

RESEARCH ARTICLE



Bianchi type-I cosmological model with Tsallis holographic dark energy in $f(R, T)$ theory of gravity

 OPEN ACCESS

Received: 04.04.2021

Accepted: 04.05.2021

Published: 20.05.2021

Citation: Sarma N (2021) Bianchi type-I cosmological model with Tsallis holographic dark energy in $f(R, T)$ theory of gravity. Indian Journal of Science and Technology 14(18): 1468-1476. <https://doi.org/10.17485/IJST/v14i18.581>

* Corresponding author.

Tel: 9401271702
sarmanitin01@gmail.com

Funding: None

Competing Interests: None

Copyright: © 2021 Sarma. This is an open access article distributed under the terms of the [Creative Commons Attribution License](#), which permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are credited.

Published By Indian Society for Education and Environment ([iSee](#))

ISSN

Print: 0974-6846

Electronic: 0974-5645

Nitin Sarma^{1*}

¹ Assistant Professor, Department of Mathematics, ADP College, Nagaon, 782002, Assam, India. Tel.: 9401271702

Abstract

Objectives: In the $f(R, T)$ theory of gravity, to examine the Bianchi type-I model with dark matter and Tsallis holographic dark energy (THDE). **Method:** To acquire exact solutions of the field equations, we used THDE density and two separate volumetric expansions. The model's physical properties are compared to current observational evidence. **Findings/ Novelty:** The universe is in accelerating stage for both volumetric expansion models and it reaches isotropy at a delayed time, according to the study of the deceleration parameter. In addition, our model acts as a lambda model.

Keywords: Bianchi type-I; THDE; $f(R, T)$ gravity; Isotropy; Lambda model

1 Introduction

The last few decades of cosmological study have resulted in a paradigm shift in our understanding of the universe. Some astrophysical findings indicate that the universe is in the course of a late cosmic acceleration stage. A new kind of energy with negative pressure, known as dark energy (DE) ⁽¹⁾, is believed to be caused for this acceleration.

Many cosmological models have been suggested by cosmologists to examine the proper existence of DE. Among these models, the holographic dark energy (HDE) model stands out because it is a worthy theory that incorporates an energy density dictated by the universe's geometric structures. Tsallis holographic dark energy (THDE) has recently been considered as a new kind of HDE model that uses Tsallis generalized entropy $S_\beta = \alpha A^\beta$ ⁽²⁾ to demonstrate the cosmological phenomena, where β denotes the parameter (non-additive) and α is a constant. Cohen et al. ⁽³⁾ demonstrated an association of S (entropy), L (cut-off IR) and Λ (cut-off UV) as ' $L^3 \Lambda^3 \leq S^{\frac{3}{4}}$ ', which when combined with $S_\beta = \alpha A^\beta$ leads to $\Lambda^4 \leq (\alpha(4\pi)^\beta) L^{2\beta-4}$. The energy density of THDE can be computed using this inequality as $\rho_T = DL^{2\beta-4}$ where D is an undefined variable ⁽⁴⁾. For $\beta = 1$, THDE simply reduces to the HDE model. Using the Hubble horizon as the system's IR cut-off, we can calculate the energy density of THDE as $\rho_T = DH^{4-2\beta}$. Many authors have recently discussed THDE in various contexts, specifically Tavayef et al. ⁽⁵⁾ studied THDE in the flat Friedmann-Robertson-Walker universe. A sign changeable interaction between THDE and DM has been investigated by Zadeh et al. ⁽⁶⁾. Sharif and Saba ⁽⁷⁾ expounded THDE model in $f(G, T)$ -gravity.

Effects of THDE in Brans-Dicke and under brane cosmology have been studied by Ghaffari et al. (8,9).

Though there are two methods to construct a cosmological model, the gravity-modified model has prompted a vast interest. The attractive feature of this method is that an accelerated universe can be induced without extracting any energy from the universe's dark components. The $f(R)$ model, $f(G)$ model, $f(R,G)$ model are some examples of gravity-modified models. The $f(R,T)$ -model described by Harko et al. (10) in 2011, is a significant and potential model. It bridges the gap between the $f(R)$ and $f(T)$ theories. Moreover, according to some astrophysical observations, the universe may have been anisotropic in the early time and later the universe becomes homogeneous and isotropic. In this case, the Bianchi type metric is appropriate for studying the evolution of the universe since it completely explains the effect of anisotropy. Even for ordinary matter, however, the Bianchi type model isotropizes at a late time. This isotropization is caused by implicit assumptions that the DE is generally isotropic. In literature, we have found that many authors have investigated different aspects of these Bianchi type models in $f(R,T)$ theories viz. Samanta and Bishi (11) investigated the Bianchi-III model for wet dark fluid in $f(R,T)$ -gravity, Tiwari et al. (12) studied the varying deceleration parameter in $f(R,T)$ -theory, Brahma and Dewri (13) investigated Bianchi type-V model in $f(R,T)$ gravity. Recently Dubey et al. (14) studied the THDE in the Bianchi-I metric by using the hybrid expansion law. Motivated by the above discussion, we want to explore the Bianchi-I model with THDE in $f(R,T)$ theory.

The following is a breakdown of the paper's structure: Gravitational field equations for $f(R,T)$ gravity are given in Section 2. Parts 3 and 4 address metric and field equations, as well as their solutions. Models for power-law and exponential law expansions are discussed in Sections 5 and 6. Graphical discussion is given in Section 7. In Section 8, we sum up our findings.

2 Gravitational field equations for f(R, T) theory

In $f(R,T)$ theory (10), the field equations are obtained from an action which is expressed as

$$S = \int \left[\frac{1}{16\pi} f(R,T) + L_m \right] \sqrt{-g} d^4x \tag{1}$$

where R is the Ricci scalar, T is the trace of the energy momentum tensor and L_m is matter Lagrangian density.

The matter's stress-energy tensor is

$$T_{ij} = - \frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}L_m)}{\delta g^{ij}} \tag{2}$$

Now taking variation of (1) with respect to g_{ij} (metric tensor), we acquire the field equations as

$$\frac{\partial f}{\partial R} R_{ij} - \frac{1}{2} f g_{ij} + (g_{ij} \square - \nabla_i \nabla_j) \frac{\partial f}{\partial R} = 8\pi T_{ij} - \frac{\partial f}{\partial T} (T_{ij} + \theta_{ij}) \tag{3}$$

where

$$\theta_{ij} = -2T_{ij} + g_{ij}L_m - 2g^{\delta\gamma} \frac{\partial^2 L_m}{\partial g^{ij} \partial g^{\delta\gamma}} \tag{4}$$

The issue of perfect fluids defined by an energy density (ρ), pressure (p_T), and four velocity (u_i) is more difficult, because there is no single description of matter Lagrangian density (L_m). Here, we consider T_{ij} is of the type

$$T_{ij} = (\rho + p_T) u_i u_j - p_T g_{ij} \tag{5}$$

where

$$L_m = -p_T, u^\delta u_\delta = 1, \text{ and } u^\delta \nabla_\gamma u_\delta = 0 \tag{6}$$

Using (5) and (6), in (4), we get

$$\theta_{ij} = -2T_{ij} - p_T g_{ij} \tag{7}$$

For the distinct option $f(R,T) = f_1(R) + f_2(T) = \lambda_1 R + \lambda_2 T$, the Equation (3) becomes

$$R_{ij} - \frac{1}{2} g_{ij} R - \left(p_T + \frac{T}{2} \right) \frac{\lambda_2}{\lambda_1} g_{ij} = \frac{(8\pi + \lambda_2)}{\lambda_1} T_{ij} \tag{8}$$

where λ_1 and λ_2 are two parameters.

3 Metric and field equations

The Bianchi type-I metric is expressed as

$$ds^2 = dt^2 - a_1^2(t) dx^2 - a_2^2(t) (dy^2 + dz^2) \tag{9}$$

For the matter energy density (ρ_m), the tensor for matter energy momentum (T'_{ij}) is

$$T'_{ij} = \text{diag} [\rho_m, 0, 0, 0] \tag{10}$$

In addition, tensor for THDE energy momentum is supposed as

$$\begin{aligned} \bar{T}_{ij} &= \text{diag} [\rho_T, -p_T^x, -p_T^y, -p_T^z] \\ &= \text{diag} [1, -\omega^x, -\omega^y, -\omega^z] \rho_T \\ &= \text{diag} [1, -\omega_T, -\omega_T, -\omega_T] \rho_T \end{aligned} \tag{11}$$

where ρ_T is the energy density of THDE, pressures and EoS parameters of THDE along x axis p_T^x and $\omega_T = \omega^x$; along y axis p_T^y and $\omega_T = \omega^y$; along z axis p_T^z and $\omega_T = \omega^z$ respectively and

$$T_{ij} = T'_{ij} + \bar{T}_{ij} \tag{12}$$

Using (9), (10), (11), (12) in (8), we get the following set of equations

$$2 \frac{\dot{a}_1}{a_1} \frac{\dot{a}_2}{a_2} + \left(\frac{\dot{a}_2}{a_2}\right)^2 = - \left(\frac{16\pi + 3\lambda_2}{2\lambda_1}\right) (\rho_m + \rho_T) + \frac{\lambda_2}{\lambda_1} p_T \tag{13}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1}{a_1} \frac{\dot{a}_2}{a_2} = \left(\frac{16\pi + 3\lambda_2}{2\lambda_1}\right) p_T - \left(\frac{\lambda_2}{2\lambda_1}\right) (\rho_m + \rho_T) \tag{14}$$

$$2 \frac{\ddot{a}_2}{a_2} + \left(\frac{\dot{a}_2}{a_2}\right)^2 = \left(\frac{16\pi + 3\lambda_2}{2\lambda_1}\right) p_T - \left(\frac{\lambda_2}{2\lambda_1}\right) (\rho_m + \rho_T) \tag{15}$$

Also, the conservation law of energy $T_{;j}^{ij} = 0$ gives

$$\dot{\rho}_m + \dot{\rho}_T + 3H(\rho_m + \rho_T + p_T) = 0 \tag{16}$$

4 Solutions of the field equations

For the metric (1), scale factor, spatial volume, the average Hubble's parameter are expressed as

$$a^3 = V = a_1 a_2^2 \tag{17}$$

$$H = \frac{1}{3} \frac{\dot{V}}{V} = \frac{1}{3} \left(\frac{\dot{a}_1}{a_1} + 2\frac{\dot{a}_2}{a_2}\right) \tag{18}$$

From (14) and (15), we get

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1}{a_1} \frac{\dot{a}_2}{a_2} = 2\frac{\ddot{a}_2}{a_2} + \left(\frac{\dot{a}_2}{a_2}\right)^2 \tag{19}$$

and it gives

$$a_1(t) = l_1^2 V^{\frac{1}{3}} \exp\left(2d_1 \int \frac{dt}{V}\right) \tag{20}$$

$$a_2(t) = l_1^{-1} V^{\frac{1}{3}} \exp\left(-d_1 \int \frac{dt}{V}\right) \tag{21}$$

where l_1, d_1 are constant.

To find parameters $a_1, a_2, \rho_T, \rho_m, p_T$ we consider two additional conditions which are given below.

(i) THDE density as

$$\rho_T = DH^{4-2\beta} \tag{22}$$

(ii) Two volumetric expansion laws as

$$V = ct^{3n} \quad (c, n = \text{constant}) \tag{23}$$

$$V = c e^{3l_2t} \quad (c, l_2 = \text{constant}) \tag{24}$$

5 Power law expansion model

Using (23) in (20) and (21), we get $a_1(t)$ and $a_2(t)$ scale factors as

$$a_1(t) = l_1^2 c \frac{1}{3} t^n \exp \left[\frac{2d_1}{c} \frac{t^{1-3n}}{(1-3n)} \right] \tag{25}$$

$$a_2(t) = l_1^{-1} c \frac{1}{3} t^n \exp \left[-\frac{d_1}{c(1-3n)} t^{1-3n} \right] \tag{26}$$

Using (25), (26) in (18), we get

$$H = \frac{n}{t} \tag{27}$$

We have considered that DE and dark matter are none mixing.

Eq. (16) provides matter’s conservation equation is

$$\dot{\rho}_m + 3H\rho_m = 0 \tag{28}$$

From (27) and (28), we get

$$\rho_m = \frac{c_1}{t^{3n}} \quad (c_1 = \text{constant}) \tag{29}$$

The THDE’s conservation equation is

$$\dot{\rho}_T + 3H(\rho_T + p_T) = 0 \tag{30}$$

The barotropic EoS is

$$\omega_T = p_T(\rho_T)^{-1} \tag{31}$$

Eqs. (22) and (27) provides, THDE density as

$$\rho_T = D \left(\frac{n}{t} \right)^{4-2\beta} \tag{32}$$

Using (31) and (32) in (30), we get ω_T as

$$\omega_T = -1 + \frac{2}{3n} (2 - \beta) \tag{33}$$

We can deduce from (32) and (33), that for $\beta = 2$, ρ_T is constant and $\omega_T = -1$.

The q (deceleration parameter) is

$$q = -1 - \frac{\dot{H}}{H^2} = \frac{1}{n} - 1 \tag{34}$$

We can deduce from (34) that the universe is in decelerating stage ($q > 0$) for $0 < n < 1$ and is in accelerating stage for $n < 0 (n > 1)$.

The Δ (anisotropy parameter) is calculated as

$$\Delta = \frac{1}{3} \sum_{i=1}^3 \left(\frac{H_i - H}{H} \right)^2 = \frac{2d_1^2}{c^2 n^2} t^{2-6n} \tag{35}$$

We can deduce from (35) that $\Delta \rightarrow 0$ when $t \rightarrow \infty (n > \frac{1}{3})$.

6 Exponential law expansion model

Using (24) in (20) and (21), we get $a_1(t)$ and $a_2(t)$ as

$$a_1 = d_1^2 c \frac{1}{3} \exp \left[l_2 t - \frac{2d_1}{3cl_2} e^{-3l_2 t} \right] \tag{36}$$

$$a_2 = d_1^{-1} c \frac{1}{3} \exp \left[l_2 t + \frac{d_1}{3cl_2} e^{-3l_2 t} \right] \tag{37}$$

Using (36) and (37) in (19) we get

$$H = l_2 \tag{38}$$

As we have considered DE and dark matter are none mixing, therefore the matter's conservation equation becomes

$$\dot{\rho}_m + 3l_2 \rho_m = 0 \tag{39}$$

It provides ρ_m as

$$\rho_m = c_2 e^{-3l_2 t} \quad (c_2 = \text{constant}) \tag{40}$$

The conservation equation for THDE is

$$\dot{\rho}_T + 3l_2 (1 + \omega_T) \rho_T = 0 \tag{41}$$

Equations (22) and (38) gives the THDE density as

$$\rho_T = D l_2^{4-2\beta} \tag{42}$$

and it denotes that ρ_T is a constant.

From (41) and (42), we get

$$\omega_T = -1 \tag{43}$$

The q (deceleration parameter) is

$$q = -1 - \frac{\dot{H}}{H^2} = -1 \tag{44}$$

The Δ (anisotropy parameter) is found as

$$\Delta = \frac{1}{3} \sum_{i=1}^3 \left(\frac{H_i - H}{H} \right)^2 = \frac{2d_1^2}{c^2 l_2^2} e^{-6l_2 t} \tag{45}$$

From (45) we can deduce that $\Delta \rightarrow 0$ when $t \rightarrow \infty$.

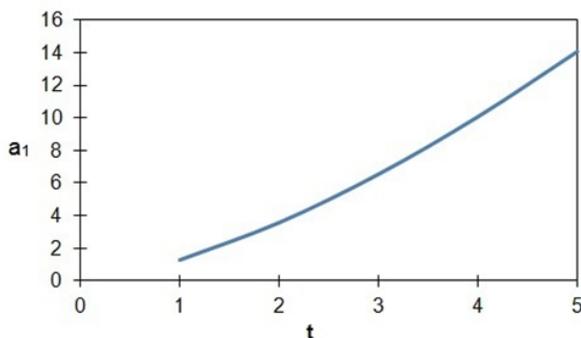


Fig 1.

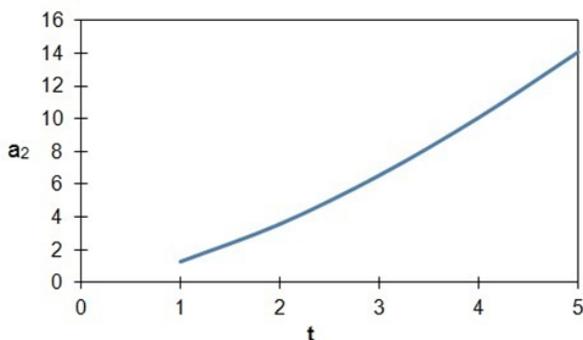


Fig 2.

7 Graphical discussions

Equations (25) and (26) give a_1 and a_2 (scale factors) for the model of power-law expansion. The graphical representation of a_1 and a_2 with time t are given by Figures 1 and 2. These show that a_1 and a_2 are increasing functions of time (t).

For the model of a power-law, the matter's energy density is given by equation (29) and its deviation with time is shown in Figure 3. From this figure, we can say that $\rho_m \rightarrow 0$ when $t \rightarrow \infty$.

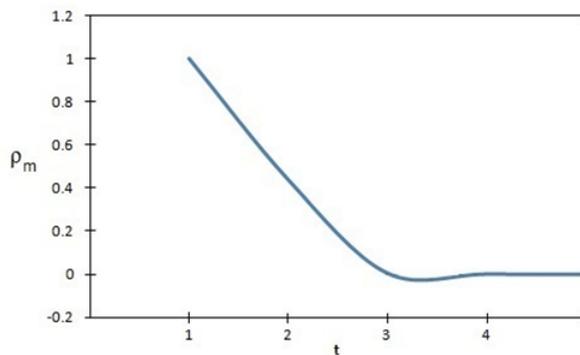


Fig 3.

In equation (27) and Figure 4, we notice that H (Hubble Parameter) is infinite at the null time and steadily decreases to zero when time passes. It means that the universe expansion rate is lower.

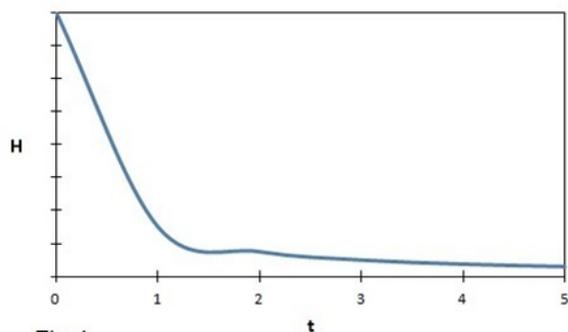


Fig 4.

The scale factors (a_1, a_2) given by equations (36) and (37) for the model of exponential expansion, are found to be increasing function of time (t) as shown in Figures 5 and 6.

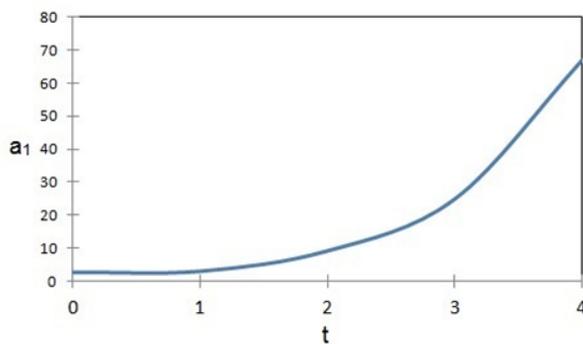


Fig 5.

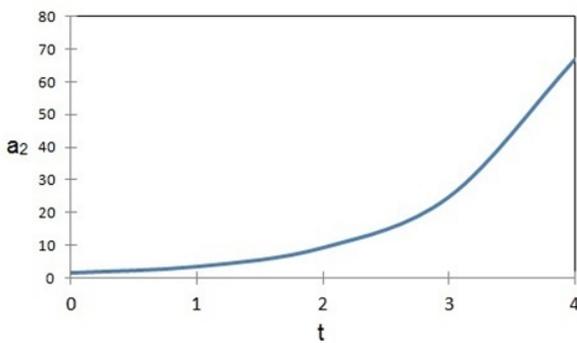


Fig 6.

Equation (40) gives the matter's energy density for the model of exponential expansion and its variation with time (t) is represented by Figure 7. The sketch of ρ_m shows that it reduces with time and reaches zero as $t \rightarrow \infty$. This is also supported by Samanta and Bishi⁽¹¹⁾.

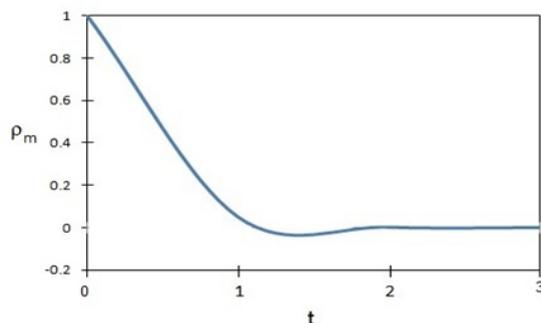


Fig 7.

8 Conclusions

In the $f(R, T)$ theory, we expounded Bianchi type-I model. We use THDE density and two volumetric expansion rules, the power law and exponential law of expansion, to give specific solutions to the field equations. As per the power law expansion, the universe is decelerating for $0 < n < 1$ and accelerating for $n > 1$ and for exponential expansion, $q = -1$ is consistent with similar supernovae Ia findings that the universe is accelerating late in times. The Δ (anisotropy parameter) is a reducing function of t (time) in both the models, and it seems to be 0 at late times. As a consequence, even in the anisotropic character of Bianchi type metric the universe achieves isotropy at late stages. For both models, the THDE's density is constant for $\beta = 2$ and matter's density tends to 0 at late times. Even so, we can extrapolate that our two models look similar to a cosmological constant as $\omega_T = -1$. The deceleration parameter is negative for both the models, so the universe is in expanding phase at the late times. Recent observations supports the results presented here, and it is possible to obtain an accelerating universe by considering THDE in $f(R, T)$ gravity, according to the study. Furthermore, we can claim that in the accelerating model, using THDE as DE in $f(R, T)$ gravity is at least phenomenological. With the help of Bianchi-I metric in $f(R, T)$ modified gravity, we hope to present a better understanding of the universe's growth.

References

- 1) Peebles PJE, Ratra B. The cosmological constant and dark energy. *Reviews of Modern Physics*. 2003;75(2):559–606. Available from: <https://dx.doi.org/10.1103/revmodphys.75.559>.
- 2) Tsallis C, Cirto L. Black hole thermodynamical entropy. *The European Physical Journal C*. 2013;73. Available from: <https://dx.doi.org/10.1140/epjc/s10052-013-2487-6>.
- 3) Cohen AG, Kaplan DB, Nelson AE. Effective Field Theory, Black Holes, and the Cosmological Constant. *Physical Review Letters*. 1999;82(25):4971–4974. Available from: <https://dx.doi.org/10.1103/physrevlett.82.4971>.
- 4) Guberina B, Horvat R, Nikolić H. Non-saturated holographic dark energy. *Journal of Cosmology and Astroparticle Physics*. 2007. Available from: <https://dx.doi.org/10.1088/1475-7516/2007/01/012>.
- 5) Tavayef M, Sheykhi A, Bamba K, Moradpour H. Tsallis holographic dark energy. *Physics Letters B*. 2018;781:195–200. Available from: <https://dx.doi.org/10.1016/j.physletb.2018.04.001>.
- 6) Zadeh MA, Sheykhi A, Bamba K, Moradpour H. Effects of anisotropy on the sign-changeable interacting Tsallis holographic dark energy. *Modern Physics Letters A*. 2020;35(09). Available from: <https://dx.doi.org/10.1142/s0217732320500534>.
- 7) Sharif M, Saba S. Tsallis Holographic Dark Energy in $f(G, T)$ Gravity. *Symmetry*. 2019;11(1):92. Available from: <https://dx.doi.org/10.3390/sym11010092>.
- 8) Ghaffari S, Moradpour H, Lobo IP, Graça JPM, Bezerra VB. Tsallis holographic dark energy in the Brans–Dicke cosmology. *The European Physical Journal C*. 2018;78(9):706. Available from: <https://dx.doi.org/10.1140/epjc/s10052-018-6198-x>.
- 9) Ghaffari S, Moradpour H, Bezerra VB, Graça JPM, Lobo IP. Tsallis holographic dark energy in the brane cosmology. *Physics of the Dark Universe*. 2019;23:100246. Available from: <https://dx.doi.org/10.1016/j.dark.2018.11.007>.
- 10) Harko T, Lobo F, Nojiri S, Odintsov SD. $f(R, T)$ gravity. *Physical Review D*. 2011;84:024020. Available from: <https://doi.org/10.1103/PhysRevD.84.024020>.
- 11) Samanta GC, Bishi BK. Geometry of the Universe Described by Wet Dark Fluid in $f(R, T)$ Theory of Gravity. *Iranian Journal of Science and Technology, Transactions A: Science*. 2017;41(1):223–230. Available from: <https://dx.doi.org/10.1007/s40995-017-0215-z>.
- 12) Tiwari RK, Beesham A, Shukla BK. Time varying deceleration parameter in $f(R, T)$ gravity: a general case. *Afrika Matematika*. 2021;p. 1–2. Available from: <https://dx.doi.org/10.1007/s13370-021-00874-w>.

- 13) Brahma BP, Dewri M. Bianchi type-V modified $f(R, T)$ gravity model in Lyra geometry with varying deceleration parameter. *Journal of Mathematical and Computational Science*. 2021;11(1):1018–1028. Available from: <https://doi.org/10.28919/jmcs/5281>.
- 14) Dubey VC, Srivastava S, Sharma UK, Pradhan A. Tsallis holographic dark energy in Bianchi-I Universe using hybrid expansion law with k-essence. *Pramana*. 2019;93(5):78. Available from: <https://dx.doi.org/10.1007/s12043-019-1843-y>.