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RESEARCH ARTICLE

Mathematical analysis on anisotropic Bianchi Type-III inflationary string Cosmological models in Lyra geometry

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Abstract

Objectives: To present a new solution to the field equations obtained for Bianchi type-III universe by using the law of variation of H, which yields constant DP. **Methods**: We study a Bianchi type-III cosmological model with a cloud strings with particles connected to them in Lyra geometry. To find the exact solutions of survival field equations we consider here that the shear scalar and scalar expansion are proportional to each other $(\sigma\alpha\theta)$ that leads to the equation $b = c^m$ and secondly we adopt the assumption considering the Deceleration Parameter q as a negative constant quantity giving the inflationary model. The geometrical and physical properties are studied and compared with the recent observational data. **Findings**: The present model starts at t=0 with 0 volume and as time progresses it expands with accelerated rate and the model shows that the present universe is particle dominated.

Keywords: Bianchi type III metric; inflation; Lyra geometry; cloud string; anisotropic

1 Introduction

It is still an interesting area of research to discover its unknown phenomenon that has yet to observe to study the ultimate fate of the universe. But till today cosmologists cannot make a final and comprehensive conclusion about the origin and evolution of universe with strong evidence. So more and more investigations are required to discover and understand the unknown phenomenon of the universe and many mysterious particles which are to be observed to study the ultimate fate of the universe. The cosmologist or researchers developed the string theory to describe the universe, its early stages and the evolution during the time. So, the study on string cosmology is becoming very interesting area for the cosmologist, because of its significant role in the study of formation and evolution of the universe at the early stages and to understand about the future evolution. In the field of the general relativity the investigation of string was generally initiated by prominent authors, Stachel [1] and Letelier [2,3]. In the recent past years many prominent authors have investigated the cosmic strings in the context of Lyra geometry since it can play a great role in describing the universe in the early stages of evolution (Kibble [4,5]) and they can give rise to density perturbations which can lead to the creation of large scale structure (galaxies) of the universe(Zel'dovich [6,7]).

The strings are crucial topological stable defects occurred due to the phase transition at the early days of the universe, when the temperature is lower than a specific temperature, known as critical temperature. The occurrence of strings inside the universe results in anisotropy within the space-time, though the strings aren't seen in the present epoch. Strings cause no damage to the cosmological models, but they can result in very interesting astrophysical effects. Because of the great position of strings in the description of the evolution of the early universe, nowadays, many prominent authors have significantly studied the string cosmology. Soon, after the big-bang, there was a breaking of symmetry during the time of phase transition and the cosmic temperature went down below some critical temperatures due to which the strings arose, as according to grand unified theories(Everett [8], Vilenkin1 [9,10]).

Though the Einstein general relativity is one of the most acceptable theory in modern era to describe the universe, it is unable to explain some of the strong unknown facts about the universe such as accelerated expansion of universe, reason behind the expansion etc. So the several researchers are trying to solve and explain those aspects of the universe by the help of different modified theories of Einstein General theory of relativity such as Weyl's theory, Brans-Dicke theory, f(R) gravity theory, f(R, T) theory, Lyra geometry, scalar tensor theory etc. Among these theories Lyra geometry is one of the most important modified theory. Inspired by the geometrization of gravitation, Weyl [11] developed a theory by geometrizing electromagnetism and gravitation, known as Weyl's theory. However, this theory was criticized and not accepted due to the condition of non-integrability of length of vector under parallel displacement. To remove this non integrability condition to H. Weyl's geometry, Lyra [12] suggested a modification by introducing a gauge function ϕ_μ into the structureless geometry to Riemannian geometry and this modified Riemannian geometry proposed by Lyra is known as Lyra's Geometry. Halford [13] constructed a theory in cosmology in Lyra geometry, and he showed that in general theory of relativity the constant $\phi_\mu$performs as cosmological constant term. Bhamra [14], Beesham [15], Singh and Singh [16,17], Rahaman et al. [18], Reddy and Rao [19,20], Yadav et al. [21], Adhav et al. [22], Reddy [23], Rao et al. [24] are the some of the prominent authors who have already constructed various cosmological models in Lyra geometry. Recently, Singh et al. [25], W. D. R. Jesus, and A. F. Santos [26], Singh and Mollah [27], Mollah et al. [28], Yadav and Bhardwaj [29], Maurya and Zia [30], A. K. Yadav [31] have studied various cosmological models in different contexts considering Lyra's geometry.

Inspired by the above discussions, here we have studied the string cosmological model with particles connected to them in Bianchi type-III universe considering Lyra geometry. The work done in this paper and findings are somewhat distinct from the earlier findings. In the sec.2, Bianchi type-III metric is presented and the field equations in Lyra geometry are derived; In the sec.3, the determinate solutions of the field equations are determined by using some plausible conditions. Physical and geometrical properties of our model with the help of graph are discussed in sec.4; In sec.5 conclusions of the paper are given.

2 The metric and field equations

We consider the Bianchi type-III metric as

$$ds^2 = a^2dx^2 + b^2e^{-2x}dy^2 + c^2dz^2 - dt^2 \tag{1}$$

Here, a, b and c are the functions of 't' alone. For the above metric let

$$x^1 = x, x^2 = y, x^3 = z \text{ and } x^4 = t \tag{2}$$

The field equations with gauge function and $8\pi G = 1, \mathrm{C} = 1$ in Lyra manifold is

$$R_{ij} - \frac{1}{2}Rg_{ij} + \frac{3}{2}\phi_i\phi_j - \frac{3}{4}g_{ij}\phi_k\phi^k = -T_{ij} \tag{3}$$

Where, ϕ_i is the displacement field vector given by

$$\phi_i = (0,0,0,\beta) \tag{4}$$

Here, β is the function of time.

The energy-momentum tensor for a cosmic string is taken as

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j \tag{5}$$

Here, $\lambda = \rho - \rho_p$ is the string tension density, ρ is the energy density and ρ_p is the particle density of the string. Also, u^i is the four velocity vector and x^i is the unit space-like vector which represents the direction of strings, and they are given by

$$x^i = \left(0,0,c^{-1},0\right) \text{ and } u^i = (0,0,0,1) \tag{6}$$

$$\text{Such that } u_i u^i = -1 = -x_i x^i \text{ and } u_i x^i = 0 \tag{7}$$

If R is the average scale factor then volume is

$$V = abc = R^3 \tag{8}$$

The expansion scalar is given by

$$\theta = u^i_{,i} = \frac{\dot{a}}{a} + \frac{\dot{b}}{b} + \frac{\dot{c}}{c} \tag{9}$$

Hubble parameter is given by

$$H = \frac{1}{3}\left(\frac{\dot{a}}{a} + \frac{\dot{b}}{b} + \frac{\dot{c}}{c}\right) \tag{10}$$

The shear scalar is given by

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{3}\left[\left(\frac{\dot{a}}{a}\right)^2 + \left(\frac{\dot{b}}{b}\right)^2 + \left(\frac{\dot{c}}{c}\right)^2 - \frac{\dot{a}\dot{b}}{ab} - \frac{\dot{b}\dot{c}}{bc} - \frac{\dot{c}\dot{a}}{ca}\right] \tag{11}$$

And the mean anisotropy parameter is

$$\Delta = \frac{1}{3}\sum_{r=1}^{3}\left(\frac{H_r - H}{H}\right)^2 \tag{12}$$

Where, $H_r(r = x, y, z)$ denotes the directional Hubble factors, and they are given by $H_x = \frac{\dot{a}}{a}, H_y = \frac{\dot{b}}{b}$ and $H_z = \frac{\dot{c}}{c}$ for the metric (1).

The field Equation (3) with the Equations (4), (5), (6) and (7) for the Equation (1) takes the form

$$\frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} + \frac{\dot{b}\dot{c}}{bc} + \frac{3}{4}\beta^2 = 0 \tag{13}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{c}}{c} + \frac{\dot{c}\dot{a}}{ca} + \frac{3}{4}\beta^2 = 0 \tag{14}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\dot{a}\dot{b}}{ab} + + \frac{3}{4}\beta^2 - \frac{1}{a^2} = \lambda \tag{15}$$

$$\frac{\dot{a}\dot{b}}{ab} + \frac{\dot{b}\dot{c}}{bc} + \frac{\dot{c}\dot{a}}{ca} - \frac{3}{4}\beta^2 - \frac{1}{a^2} = \rho \tag{16}$$

$$\frac{\dot{a}}{a} - \frac{\dot{b}}{b} = 0 \tag{17}$$

Here the overhead dots represent the order of differentiation w. r. t. time 't'.

3 Solutions of the field equations

Solving Equation (17), we have

$$a = rb \tag{18}$$

Here r is the integration constant. With generality, we can take $r = 1$.

And using it, (18) can be written as,

$$a = b \tag{19}$$

Thus using relation (19) the field Equations (13), (14), (15) and (16) reduces to

$$\frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} + \frac{\dot{b}\dot{c}}{bc} + \frac{3}{4}\beta^2 = 0 \tag{20}$$

$$2\frac{\ddot{b}}{b} + \frac{\dot{b}^2}{b^2} + \frac{3}{4}\beta^2 - \frac{1}{b^2} = \lambda \tag{21}$$

$$\frac{\dot{b}^2}{b^2} + 2\frac{\dot{b}\dot{c}}{bc} - \frac{3}{4}\beta^2 - \frac{1}{b^2} = \rho \tag{22}$$

We have 3 highly nonlinear independent differential Equations (20), (21) and (22) with variables b, c, λ, β and ρ which are unknown. So to obtain the exact solutions of above equations we must have two extra conditions. So here we used the following two physically plausible conditions:

Here, we take the assumption that the shear scalar and expansion scalar are proportional to each other $(\sigma\alpha\theta)$ which leads to the equation

$$b = c^m \tag{23}$$

Here $m \neq 0$ is a constant.

This is based on observations of velocity and red-shift relation for an extragalactic source which predicted that the Hubble expansion is 30 percent isotropic, which is supported by the works of Thorne [32], Kantowski and Sachs [33], Kristian and Sachs [34]. In particular, it can be said that $\frac{\sigma}{H} \geq 0.30$, where σ and H are respectively shear scalar and Hubble constant. Also, Collins et al. [35] has shown that if the normal to the spatially homogeneous line element is congruent to the homogeneous hyper-surface then $\frac{\sigma}{\theta} =$ constant , θ being the expansion factor.

Secondly we adopt the assumption proposed by Berman [36] about the variation of hubble's parameter H, which gives constant DP in the model as

$$q = -\frac{R\ddot{R}}{\dot{R}^2} = (\text{constant}) \tag{24}$$

When h is negative then the model universe expand with acceleration and when q is positive then the model universe contract with deceleration. Although the present observations like CMBR and SNe Ia suggested the negative value of q but it can be remarkably state that they are not able to deny about the decelerating expansion (positive q) of universe. This is the most suitable condition to explore the physically meaningful solutions of the above field equations.

The scale factor R admits the solution-

$$R = (ht + k)^{\frac{1}{1+q}}, \quad q \neq -1 \tag{25}$$

Here $h \neq 0$ and k are integration constants.

Using the Equations (8), (19), (23) and (25), we get,

$$a = b = (ht + k)^{\frac{3m}{(1+q)(2m+1)}} \tag{26}$$

$$c = (ht+k)^{\frac{3}{(1+q)(2m+1)}} \tag{27}$$

Without loss of generality we take h=1 and k=0 then (26), (27) becomes

$$a = b = t^{\frac{3m}{(1+q)(2m+1)}}, \quad c = t^{\frac{3}{(1+q)(2m+1)}} \tag{28}$$

Using (28) the metric (1) can be reduced to

$$ds^2 = t^{\frac{6m}{(1+q)(2m+1)}}\left(dx^2 + e^{-2x}dy^2\right) + t^{\frac{6}{(1+q)(2m+1)}} dz^2 - dt^2 \tag{29}$$

This gives the geometry of the metric (1).

4 Physical and geometrical parameters

We obtained some of the important physical and geometrical parameters that are useful for the discussion on the evolution of the universe.

Using (28) in (22) we obtained ρ as

$$\rho = \frac{3(m+1)(2-q)}{(1+q)^2(3m+1)t^2} - t^{-\frac{6m}{(1+q)(2m+1)}} \tag{30}$$

From (19) and (20) using (28) we obtained

$$\lambda = \frac{3(m-1)(2-q)}{(1+q)^2(3m+1)t^2} - t^{-\frac{6m}{(1+q)(2m+1)}} \tag{31}$$

From (30), (31) we obtained the ρ_p as

$$\rho_p = \frac{6(2-q)}{(1+q)^2(3m+1)t^2} \tag{32}$$

The gauge function β is obtained as

$$\beta^2 = \frac{4\left[(m+1)(2m+1)(1+q) - 3\left(m^2+m+1\right)\right]}{(1+q)^2(3m+1)^2t^2} \tag{33}$$

The spatial volume, scalar expansion, Hubble parameter, shear scalar and mean anisotropy parameter of the model are

$$V = t^{\frac{3}{1+h}} \tag{34}$$

$$\theta = \frac{3}{(1+h)t} \tag{35}$$

$$H = \frac{1}{(1+h)t} \tag{36}$$

$$\sigma = \frac{\sqrt{3}(m-1)}{(1+h)(2m+1)t} \tag{37}$$

$$\Delta = \frac{2(m-1)^2}{(2m+1)^2} = \text{Const.} \tag{38}$$

5 Interpretations of the solutions

The Equation (29) represents the Bianchi type-III anisotropic cosmological model with strings in Lyra geometry. The physical and geometrical behavior of model for $-1 < h < 0$ are discussed as

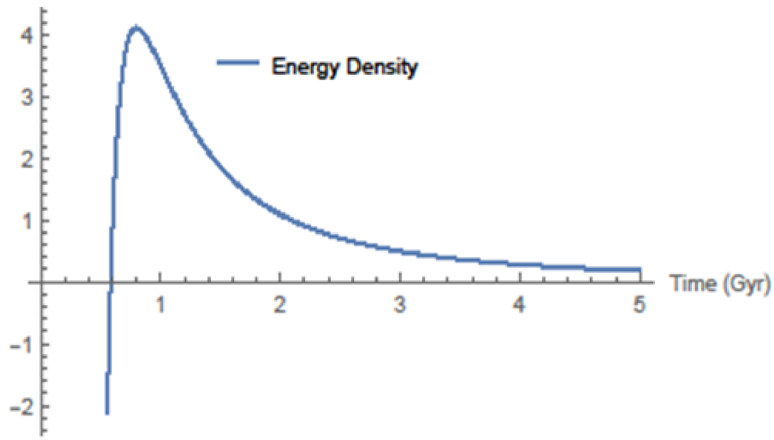

Fig 1. Energy density Vs. Time

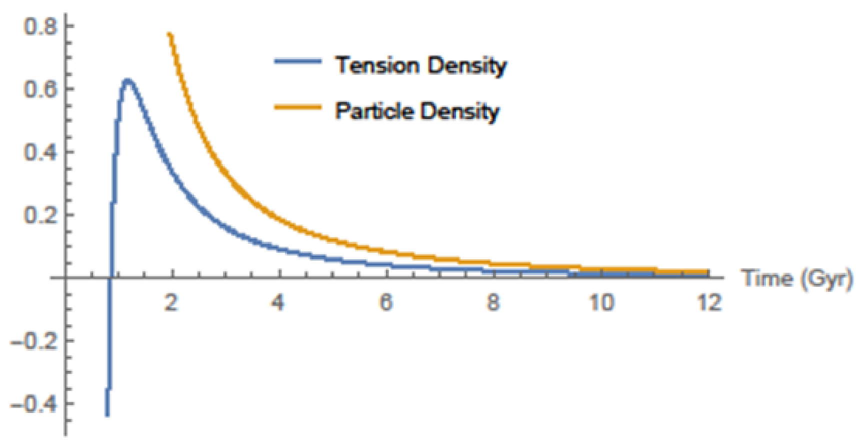

Fig 2. Tension density, Particle density Vs. Time

- From the expressions of energy density ρ and tension density λ given by Equations (30) and (31), we have observed that both of them are negative at the initial epoch of time but as the time progresses they changes sign from negative to positive and then decreases gradually and finally become zero when $t \to \infty$. Figure 1 presents the variations of energy density with time t, which clearly indicate that at infinite time, $\rho \to 0$. Again, the nature of the variations of tension density λversus time t is shown by Figure 2. From this we can conclude that initially when $t \to 0$, $\lambda$is negative but with the passage of cosmic time it changes sign from negative to positive and finally at infinite time it becomes zero, which is supported by Letelier[2,3]
- For the model universe, the expression of particle density ρ_p is found as the Equation (32) and its variations versus cosmic time is shown in Figure 2. Which shows that $\rho_p$is always positive which decreases from $\rho_p = \infty$ as t = 0 to $\rho_p = 0$ whenever $t \to \infty$. Also, Figure 2 depicts that the tension density diminishes more quickly than the particle density, therefore with the passage of time string will disappear leaving the particles only. Hence, our model is realistic one. And it is also seen that $\frac{\rho_p}{|\lambda|} > 1$, that shows that tension density of string diminishes faster than particle density. This tells us that the late

universe is particle dominated.

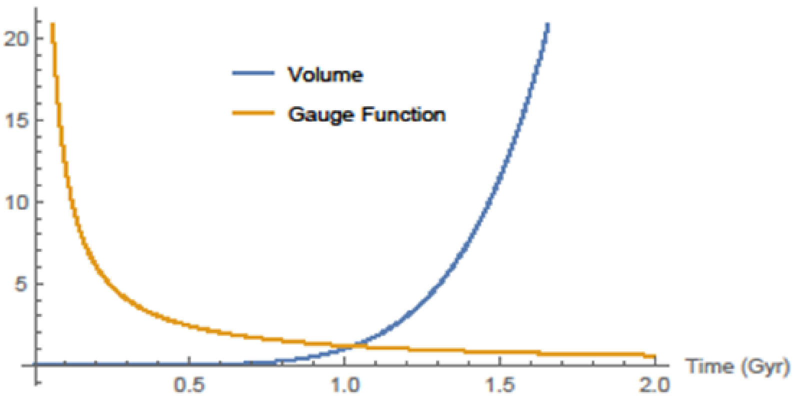

Fig 3. Volume, Gauge Function Vs. Time

- In this model universe, at the initial epoch of time, the gauge function β^2 given by Equation (33) is found to be infinite and it decreases with the increase of time. Finally, the gauge function $\beta^2 \to 0$ *when* $t \to \infty$.
- The volume for this model increases as time increases. The expression of volume V as obtained in Equation (34) shows that the model universe begin with initial singularity at t = 0 from V = 0 i.e. our model universe starts from zero volume at t = 0 and as time increases it expands and also when $t \to \infty$, $V \to \infty$. So, for $1+q > 0$ the model shows that the universe is expanding with accelerated rate.

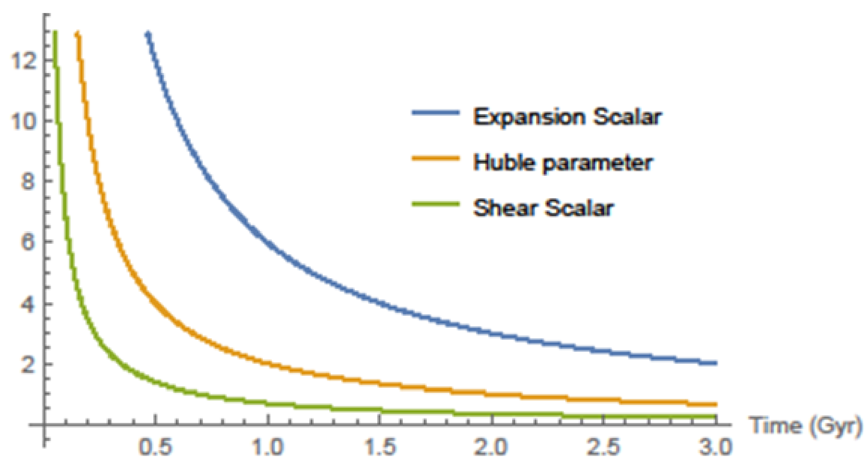

Fig 4. Expansion scalar, Hubble parameter, Shear scalar Vs. Time

- From the expansion scalar and Hubble parameter for the model (29), at $t = 0$, the θ and H both are infinite and as the time progresses gradually they decrease and finally θ and H become 0 when t is infinite. Hence, the model shows that the universe expands with time but the rate of expansion slower as the increases of time and the expansion stops at $t \to \infty$. Again, it is seen that $\frac{dH}{dt} = -\frac{1}{(1+h)t^2} = 0$, when t approaches infinity and this implies the greatest value of Hubble's Parameter and accelerated expansion of the universe. These behaviors of the model are presented in Figure 4
- In Equation (37) and Figure 4. It is seen that the value of the shear scalar $\sigma \to \infty$ at initial epoch and it decreases as the time increases and become zero at late universe showing that the universe obtained here is shear free in the late time.

- From Equation (38) the mean anisotropy parameter $\Delta = \text{constant}(\neq 0)$ for $m \neq 1$ and $\Delta = 0$ for $m = 1$. Also as $t \to \infty$ the value of $\frac{\sigma^2}{\theta^2} = \frac{(m-1)^2}{3(2m+1)^2} = \text{constant}(\neq 0)$ for $m \neq 1$ and $\frac{\sigma^2}{\theta^2} = 0$ for $m = 1$ From both statements we can conclude that this model is anisotropic for large value of t when $m \neq 1$ but it is isotropic for m=1.

6 Conclusions

In this article, we have attempted to present a new solution to the field equations obtained for Bianchi type-III universe in Lyra geometry by using the law of variation of Hubble's parameter H which yields constant DP. This variational law for H in Equation (24) explicitly determine the values of the average scale factors(R). So here we have constructed a Bianchi type-III cosmological model attached to strings in Lyra geometry, which is an anisotropic and inflationary model. The physical and geometrical parameters which are very important in the description of cosmological models have been obtained and discussed. The model starts at t = 0 with volume 0 and it expand with acceleration in which the strings disappear leaving the particles only in the late universe giving particle dominated universe which agrees with the present observational data. The model is expanding, anisotropic for $m \neq 1$ at late universe, accelerating, non-shearing and admits initial singularity at t=0, that also agree with the present day observational data. Through this study, we hope to present a better knowledge of the cosmological evolution of the present universe with the help of Bianchi type-III universe in Lyra geometry.

Financial disclosure/conflict of interest

The authors declare that there was no financial aid received and no conflict of interest associated with this research work.

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