

## RESEARCH ARTICLE



# Mixed Method for Model Order Reduction Using Meta Heuristic Harris Hawk and Routh Hurwitz Array Technique

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## Abstract

**Abstract:** A physical system is of higher-order and it is hectic for researchers to understand these systems in higher mathematical form. So, there is a requirement for systematic conversion of higher-order into a lower order. The lower order approximately gives the same result as that of the higher-order by preserving the important properties of higher-order. But the lower order retains some approximation error. **Objective:** The objective is to optimise the reduced-order by minimizing the integral square error between the higher-order system (HOS) and the lower-order system (LOS). **Methodology:** For the optimization process the novel harris hawk hunting behaviour is optimized. It is applied to find the unknown numerator by applying the novel algorithm. The denominator parameter is obtained by the Routh Hurwitz Array technique. **Finding:** The proposed technique is applied on a linear time-invariant single input single output system of higher-order which is randomly selected from the literature. To justify the proposed technique, the result obtained is compared with the result available in the literature. The comparison is based on the step response characteristics of the diminished order with original and result accessed from literature. The response indices such as integral square, integral absolute, integral time absolute errors are also compared. The error gets minimized and results improved as associated with the result presented in the literature.

**Keywords:** Harris Hawk Optimization; Routh Array Technique; Integral square error; step response characteristics; reduced order

## 1 Introduction

MOR is motivated by the need for increased system complexity. Understanding the complex system is not easy and design of the system also very cumbersome. Interestingly, due to its approximate response and preserving the important characteristics of the HOS, it becomes a wide area of research including control, power, chemical and mechanical, design engineering with many more. The varieties of MOR approaches are accessible from the literature and each has quite a unique approach. These techniques only differ with system design characteristics as stability, matching steady-state value, frequency and time response. They all maintain a mutual goal of diminishing the HOS.

Research are still going on to identify the more effective method for simplification.

The mixed method basically uses two methods for the reduced order finding in order to improve the response of approximation. In recent time the method is integrated with the nature motivated optimization methods. The traditional procedures such as particle swarm optimization(PSO)<sup>(1)</sup>, in this the advantage of the Eigen spectrum investigation and the error minimization by PSO. It gives the advantages of retain the steady state value of the original system. The algorithms based on the genetic process is genetic algorithm which is traditional and effective combined with other traditional methods as well as the nature inspired algorithms in order to obtained the lower order. The application of GA in MIMO is well illustrated in<sup>(2)</sup>, in which the 10<sup>th</sup> order two input two output practical power system model is diminished in 3<sup>rd</sup> order. The important methods are Eigen permutation for finding the numerator parameter with Jaya algorithm for finding the denominator parameter<sup>(3)</sup>, cuckoo algorithm<sup>(4)</sup>, fuzzy c-means<sup>(5)</sup>. Physics based algorithm such as big bang big crunch with time moment matching<sup>(6)</sup>, bat algorithm<sup>(7)</sup> stability equation with genetic algorithm<sup>(8)</sup>, Routh pade approximation with the harmony search algorithm<sup>(9)</sup>, Invasive weed optimization for MIMO system<sup>(10)</sup>.

In recent, the study based on the behavior of natural process, animals, physics, genetic and swam based algorithms are in development and improvement phases. The behavior of hunting animal converted into a systematic mathematical procedure via making a rigorous study. Some of the important algorithm based on the process are grey wolf in which a group of wolf encircles the prey, blue whale in which 7-8 whales encircle the prey by making mimic sound in spiral formation, harris hawk optimization, the hawk searches the prey from a height like a high tree or pole and make some glide attack on prey in order to catch it and many more. The Figure 1 shows the classification of optimization techniques.

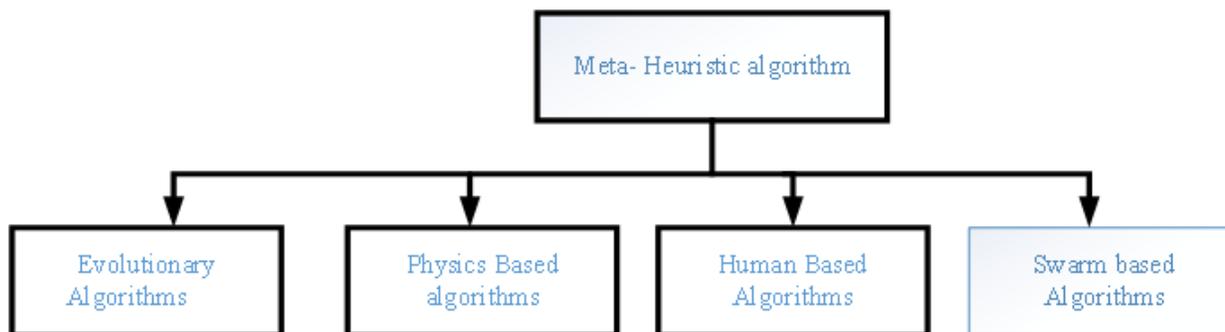


Fig 1. Classification of Meta-heuristic algorithm

The paper is based on the harris hawk optimization. The section compares the swarm based algorithm and justifying the selection for implementing HHO.

Table 1. Swarm base algorithms developed in Literature with advantage and disadvantage

Algorithms	Inspiration	Advantages	Disadvantages
Particle swam opti- mization <sup>(11)</sup>	Bird flock	Simple ,effective	Depend on stochastic process like evolu- tionary programming.
Cuckoo algorithm <sup>(12)</sup>	Cuckoo	The number of parameters to be tuned is less than GA and PSO, and thus it is potentially more generic to adapt to a wider class of optimization problems	Complex, the step length is heavy- tailed, and any large step is possible.
Fruit fly optimiza- tion <sup>(13)</sup>	Fruit fly	Easy and execution speed will be faster	The stability of the fruit fly swarm search route is related to fruit fly quantity. The swarm with fewer fruit fly numbers will have disadvantages of an unstable search route and a slower convergence speed;
Marriage in Honey bee optimization algorithm <sup>(14)</sup>	Honey Bee	Algorithm preserved concepts and achieve the good performance.	Multi behavior; Mating process is hard to observe

Continued on next page

Table 1 continued

Dolphin Partner optimization <sup>(15)</sup>	Dolphin	It has rapid and niche character and good adaptability for different objective functions.	The particle exchange only global best positions, the fitness is ignored
Dolphin Echolocation <sup>(16)</sup>	Dolphin	Affordable computer cost Parameter is better to be chosen according to the size of search space. Capability of adopting itself by type of problem	The time lapse between click and echo enables the dolphin to evaluate the distance from the object; the varying strength of the signal as it is received on the two sides of the dolphin's head enabling him to evaluate the direction
Artificial fish swarm algorithm <sup>(17)</sup>	Fish swarm and social behaviors	High convergence speed, flexibility, fault tolerance and high accuracy.	High complexity, lack of balance between and local search, lack of benefiting from experience of group members for next movement
Bat Inspired algorithm <sup>(18)</sup>	Bat herd	Potentially powerful, simple.	Implementation is complicated. Solution is not depend on the quality of solutions
Termite algorithm <sup>(19)</sup>	Termite colony	Decisions making is good	Random in the search space, trajectory are biased.
Ant colony optimization <sup>(20)</sup>	Ant colony	Positive feedback, distributed computation, Rapid discovery of solutions	Premature convergence
Wasp swarm algorithm <sup>(21)</sup>	Parasitic wasp	The quality of best solution is always high	Chances in falling into local minima caused by saturation of Local search
Firefly algorithm <sup>(22)</sup>	Firefly	Convergence makes quickly and global optimization achieve naturally	The algorithms stops when the variations of functions values is less than a given tolerance $\leq 10^{-5}$
Hunting search <sup>(23)</sup>	Group search	Preserves the history of past vectors	Depend on the corporation of members the optimum solution is static and does not change it position
Whale algorithm <sup>(24)</sup>	Whale bubble net strategy	Success rate of solving problem is high, high exploration ability due to position updating parameters	Search space is large
Grey wolf optimization <sup>(25)</sup>	Grey wolf herd	Exploration ability is high.	Based on social hierarchy. Prone to stagnation in local solutions
Harris Hawk optimization <sup>(26)</sup>	Hawk behavior	Capable of finding excellent solutions due to cooperative behavior and chasing the prey.	Depend on the energy of prey.

The harris hawk is one of the Eagle variety and its behavior study converted into the meta heuristic algorithm<sup>(26)</sup>. The algorithm is swarm based as number of hawk try to hunt the prey. The hunting of hawk based on the agility and speed with sturdy feed and sharp talons to grab the prey specially rabbit, squirrels. The behavior of hawk is directly depending on the events occurs when the prey tries to escape the hunt. In this manuscript, HHO is used to in model reduction field. Therefore, this paper extends the approach of this algorithm in MOR field along with Routh Hurwitz Array (RHA) making it a mixed method for MOR. Hence unification of HHO and RHA is obtained proving that HHO is suitable for stability preservation methods.

The paper is separated into six sections. Starting from the introduction and followed by the statement of problem, methodologies, implementation in numerical examples with discussion and the last conclusion of the paper is given further references are listed

## 2 Problem Statement

### 2.1 For LTI SISO systems

The SISO system transfer function with unknown order of may be represented by the following Equation

$$G_n(s) = \frac{N_{n-1}(s)}{D_n(s)} = \frac{\sum_{a=0}^{n-1} N_i s^i}{\sum_{a=0}^n D_i s^i} \tag{1}$$

$N_i$  is the numerator and  $D_i$  is denominator constants of the original system. In some cases,  $N_0 = D_0$  for the steady-state output result to a unit step input will be unity. To find the unknown scalar constant of the ROS  $m^{\text{th}}$  ( $m < n$ ) from the OHOS. The

obtained reduced-order has the following transfer function in Eq. (2)

$$R_n(s) = \frac{Nr_{n-1}(s)}{Dr_n(s)} = \frac{\sum_{a=0}^{m-1} Nr_i s^i}{\sum_{a=0}^m Dr_i s^i} \tag{2}$$

### 3 Methodologies

#### 3.1 Harris Hawk Optimization

Harris Hawk optimization (HHO) is based on the studies of hawk behavior usually in the period of hunting. The study is done by Louis Lefebvre. The mathematical implementation using the algorithm is Mirjili.<sup>(26)</sup> The behavior of hunting and chasing patterns for capture if pray in nature is known as surprise pounce. The searching of prey is a task done by the predator using the highest point of the area such as standing on top of trees or flying in the sky. The attack of the hawk on prey is called a pounce. As the prey is spotted another member is informed by visual displaying or vocalization. The HHO is divided into three-phase naming exploration, the transition from exploration to exploitation and exploitation phase. The exploitation stage is separated into four stages namely soft besiege, hard besiege, soft besiege with advanced quick dives, hard besiege with progressive speedy dives.

##### 3.1.1 The Exploration Phase

To start this phase, the Hawk reaches on the peak of tree/pole/top of hill in order to trace the prey and also consider the other of Hawks positions. Situation of  $q \leq 0.5$  or branch on random giant trees for situation of  $q \geq 0.5$ . The condition ids modelled as

$$x(t+1) = \begin{cases} x_{\text{rand}}(t) - r_1 |X_{\text{rend}}(t) - 2r_2 X(t)| & q \geq 0.5 \\ (X_{\text{prey}}(t) - X_m(t) - r_3(LB + r_4(UB-LB))) & q \leq 0.5 \end{cases} \tag{3}$$

$X(t+1)$  is position vector of the hawk in succeeding iteration  $t$ .  $X_{\text{prey}}(t)$  is the present position vector of hawks  $r_1, r_2, r_3, r_4$  and  $q$  are the random number confidential (0,1) upgraded with iteration. LB is the lower bounds and UB is upper bounds of numbers.  $X_{\text{rand}}(t)$  arbitrarily hawk from the present population.  $X_m$  is the average position of the current population of hawks. The primary rule creates solutions based on a random position. In second rule of Eq. (3), the variance between the best position and the average location of the group plus an arbitrarily climbed factor depending on the number of variables. The scaling factor  $r_3$  increases the random nature of regulation once  $r_4$  adjacent value to 1 and comparable distribution designs. Random factor scaling coefficients increase pattern diversification and explore various feature regions. The rules for buildings are capable of mimicking the actions of a hawk. The hawk's average location is obtained using Eq. (4):

$$X_m(t) = \frac{1}{M} \sum_{i=1}^M X_i(t) \tag{4}$$

$X_m(t)$  is obtaining by Equation (4).  $X_i(t)$  designates the position of individual hawk in iteration  $t$  and  $N$  signifies the number of hawks.

##### 3.1.2 Conversion from Exploration to Exploitation

The exploration to exploitation changes between exploitation performances founded on the absconding energy of the prey. The energy of a prey reduces throughout the escaping. The energy of the prey is modelled as in Eq. (5)

$$E = 2E_0 \left(1 - \frac{t}{T}\right) \tag{5}$$

$E$  designate the absconding energy of prey.  $T$  the maximum number of iteration and  $E_0$  initial state of energy.

##### 3.1.3 Exploitation phase

The process begins by surprise and the imagined prey of the previous stage is hostile. Preys are trying to get out of the case. The probability of fleeing from the prey is ( $r < 0.5$ ) or not to escape efficaciously ( $r \geq 0.5$ ). The hawk executes rough or soft besieges in relation to prey activity to capture the prey. Based on the vitality of the prey, the hawk encircles around the beast in various ways. The hawk getting closer to the desired prey to maximize its odds of cooperating in killing the rabbit. The gentle assault begins and the rough assault takes place.

### 3.1.4 Besiege occurs

#### A. Soft besiege

The prey has energy and try to escape using random confusing jumps. The value for escaping energy must be  $r \geq 0.5$  and  $E \geq 0.5$ . If the value is below as stated, the prey unable to jump. Hawk encircles prey gently to make it more tired and achieve the surprise dive. This conduct is modelled by subsequent rules represented in Eq (6) and Eq. (7)

$$X(t + 1) = \Delta X(t) - E |J^* X_{prey}(t) - X(t)| \tag{6}$$

$$\Delta X(t) = X_{prey}(t) - X(t) \tag{7}$$

#### B. Hard Besiege

The prey is exhausted and has less energy when value  $r \geq 0.5$  and  $E \geq 0.5$ . The Hawks barely enclose the intended prey to finally achieve the shock pounce. The present locations are updated as per Eq. (8)

$$X(t + 1) = X_{prey}(t) - E |\Delta X(t)| \tag{8}$$

### 3.1.5 Soft besiege with progressive rapid dives

To catch the prey, the Hawk, decide their subsequent move founded on Eq. (9)

$$Y = X_{prey}(t) - E |J^* X_{prey}(t) - X(t)| \tag{9}$$

Dive is founded on the LF-based designs using the law represented in Eq. (10)

$$Z = Y + S \times LF(D) \tag{10}$$

D dimension problem and s is a random vector by size 1XD and LF is the levy fight function, and calculated as in Eq. (11)

$$LF(x) = 0.01 \times \frac{u \times \sigma}{|v|^{\frac{1}{\beta}}}, \sigma = \left( \frac{\Gamma(1 + \beta) \times \sin\left(\frac{\pi\beta}{2}\right)}{\Gamma\left(\frac{1 + \beta}{2}\right) \times \beta \times 2^{\left(\frac{\beta - 1}{2}\right)}} \right)^{\frac{1}{\beta}} \tag{11}$$

u,v are random values inside (0, 1),  $\beta$  is a constant set to 1.5

The last tactic for apprising the locations of hawks. The soft besiege stage can be achieved and given in Eq. (12)

$$X(t + 1) = \begin{cases} Y & \text{if } F(Y) < F(X(t)) \\ Z & \text{if } F(Z) < F(X(t)) \end{cases} \tag{12}$$

The Y and Z are obtained using the Eq. (11) and Eq. (12)

### 3.1.6 Hard besiege with progressive rapid dives

The prey has not adequate energy  $|E| < 0.5$  and  $r < 0.5$ . To escape and hard besiege is built earlier the surprise pounce to catch and kill the prey. The condition on the prey side is comparable to that of soft besiege except this time, the hawk seeks to reduce the difference between their regular position and the fleeing target.

$$X(t + 1) = \begin{cases} Y & \text{if } F(Y) < F(X(t)) \\ Z & \text{if } F(Z) < F(X(t)) \end{cases} \tag{13}$$

The Y and Z are gained by the Equation. (14) and Equation. (15)

$$Y = X_{prey}(t) - E |J^* X_{prey}(t) - X_m(t)| \tag{14}$$

$$Z = Y + S \times LF(D) \tag{15}$$

### 3.2 Routh Hurwitz Array

The abridged denominator can be achieved by the Routh stability array of the denominator polynomial. For convenience the even and odd portions are separated

$$D(s) = \sum_j b_{1,j+1} s^{n-2j} + \sum_k b_{2,k+1} s^{n-(2k+1)}$$

$$j = 0, 1, 2, \dots, n/2 \text{ and } k = 0, 1, 2, \dots, (n-2)/2 \text{ for } n \text{ even} \tag{16}$$

$$j = 0, 1, 2, \dots, (n-1)/2 \text{ and } k = 0, 1, 2, \dots, (n-1)/2 \text{ for } n \text{ odd}$$

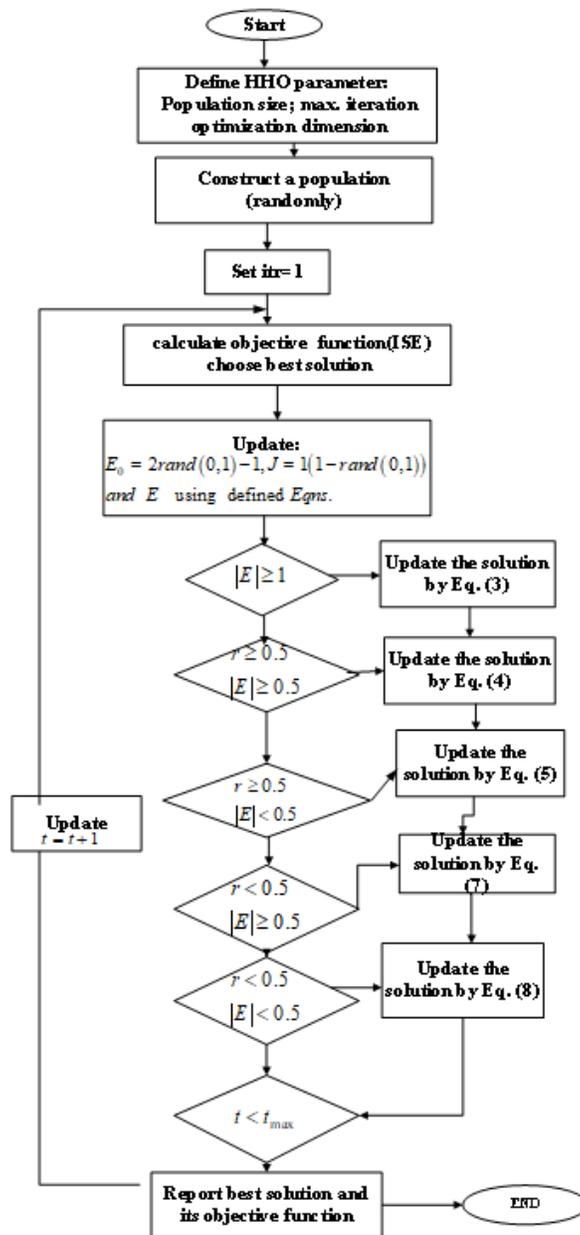


Fig 2. Flow Chart of Harris Hawk Optimization

Now, the Routh- Horwitz stability array is moulded for the denominator polynomial

$$\begin{array}{ccccccc}
 b_{11} & b_{12} & b_{13} & b_{14} & \dots & & \\
 b_{21} & b_{22} & b_{23} & b_{24} & \dots & & \\
 b_{31} & b_{32} & b_{33} & & & & \\
 \dots & & & & & & \\
 b_{n,1} & & & & & & \\
 b_{n+1,1} & & & & & & 
 \end{array}$$

The well-known routh algorithm for the overhead array

$$b_{i,j} = b_{i-2,j+1} - (b_{i-2}, b_{i-1,j+1}) / b_{i-1,1} \tag{17}$$

Where  $i > 3$  and  $1 < j < [(n - i + 3)/2]$ ,  $[\cdot]$  stands for the integral part of the quantity . A polynomial of lower order  $r$  may be easily constructed with the  $(n + 1 - r)^{th}$  and  $(n + 2 - r)^{th}$  rows of the above array

$$\begin{aligned}
 D_r(s) &= b_{(n+1-r),1}s^r + b_{(n+2-r),1}s^{r-1} + b_{(n+1-r),2}s^{r-2} + \dots \\
 &= d_0 + d_1s + \dots + d_r s^r
 \end{aligned}
 \tag{18}$$

The response indices *i.e.*, Integral Square Error (ISE) penalizes the larger errors more than the smaller error. This gives a more conservative response and the reduced system return faster to the set point. The objective function considered is ISE and represented in Equation (19)

$$ISE = \int_0^\infty [G(t) - G_r(t)]^2 dt \tag{19}$$

### 4 Result analysis and Discussion

**Example 1:** The fourth-order transfer function is randomly picked for the implementation of the aimed technique represented in Equation. (20)

$$G(s) = \frac{s^3 + 7s^2 + 24s + 24}{s^4 + 10s^3 + 35s^2 + 50s + 24} \tag{20}$$

The reduced second order denominator polynomial of the system represented in Equation (21) using the routh approximation method

$$\tilde{D}_r(s) = s^2 + 1.6556s + 0.7944$$

The coefficient of the unknown numerator parameter is obtained using the harris hawk optimization taking the values as mentioned in Table 1 for Example 1  $N_r(s) = 0.8135s + 0.7942$  .

So , the obtained reduced order of Example 1 from Equation (20) is given in Equation (21)

$$R(s) = \frac{N_r(s)}{D_r(s)} = \frac{0.8135s + 0.7942}{s^2 + 1.6556s + 0.7944} \tag{21}$$

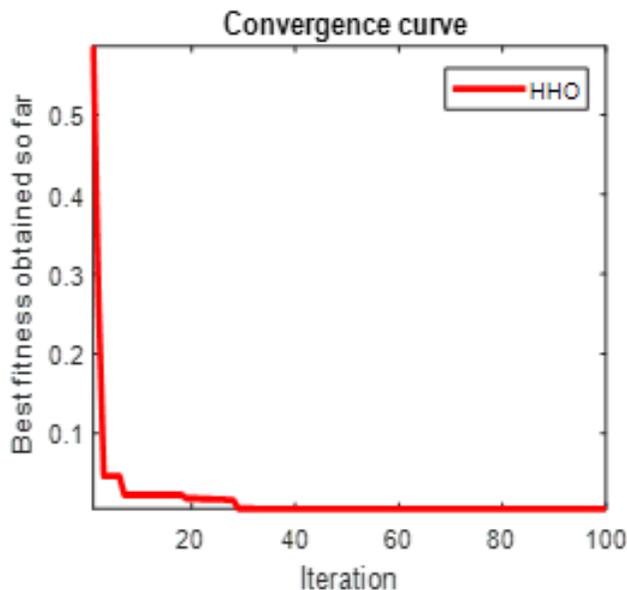


Fig 3. Convergence curve of Example 1 using the HHO

The parameters used for obtaining the numerator part from HHO is listed in Table 1

Table 2. Parameter values of for Example 1

Name	Values of Example 1
Dim	2 (N1, N2)
N	30
Rabbit Energy/Best Fitness of HHO	0.0050304
T'	100
Ub	[0.5,0.7]
LB	[0.9000, 0.7]
Elapsed Time	1551.326298 seconds

The integral square error is 0.000245 and the proposed method. To avoid ambiguity only the response of the few reduction techniques is revealed in Figure 2. Table 2 give the comparative analysis of the projected diminished order using the novel technique and diminished order available in literature. The ISE, IAE and ITAE of the response indices is improved.

Table 3. Response error indices of, proposed and 2nd -order available in the literature

Author/Year/Method	ROM	Response indices		
		ISE	IAE	ITAE
Original		-	-	-
Proposed with algorithm	$\frac{0.8135s+0.7942}{s^2+1.6556s+0.7944}$	0.000245	0.04087	0.1544
Sambariya; 2016; RA+CSA (27)	$\frac{0.8130s+0.7945}{s^2+1.6560s+0.7947}$	0.0002455	0.04279	0.2264
Desai; 2013; BBBC+RA (28)	$\frac{0.8085s+0.7944}{s^2+1.65s+0.7944}$	0.0002835	0.04466	0.2217
Parmar; 2007; FDA+ESA (29)	$\frac{0.6667s+4}{s^2+5s+4}$	0.0002637	0.02613	0.06642
Sikander (30); 2015; CSA	$\frac{0.7751s+1.258}{s^2+2.12s+1.258}$	0.000132	0.02739	0.1224

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*Table 3 continued*

Sikander; 2015; SE+PSO <sup>(31)</sup>	$\frac{0.7528s+0.6952}{s^2+1.458s+0.6997}$	0.001519	0.1471	1.348
Sikander; 2015; SE+FDA <sup>(32)</sup>	$\frac{0.6997s+0.6997}{s^2+1.45771s+0.6997}$	0.00278	0.1319	0.5537
Sikander; 2016 <sup>(33)</sup>	$\frac{0.7423s+0.6957}{s^2+1.458s+0.6997}$	0.001536	0.1443	1.239
Sambariya; 2016; RSA+SE <sup>(34)</sup>	$\frac{20.57143s+24}{35s^2+50s+24}$	0.01307	0.2319	0.767
Sambariya <sup>(35)</sup> ; Routh array; 2016	$\frac{246.852s+288}{70s^2+300s+288}$	0.3217	0.8988	2.04
Narwal; 2016; MCA <sup>(36)</sup>	$\frac{0.7840s+2.1215}{s^2+3.1213s+2.1213}$	0.0002128	0.03058	0.1092
Narwal;2015; SE+CSO <sup>(37)</sup>	$\frac{0.7597s+0.6997}{s^2+1.4577s+0.6997}$	0.001991	0.1108	0.5743
Lucas; 1983; FD <sup>(38)</sup>	$\frac{0.833s+2}{s^2+3s+2}$	0.0003284	0.03205	0.0925
Howitt; 1990; <sup>(39)</sup>	$\frac{0.81796.s+0.78411}{s^2+1.64068s+0.78411}$	0.0003053	0.04576	0.2311

The error obtained from the proposed method is very less than compared to the methods available in literature. The algorithm based on swarm, physics and traditional methods are compared. The particle swarm optimization with stability equation in <sup>[31]</sup>, cuckoo search algorithm <sup>[30]</sup>, Cuckoo search algorithm with SE <sup>[34]</sup> are compared along with the traditional methods and mixed methods and proposed result is better. This proves that the HHO is effective in MOR field. The Table 4 shows the step response characteristics of the proposed reduced order and reduced order available from literature.

**Table 4.** Step response characteristics of proposed 2<sup>nd</sup> order and 2<sup>nd</sup> order available in literature

Author/Year/Method	Step Response Characteristics			
	ST	RT	Peak	PT
Original	3.9308	2.2603	0.9990	6.8847
Proposed with algorithm	3.6289	2.2753	1.0023	6.0082
Sambariya;2016; RA+CSA <sup>(35)</sup>	3.6319	2.2767	1.0022	6.0624
Desai; 2013; BBBC+RA <sup>(28)</sup>	3.6199	2.2785	1.0027	5.9728
Parmar; 2007; FDA+ESA <sup>(29)</sup>	4.0176	2.2646	0.9993	7.3222
Sikander <sup>(30)</sup> ; 2015; CSA	3.6722	2.2409	1.0002	6.9078
Sikander; 2015 <sup>(31)</sup> ; SE+PSO	3.1669	2.1574	1.0072	4.9273
Sikander; 2015 <sup>(32)</sup> ; SE+FDA	3.4104	2.3011	1.0107	5.2442
Sikander; 2016 <sup>(33)</sup>	3.2143	2.1850	1.0073	4.9905
Sambariya; 2016 <sup>(34)</sup> ; RSA+SE	3.4554	2.3769	0.9727	5.2223
Sambariya <sup>(35)</sup> ; Routh array; 2016	2.0937	0.5040	1.0382	1.2688
Narwal ; 2016 <sup>(32)</sup> ; MCA	4.0867	2.3373	1.0001	11.2454
Narwal; 2015 <sup>(35)</sup> ; SE+CSO	3.1562	2.1514	1.0139	4.8652
Lucas ; 1983 <sup>(38)</sup> ; FD	4.0642	2.3197	0.9992	7.3222
Howitt; 1990 <sup>(39)</sup> ;	3.5769	2.2548	1.0030	5.8944

The proposed reduced-order using the HHO and RHA is compared in Table 3. The proposed algorithms are better than reduced-order available results. The proposed method proves its effectiveness. Table 4 gives the step response analysis of the reduced order. Figure 2 is the convergence curve of the best fitness vs. the number of iterations. The best fitness obtained for example 1 in 100 iterations is 0.0050304. As per the theoretical values  $|E| < 0.5$  and  $r < 0.5$ . The hawk attacked the rabbit (prey) and the target of hunting is completed as the fitness of the rabbit is 0.0050304 and less than 0.5. The proposed reduced-order using the HHO and RA is represented in Eq. (21) The comparable step response of the tactical 2<sup>nd</sup> second order in red near amplitude value 1 and second-order present in the literature is given in Figure 3. The proposed order has a steady state value of 1 better than available in the literature.

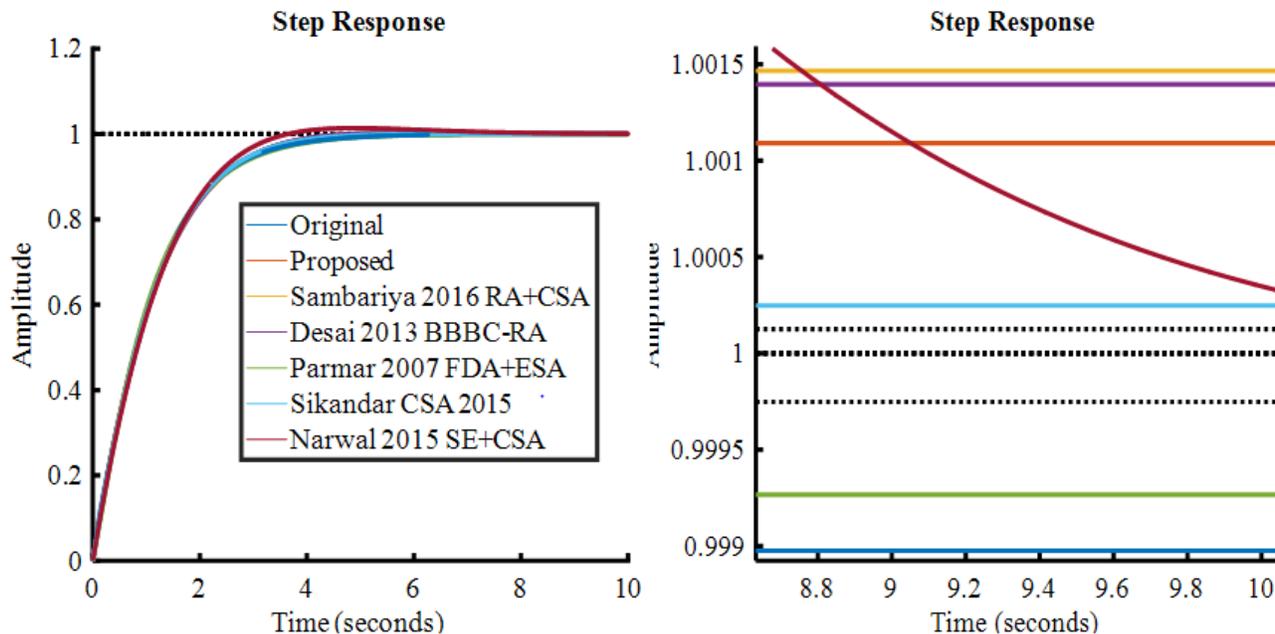


Fig 4. Step response characteristics of Example 1 compared from literature

## 5 Conclusion

The manuscript presented a novel method to reduce a higher order system. The novel method consists the hunting behaviour of harris hawk and escaping of the prey in a systematic manner and Routh Hurwitz array. The proposed method implemented on an LTI SISO system. Tables 3 and 4 shows the effectiveness of the method as results get improved. The error minimization “ISE” between HOS and ROM is optimized by HHO. The important physiognomies of the system get preserved as the proposed order follows the definition of model order reduction. The application of the projected scheme may be extended to MIMO system.

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