\eta\to\infty} Ax_\eta = \lim_{\eta\to\infty} Sx_\eta = t$.

Example 2.7.1:*Let* $X = [1,10]$ Define $d^*: X\times X \to R^+$ by $d^*(\alpha,\beta) = e^{|\alpha-\beta|}$ then (X,d^*) is MMS. The mappings A and S are defined as

$$\mathrm{A}(\alpha) = \begin{cases} 1 \text{ for } 1 \leq \alpha \leq 2 \text{ and } \alpha = 3 & \\ 4 & \text{for } \alpha > 3 \\ \frac{3\alpha-1}{5} & \text{for } \alpha \in (2,3) \end{cases} \quad \text{and } \mathrm{S}(\alpha) = \begin{cases} 2 \text{ for } 1 \leq \alpha \leq 2 \\ \frac{2\alpha+1}{5} \text{ for } \alpha > 2 \end{cases}$$

Let $(\alpha_\eta\} = \left(2+\frac{1}{\eta}\right\}\ for\ \eta\ > 0$. Then it is easy to see that the pair (A,S) is not compatible but A is S-absorbing.

Definition 2.8: Mappings $S\ and\ A$ of multiplicative metric space (X,d^*) are said to be semi compatible if $\lim_{\eta\to\infty} d^*(SA\alpha_\eta, A\zeta) = 1$ for all $\zeta > 0$ whenever $\{\alpha_\eta\}$ is a sequence in X such that $\lim_{\eta\to\infty} S\alpha_\eta = \lim_{\eta\to\infty} A\alpha_\eta = \zeta$ for some $\zeta \in X$.

Example 2.8.1: Let (X,d^*) be a multiplicative metric space where X=[0,1] and $d^*(\alpha,\beta) = e^{|\alpha-\beta|}$. We define the functions S and A by

$$\mathrm{S}(\alpha) = \begin{cases} \frac{1}{6} - \alpha \text{ if } 0 \leq \alpha \leq \frac{1}{12} \\ \frac{1}{12} \text{ if } \frac{1}{12} < \alpha \leq 1 \end{cases} \qquad \mathrm{A}(\alpha) = \begin{cases} \frac{1}{12} \text{ if } 0 \leq \alpha \leq \frac{1}{12} \\ \frac{1}{10} \text{ if } \frac{1}{12} < \alpha \leq 1 \end{cases}$$

Let $\alpha_\eta = \frac{1}{12} - \frac{1}{\eta}\ for\ \eta \geq 1$, then the pair (S,A) is semi- compatible.

Definition 2.9: A pair of self maps (S,A) of multiplicative metric space (X,d^*) is said to be reciprocally continuous if $\lim_{\eta\to\infty} d^*(AS\alpha_\eta, A\zeta) = 1$ and $\lim_{\eta\to\infty} d^*(SA\alpha_\eta, S\zeta) = 1$whenever there exists a sequence $\{\alpha_\eta\}\ in\ X\ such$ that $\lim_{\eta\to\infty} S\alpha_\eta = t$, $\lim_{n\to\infty} A\alpha_\eta = t$ for some $t \in X$.

Example 2.9.1: Let (X,d^*) be a multiplicative metric space where $X = [-1,1]$ and $d^*(\alpha,\beta) = e^{|\alpha-\beta|}$.We define mappings $A$and S by

$$\mathrm{A}(\alpha) = \begin{cases} \frac{1}{5} \text{ if } -1 \leq \alpha < \frac{1}{6} \\ \frac{1}{6} \text{ if } \frac{1}{6} \leq \alpha \leq 1 \end{cases} \quad \text{and } \mathrm{S}(\alpha) = \begin{cases} \frac{1}{5} \text{ if } -1 \leq \alpha < \frac{1}{6} \\ \frac{6\alpha+5}{36} \text{ if } \frac{1}{6} \leq \alpha \leq 1 \end{cases}$$

Let the sequence $\alpha_\eta = \frac{1}{6} + \frac{1}{\eta}\ for\ \eta \geq 1$ then the pair (A,S) is reciprocally continuous.

Now some common fixed point theorems are to be established using different conditions in multiplicative metric space.

3 Main Results

3.1 Theorem: The mappings S, T, A and B defined in a complete MMS holding the conditions:

3.1.1 $\mathrm{S(X)} \subseteq \mathrm{B(X)}, \mathrm{T(X)} \subseteq \mathrm{A(X)}$

3.1.2 $d^*(\mathrm{S}\alpha,\mathrm{T}\beta) \le \{d^*(\mathrm{A}\alpha,\mathrm{B}\beta)\cdot d^*(\mathrm{A}\alpha,S\alpha)\cdot d^*(\mathrm{B}\beta,\mathrm{T}\beta)\cdot d^*(S\alpha,\mathrm{B}\beta)\cdot d^*(\mathrm{A}\alpha,\mathrm{T}\beta)\}^{\frac{\lambda}{6}}$ where $\lambda \in \left(0.\frac{1}{2}\right)$ for all $\alpha,\beta \in X$

3.1.3 If the mappings S *and* T are point wise A-absorbing and point wise B-absorbing respectively.

3.1.4 If the range of one of the mappings S,T,A *or* B is a complete subspace of X then A,B,S *and* T have a unique common fixed point in X.

Following discussion is useful in proving of Theorem 3.1

Since $S(X) \subseteq B(X)$, consider a point $\alpha_0 \in X$, there exists $\alpha_1 \in X$ such that $S\alpha_0 = B\alpha_1 = \beta_0$. For this α_1 *there exists* $\alpha_2 \in X$ such that $T\alpha_1 = A\alpha_2 = \beta_1$. Continuing this process we get $S\alpha_{2\eta} = B\alpha_{2\eta+1} = \beta_{2\eta}(say)and\ T\alpha_{2\eta+1} = A\alpha_{2\eta+2} = \beta_{2\eta+1}(say)$.

Now we can define $(\beta_\eta\}$ in X, we obtain

$$d^*(\beta_{2\eta},\beta_{2\eta+1}) \le \left\{\begin{matrix} d^*(A\alpha_{2\eta},B\alpha_{2\eta+1})\cdot d^*(\mathrm{A}\alpha_{2\eta},S\alpha_{2\eta})\cdot d^*(\mathrm{B}\alpha_{2\eta+1},\mathrm{T}\alpha_{2\eta+1}) \\ d^*(S\alpha_{2\eta},B\alpha_{2\eta+1})\cdot d^*(\mathrm{A}\alpha_{2\eta},\mathrm{T}\alpha_{2\eta+1}) \end{matrix}\right\}^{\frac{\lambda}{6}}$$

$$\le (d^*(\beta_{2\eta-1},\beta_{2\eta}).d^*(\beta_{2\eta-1},\beta_{2\eta}).d^*(\beta_{2\eta},\beta_{2\eta+1}).d^*(\beta_{2\eta},\beta_{2\eta}).d^*(\beta_{2\eta-1},\beta_{2\eta+1})\}^{\frac{\lambda}{6}}$$

$$\le \left(d^{*3}(\beta_{2\eta-1},\beta_{2\eta}).d^{*2}(\beta_{2\eta},\beta y_{2\eta+1})\right\}^{\frac{\lambda}{6}}$$

this implies that

$d^*(\beta_{2\eta},\beta_{2\eta+1}) \le d^{*\frac{3}{2}\left(\frac{\lambda}{3-\lambda}\right)}(\beta_{2\eta-1},\beta_{2\eta})$.

Let $\frac{3}{2}\left(\frac{\lambda}{1-\lambda}\right) = h$, then

$d^*(\beta_{2\eta},\beta_{2\eta+1}) \le d^{*^h}(\beta_{2\eta-1},\beta_{2\eta})$.

We also obtain $d^*(\beta_{2\eta+1},\beta_{2\eta+2}) \le d^{*^h}(\beta_{2\eta},\beta_{2\eta+1})$.

Therefore $d^*(\beta_\eta,\beta_{\eta+1}) \le d^{*^h}(\beta_{\eta-1},\beta_\eta) \le \ldots d^{*^{h^\eta}}(\beta_1,\beta_0)$ *for all* $\eta \ge 2$.

Let $m,\eta \in N$ such that $m \ge \eta$, then we get

$$d^*(\beta_m,\beta_\eta) \le d^*(\beta_m,\beta_{m-1}).d^*(\beta_{m-1},\beta_{m-2})\ldots\ldots d^*(\beta_{\eta+1},\beta_\eta)$$

$$\le d^{*^{h^{m-1}}}(\beta_1,\beta_0).d^{*^{h^{m-2}}}(\beta_1,\beta_0)\ldots\ldots d^{*^{h^\eta}}(\beta_1,\beta_0)$$

$$\le d^{*\frac{h^\eta}{1-h}}(\beta_1,\beta_0)$$

this implies $d^*(\beta_m,\beta_\eta) \to 1$ as $\eta \to \infty$.

Hence $(\beta_\eta\}$ is multiplicative Cauchy sequence.

By the completeness of X, there exists $\zeta \in X$ such that $(\beta_\eta\} \to \zeta$ *as* $\eta \to \infty$. Moreover, $(S\alpha_{2\eta}\} = (B\alpha_{2\eta+1}\} = (\beta_{2n}\}$ and $(T\alpha_{2\eta+1}\} = (A\alpha_{2\eta+2}\} = (\beta_{2\eta+1}\}$ are sub sequences of $(\beta_\eta\}$, consequently $S\alpha_{2\eta}$, $B\alpha_{2\eta+1}$, $T\alpha_{2\eta+1}$ $A\alpha_{2\eta+2}$ *convege to* ζ *as* $\eta \to \infty$.

Proof of Theorem 3.1

Let $A(X)$ be the range of X being a complete subspace, then there exists a point Au such that $\lim_{\eta\to\infty} A\alpha_{2\eta} = Au$. By condition 3.1.4 we get $T\alpha_{2\eta+1} \to Au$, $S\alpha_{2\eta-2} \to Au, B\alpha_{2\eta} \to Au$ *as* $\eta \to \alpha$in view of discussion $Au = \zeta$.

Put $\alpha = u$, $\beta = \alpha_{2\eta+1}$ in 3.1.2 we have

$$d^*(Su,T\alpha_{2\eta+1}) \quad \le (d^*(Au,B\alpha_{2\eta+1}).d^*(Au,Su).d^*(B\alpha_{2\eta+1},T\alpha_{2\eta+1}).d^*(Su,B\alpha_{2\eta+1}).d^*(Au,T\alpha_{2\eta+1})\}^{\frac{\lambda}{6}}$$

letting $\eta \to \infty$we obtain

$$d^*(Su,Au) \le (d^*(Au,Au).d^*(Au,Su).d^*(Au,Au).d^*(Su,Au).d^*(Au,Au)\}^{\frac{\lambda}{6}}$$

this gives

$d^*(Au,Su) \leq d^{*\frac{\lambda}{3}}(Au,Su)$, a contradiction to the definition of 2.1
which implies $Au = Su$.
Since $\mathrm{S(X)} \subseteq \mathrm{B(X)}$ then there exists $w \in \mathbb{X}$ such that $Au = Bw$.
Put $\alpha = u,\ \beta = w$ in the inequality we get

$$d^*(Su,Tw) \leq \left(d^*(Au,Bw).d^*(Au,Su).d^*(Bw,Tw).d^*(Su,Bw).d^*(Au,Tw)\right\}^{\frac{\lambda}{6}}$$

$$d^*(Au,Tw) \leq d^{*\frac{\lambda}{3}}(Au,Tw)$$

this implies $Au = Tw$.
Thus we have $Su = Au = Tw = Bw$.
Since S is a point wise A-absorbing *this makes*

$$d^*(Au,ASu) \leq d^{*R}(Au,Su)$$

this implies $Au = ASu = AAu$.
By putting $\alpha = Au, \beta = w$ in condition 3.1.2

$$d^*(SAu,Tw) \leq \left(d^*(AAu,Bw).d^*(AAu,SAu).d^*(Bw,Tw).d^*(SAu,Bw).d^*(AAu,Tw)\right\}^{\frac{\lambda}{6}}$$

$$d^*(SAu,Au) \leq d^{*\frac{\lambda}{3}}(SAu,Au)$$

this implies $SAu = Au$.
Therefore $ASu = SAu = Au$.
Similarly T is a point wise B-absorbing
implies $d^*(Bw,BTw) \leq d^{*R}(Bw,Tw)$
and implies $Bw = BTw$.
This gives $Bw = BTw = BBw = Tw = Au$.
Thus we have $BAu = Au$.
Now we claim $TTw = Tw$ for this put $\alpha = u,\ \beta = Tw$ in condition then we get

$$d^*(Su,TTw) \leq \left(d^*(Au,BTw).d^*(Au,Su).d^*(BTw,TTw).d^*(Su,Bw).d^*(Au,TTw)\right\}^{\frac{\lambda}{6}}$$

this gives$d^*(Au,TTw) \leq d^{*\frac{\lambda}{3}}(Au,TTw)$
this gives $Au = TTw = T(Tw) = TAu$
and this gives $TAu = Au$.
Therefore $AAu = SAu = BAu = TAu = Au$.
Thus Au is a common fixed point of $S,T,A,$ *and* B and ζ is common fixed point of the mappings S,T,A and B. The uniqueness of the fixed point can be easily verified. The proof is similar when $T(X)$, *or* $B(X)$ *or* $S(X)$ is assumed to be complete subspace of X.

The following example satisfies all the properties of Theorem 3.1.

3.1.6 Example: Let $(\mathbb{X},d^*)$ be MMS where $\mathbb{X} = [0,8]$ and $d^* = e^{|\alpha-\beta|}$ where $\alpha,\beta \in \mathbb{X}$.
The self mappings A,B,S *and* T defined as

$$\mathrm{A}(\alpha) = \mathrm{B}(\alpha) = \alpha \text{ if } \alpha \in [0,8]$$

$$\mathrm{S}(\alpha) = \begin{cases} \alpha \text{ if } \alpha \in (01) \\ 2 \text{ if } \alpha \in [1,8] \end{cases} \qquad \mathrm{T}(\alpha) = \begin{cases} 1 \text{ if } \alpha \in (0,1) \\ 2 \text{ if } \alpha \in [1,8] \end{cases}$$

where $\alpha = 2$ *common fixed point* for the four mappings $S,T,A,$ *and* B

3.2 Theorem: Let $S,T,$ A *and* B be self mappings of a complete multiplicative metric space, they satisfying the following conditions 3.1.1 and 3.1.2 together with

3.2.1T be a point wise B -absorbing

3.2.2 the pair of mappings (S,A) is reciprocally continuous and compatible then the mappings $S,T,A\ and\ B$ have a unique common fixed point in X.

Proof

Using the condition 3.2.2 we have $\lim_{\eta\to\infty} SA\alpha_{2\eta} = S\zeta$ and $\lim_{\eta\to\infty} AS\alpha_{2\eta} = A\zeta$ and $\lim_{\eta\to\infty} d^*\left(SA\alpha_{2\eta}, AS\alpha_{2\eta}\right) = 1$.

This gives $S\zeta = A\zeta$.

Put $\alpha = \zeta\ and\ \beta = \alpha_{2\eta+1}$ in condition 3.1.2 then

$$d^*\left(S\zeta, T\alpha_{2\eta+1}\right) \le \left(d^*\left(A\zeta, B\alpha_{2\eta+1}\right).d^*\left(A\zeta, S\zeta\right).d^*\left(B\alpha_{2\eta+1}, T\alpha_{2\eta+1}\right).d^*\left(S\zeta, B\alpha_{2\eta+1}\right).d^*\left(A\zeta, T\alpha_{2\eta+1}\right)\right\}^{\frac{\lambda}{6}}$$

letting $\eta\to\infty$ we get,

$$d^*\left(A\zeta,\zeta\right) \le \left(d^*\left(A\zeta,\zeta\right).d^*\left(A\zeta,A\zeta\right).d^*\left(\zeta,\zeta\right), d^*\left(A\zeta,\zeta\right).d^*\left(A\zeta,\zeta\right)\right\}^{\frac{\lambda}{6}}$$

this gives

$d^*\left(A\zeta,\zeta\right) \le d^{*\frac{\lambda}{2}}\left(A\zeta,\zeta\right)$ is a contradiction.

This gives $A\zeta = \zeta$.

$Therefore\ A\zeta = S\zeta = \zeta$.

Since $S(X)B(X)$ then there exists a point u in X such that $S\alpha_{2\eta} = Bu$

letting $\eta\to\infty$ this gives

$\zeta = \lim_{\eta\to\infty} S\alpha_{2\eta} = Bu$, this gives $\zeta = Bu$.

Now put $\alpha = \zeta$, $\beta = u$ in condition 3.1.2 then we get

$$d^*\left(S\zeta, Tu\right) \le \left(d^*\left(A\zeta, Bu\right), d^*\left(A\zeta, S\zeta\right), d^*\left(Bu, Tu\right), d^*\left(S\zeta, Bu\right), d^*\left(A\zeta, Tu\right)\right\}^{\frac{\lambda}{6}}.$$

On using $A\zeta = S\zeta = \zeta$, we get

$$d^*(\zeta, Tu) \le \{d^*(\zeta, Tu)\}^{\frac{\lambda}{3}}$$

implies $\zeta = Tu$.

Therefore $S\zeta = A\zeta = Bu = Tu = \zeta$.

Again on using B-absorbing nature of the pair (B,T) , we have

$d^*(Bu, BTu) \le d^{*R}(Bu, Tu)$, a contradiction in view of $Bu = Tu = \zeta$.

Therefore $Bu = BTu = Tu$ and implies $B\zeta = \zeta$.

Put $\alpha = \zeta,\ \beta = \zeta$ in the condition 3.1.2 and on simplification this leads to

$d^*(\zeta, T\zeta) \le d^{*\frac{\lambda}{3}}(\zeta, T\zeta)$

implies $T\zeta = \zeta$.

Consequently $S\zeta = T\zeta = B\zeta = A\zeta = \zeta$, proving that the point ζ is a common fixed point of $S,T,A\ and\ B$.

The uniqueness of the fixed point can be easily proved.

The following example supports the conditions of Theorem 3.2.

3.2.3 Example: Let (X,d^*) be MMS where $X = [0,2]$ and $d^* = e^{|\alpha-\beta|}$ and four self mappings defined as

$$\mathrm{A}(\alpha) = \begin{cases} 1 \text{ if } \alpha \in [0,1) \\ 6/4 \text{ if } \alpha \in [1,2) \\ 5/4 \text{ if } \alpha = 2 \end{cases} \quad , \quad \mathrm{B}(\alpha) = \begin{cases} 1/4 \text{ if } \alpha \in [0,1) \\ 6/4 \text{ if } \alpha \in [1,2) \\ 1 \text{ if } \alpha = 2 \end{cases}$$

$$\mathrm{S}(\alpha) = \frac{6}{4} \text{ if } \alpha \in [0,2] \quad \text{and} \quad \mathrm{T}(\alpha) = \begin{cases} 5/4 \text{ if } \alpha \in [0,1) \\ 6/4 \text{ if } \alpha \in [1,2] \end{cases}$$

After verifying the conditions of Theorem 3.2 in a routine manner $\alpha = \frac{6}{4}$ is arrived as the unique common fixed point.

3.3 Theorem

Let S, T A *and* B be four self mappings of a complete multiplicative metric space (X, d^*) satisfying the conditions 3.1.1, 3.1.2 along with

3.3.1 The pair (A,S) reciprocally continuous with semi-compatible and T be Point wise B-absorbing or (B, T) is reciprocally continuous and semi compatible with S be point wise A-absorbing then A, B, S *and* T have unique common fixed point.

Proof: Since the pair of maps (A, S) is reciprocally continuous and semi compatible then we have $\lim_{\eta\to\infty} SA\alpha_\eta = S\zeta$. $\lim_{\eta\to\infty} AS\alpha_\eta = A\zeta$ and $\lim_{\eta\to\infty} d^*(AS\alpha_\eta, S\zeta) = 1$.

Hence we get $S\zeta = A\zeta$.

Now we claim $S\zeta = \zeta$ for this Put $\alpha = \zeta$, $\beta = \alpha_{2\eta+1}$ in contraction condition 3.1.2.

letting $\eta \to \alpha$, we obtain

$$d^*(S\zeta, \zeta) \le (d^*(S\zeta, \zeta).d^*(S\zeta, S\zeta).d^*(\zeta, \zeta).d^*(S\zeta, \zeta).d^*(S\zeta, \zeta)\}^{\frac{\lambda}{6}}$$

this gives

$d^*(S\zeta, \zeta) \le d^*(S\zeta, \zeta)^{\frac{\lambda}{2}}$ is a contradiction, implies $S\zeta = \zeta$.

Hence we get $S\zeta = A\zeta = \zeta$.

Since $S(X) \subseteq B(X)$ there exists a point $u \in X$ such that $S\alpha_{2\eta} = Bu\ letting$ $\eta \to \infty$ we get $\lim_{\eta\to\infty} S\alpha_{2\eta} = \zeta = Bu$.

To claim $Tu = \zeta$ substitute $\alpha = \zeta$, $\beta = u$ in contraction condition 3.1.2

$$d^*(S\zeta, Tu) \le (d^*(A\zeta, Bu).d^*(A\zeta, S\zeta).d^*(Bu, Tu).d^*(S\zeta, Bu).d^*(A\zeta, Tu)\}^{\frac{\lambda}{6}}$$

this gives

$d^*(\zeta, Tu) \le (Max(d^*(\zeta, \zeta).d^*(\zeta, \zeta).d^*(\zeta, Tu).d^*(\zeta, \zeta).d^*(\zeta, Tu)\}\}^{\frac{\lambda}{6}}$

this implies

$d^*(\zeta, Tu) \le d^*(\zeta, Tu)^{\frac{\lambda}{3}}$ is a contradiction, Hence we get $Tu = \zeta$.

Therefore $S\zeta = A\zeta = Bu = Tu = \zeta$.

Again on using the point wise B-absorbing nature of $T\ there\ exists\ a\ real\ numberR > 0$ such that $d^*(Bu, BTu) \le d^{*R}(Bu, Tu)$ implies $Bu = BTu$ $,that\ is\ \zeta = B\zeta$.

Put $\alpha = \zeta, \beta = \zeta$ in the condition 3.1.2 $resultsT\zeta = \zeta$.

Thus $A\zeta = B\zeta = T\zeta = S\zeta = \zeta$, giving ζ is a common fixed point of S, T, A *and* B .The uniqueness of the common fixed can be easily proved.

The following example satisfies all the conditions of Theorem 3.3.

3.3.4 Example: Let $(\mathbb{X}, d^*)$ be MMS where $\mathbb{X} = [1, 18]$ and $d^* = e^{|\alpha-\beta|}$. Define four self mappings $\mathscr{A}, \mathscr{B}, \mathscr{S}$ and $\mathscr{T}$.

$$A(\alpha) = \begin{cases} 1 \text{ if } \alpha = 1 \\ 10 \text{ if } 1 < \alpha \le 4 \\ \frac{\alpha+1}{3} \text{ if } \alpha \in (4, 18] \end{cases} \qquad B(\alpha) = \begin{cases} 1 \text{ if } \alpha = 1 \\ 5 \text{ if } \alpha \in (1, 18] \end{cases}$$

$$S(\alpha) = \begin{cases} 1 \text{ if } \alpha = 1 \\ 5 \text{ if } \alpha \in (1, 4] \\ 1 \text{ if } \alpha \in (4, 18] \end{cases} \qquad T(\alpha) = \begin{cases} 1 \text{ if } \alpha = 1 \\ 2 \text{ if } \alpha \in (1, 18] \end{cases}$$

After simple verification of the conditions of Theorem 3.3, $\alpha = 1$ is emerged as the unique common fixed point for the four self mappings.

4 Conclusion

This study is focussed on proving three common fixed point results: In Theorem 3.1 the concept of point wise absorbing mappings together with reciprocally continuous mappings is applied. In Theorem 3.2, the notion of point wise absorbing mapping is used along with reciprocally continuous and compatible mappings is discussed. Finally, in the last Theorem 3.3, the concept of absorbing mappings is applied along with reciprocally continuous and semi compatible mappings. Further, all the results are substantiated with suitable examples.

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