INDIAN JOURNAL OF SCIENCE AND TECHNOLOGY

RESEARCH ARTICLE

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OPEN ACCESS

Received: 18.07.2021

Accepted: 23.09.2021

Published: 27.10.2021

Citation: Vembarasi G, Gowri R (2021) Super Root Cube of Cube Difference Labeling of Some Special Graphs. Indian Journal of Science and Technology 14(35): 2778-2783. https://doi.org/10.17485/IJST/v14i35.1336

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Funding: None

Competing Interests: None

Published By Indian Society for Education and Environment (iSee)

ISSN
Print: 0974-6846
Electronic: 0974-5645

Super Root Cube of Cube Difference Labeling of Some Special Graphs

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Abstract

Background/Objectives: This study gives an extended and the new kinds of super root cube of cube difference labeling of some graphs are obtained. **Methods/ Findings:** We derive super root cube of cube difference labeling of path related graph and analyzed cycle related graphs.

Keywords: Triangular Snake T_n; Cycle graph C_n; Crown C_n©K_1; pendent edge to both sides of each vertex of a path Pn; supr root cube of cube difference labeling of graphs.

1 Introduction

All graphs $G=(V(G),E(G))$ with p vertices and q edges we mean a simple connected and undirected graph. In 2012, J. Shiama [1], studied square difference labeling of some graphs. In 2013, J. Shiama [2], introduced the concept of cube difference labelings and investigated the labelings for certain graphs. S.Sandhya et.al [3], was initiated the concept of root square mean labeling of graphs. In 2016, M. Kannan et.al [4], introduced the concept of super root square mean labeling of disconnected graphs are discussed. In 2017, R.Gowri and G.Vembarasi [5], was discussed root cube mean labeling of graphs. R.Gowri and G.Vembarasi [6], extended the new concept of root cube difference labeling of graphs are introduced in 2018. In 2019, S.Kulandhai Theresa and K.Romila [7], was discussed the concept of cube root cube mean labeling of graphs are introduced. In 2020, R.Gowri and G.Vembarasi [8] recently introduced the concept of root cube of cube difference labeling of graphs. Likewise, many authors have discussed this topic in their work. In this study we discuss about the super root cube of cube difference labeling and investigate certain families of graphs.

2 Preliminaries

Definition 2.1 [5]

The graph obtained by joining a simple pendent edge to each vertex of a path is called a Comb graph.

Definition 2.2 [5]

A walk in which vertices are called a path. A path on n vertices is denoted by P_n.

Definition 2.3[5]

The product graph $P_2 \times P_n$ is called a Ladder and it is denoted by L_n .

Definition 2.4[9]

A graph G with p vertices and q edges then $f : V(G) \to \{1,2,\ldots,p+q\}$ be an injective function. For each edge e=uv . Let $f^*(e = uv) = \left\lceil \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rceil$ (or) $\left\lfloor \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rfloor$ then f is called a super root square mean labeling if $f(v) \cup \{f^*(e) : e \in E(G)\} = \{1,2,\ldots,p+q\}$. A graph that admits a super root mean labeling is called a super root mean graph.

Definition 2.5[2]

Let $G = (V(G),E(G))$ be a graph. G is said to be a cube difference labeling if there exists a injective function $f : v(G) \to \{0,1,\ldots,p-1\}$ such that the induced function $f^* : E(G) \to N$ is given by $f^*(uv) = \left|[f(u)]^3 - [f(v)]^3\right|$ is injective.

Definition 2.6[6]

Let $G = (V(G),E(G))$ be a graph. G is said to be a cube difference labeling if there exists a injective function $f : v(G) \to \{0,1,\ldots,p-1\}$ such that the induced function $f^* : E(G) \to N$ is given by $f^*(uv) = \left\lceil \sqrt{|\,[f(u)]^3 - [f(v)^3]\,|} \right\rceil$ (or) $\left\lfloor \sqrt{|[f(u)]^3 - [f(v)]^3|} \right\rfloor$ is injective.

3 Super Root Cube Of Cube Difference Labeling of Graphs

Definition 3.1

A graph G with p vertices and q edges then $f : V(G) \to \{1,2,\ldots,p+q\}$ be an injective function. For each edge $e = uv$. Let $f^*(e = uv) = \left\lceil \sqrt{\left|(f(u)^3 - f(v)^3)^{\frac{1}{3}}\right|} \right\rceil$ (or) $\left\lfloor \sqrt{\left|(f(u)^3 - f(v)^3)^{\frac{1}{3}}\right|} \right\rfloor$, then f is called a super root cube of cube difference labeling if $f(v) \cup \{f^*(e) : e \in E(G)\} = \{1,2,\ldots,p+q\}$. A graph is called a super root cube of cube difference labeling.

Theorem 3.2

Triangular Snake T_n is a super root cube of cube difference labeling of graph.

Proof : A Triangular Snake T_n is obtained from a path $u_1,u_2,\ldots,u_n$ by joining u_i and u_{i+1} to a new vertex v_i for $1 \leq i \leq n$ That is every edge of a path is replaced by a triangular C_3 . Define the function $f : V(G) \to \{1,2,\ldots,p+q\}$ by

$$\begin{aligned} f(u_i) &= 2i, && \text{for} \quad 1 \leq i \leq n \\ f(v_i) &= 2i-1, && \text{for} \quad 1 \leq i \leq n. \end{aligned}$$

And the induced edge labeling function $f^* : E(G) \to N$ defined by

$$f^*(e = uv) = \left\lceil \sqrt{\left|(f(u)^3 - f(v)^3)^{\frac{1}{3}}\right|} \right\rceil$$

Then the edge sets are,

$$f^*(u_iu_{i+1}) = \sqrt{\left|(24i^2+24i+8)^{\frac{1}{3}}\right|}, \quad \text{for } 1 \leq i \leq n-1$$

$$f^*(u_{n-1}u_n) = \sqrt{\left|(24n^2-24n+8)^{\frac{1}{3}}\right|}$$

$$f^*(u_iv_i) = \sqrt{\left|(12i^2-6i+1)^{\frac{1}{3}}\right|}, \quad \text{for } 1 \leq i \leq n$$

$$f^*(u_nv_n) = \sqrt{\left|(12n^2-6n+1)^{\frac{1}{3}}\right|}$$

$$f^*(u_{i+1}v_i) = \sqrt{|\,(36i^2+18i+9)^{\frac{1}{3}}\,|} \quad \text{for } 1 \le i \le n$$

$$f^*(u_{n+1}v_n) = \sqrt{\left|(36n^2+90n+63)^{\frac{1}{3}}\right|}$$

Hence the graph G is a Super root cube of cube difference labeling.

Example 3.3

Super root cube of cube difference labeling of T_4 is given below.

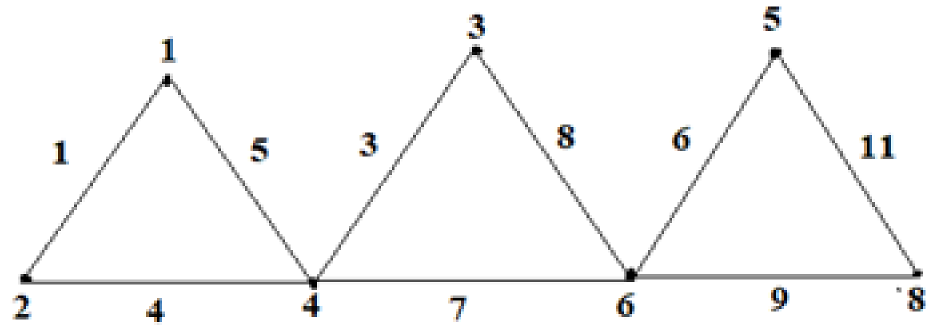

Fig 1.

Theorem 3.4

The Cycle graph C_n is a Super root cube of cube difference labeling.

Proof : A closed path is called a cycle. A cycle on n vertices is denoted by C_n graph with vertices $u_1, u_2, \ldots, u_n$ and the edges $e_1, e_2, \ldots, e_m$ Define the function $f : V(G) \to \{1, 2, \ldots, p+q\}$ by

$$f(u_i) = i \qquad \text{for } 1 \le i \le n$$

And the induced edge labeling function $f^* : E(G) \to N$ defined by

$$f^*(e = uv) = \left[\sqrt{\left|(f(u)^3 - f(v)^3)^{\frac{1}{3}}\right|}\right]$$

Then the edges labels are,

$$f^*(u_iu_{i+1}) = \sqrt{(3i^2+3i+1)^{\frac{1}{3}}}, \quad \text{for } 1 \le i \le n-1$$

$$f^*(u_{n-1}u_n) = \sqrt{(3n^2-3n+1)^{\frac{1}{3}}}$$

$$f^*(u_nu_1) = \sqrt{(n^3-1)^{\frac{1}{3}}}$$

Hence the graph C_n is a Super root cube of cube difference labeling.

Example 3.5

The following is an example for C_7 is a Super root cube of cube difference labeling of graph.

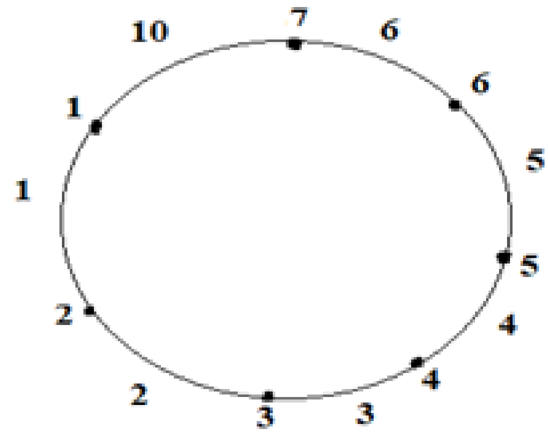

Fig 2.

Theorem 3.6

The crown $C_n\Theta K_1$ is a Super root cube of cube difference labeling of graph.

Proof: Let C_n be the Cycle $u_1u_2..u_nu_1$ and v_i be the pendant vertices adjacent to $u_i, 1 \leq i \leq n$. Define the function $f : V(C_n\Theta K_1) \rightarrow \{1,2,\ldots,p+q\}$ by

$$f(u_i) = i,$$
$$\text{for } 1 \leq i \leq n$$
$$f(v_i) = i+6,$$
$$\text{for } \quad 1 \leq i \leq n.$$

And the induced edge labeling function $f^* : E(G) \rightarrow N$ defined by

$$f^*(e = uv) = \left[\sqrt{\left|(f(u)^3 - f(v)^3)^{\frac{1}{3}}\right|}\right]$$

Then the edge sets are,

$$f^*(u_iu_{i+1}) = \sqrt{\left|(24i^2 + 24i + 8)^{\frac{1}{3}}\right|}, \quad \text{for } 1 \leq i \leq n-1$$
$$f^*(u_{n-1}u_n) = \sqrt{\left|(24n^2 - 24n + 8)^{\frac{1}{3}}\right|}$$
$$f^*(u_iv_i) = \sqrt{\left|(12i^2 - 61 + 1)^{\frac{1}{3}}\right|}, \quad \text{for } 1 \leq i \leq n$$
$$f^*(u_nv_n) = \sqrt{\left|(12n^2 - 6n + 1)^{\frac{1}{3}}\right|}$$
$$f^*(u_{i+1}v_i) = \sqrt{\left|(36i^2 + 18i + 9)^{\frac{1}{3}}\right|}, \quad \text{for } 1 \leq i \leq n$$
$$f^*(u_{n+1}v_n) = \sqrt{\left|(36n^2 + 90n + 63)^{\frac{1}{3}}\right|}$$

Hence the graph G is a Super root cube of cube difference labeling.

Example 3.7

Super root cube of cube difference labeling of $C_n \Theta K_1$ is given below.

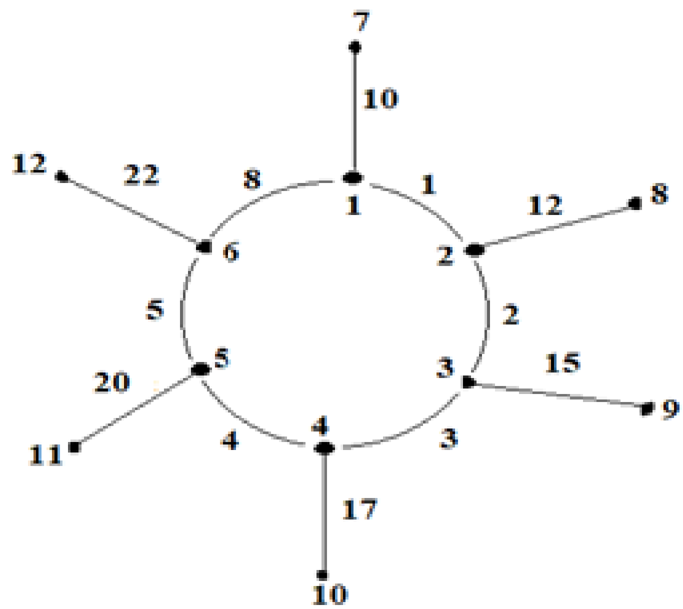

Fig 3.

Theorem 3.8

Let G be a graph obtained by attaching a pendant edge to both sides of each vertex of a path P_n. Then G is a Super root cube of cube difference labeling of graph only if $n \geq 5$. .

Proof: Let G be path P_n. The graph obtained by attaching pendant edges to both sides of each vertex. Let x_i, y_i and z_i for $1 \leq i \leq n$ be the new vertices of G. Define the function $f : V(G) \to \{1, 2, \ldots, p+q\}$ by

$$f(x_i) = 3i - 1 \quad \text{for } 1 \leq i \leq n$$

$$f(y_i) = 3i \quad \text{for } 1 \leq i \leq n$$

$$f(z_i) = 3i - 2 \quad \text{for } 1 \leq i \leq n$$

And the induced edge labeling function $f^* : E(G) \to N$ defined by

$$f^*(e = uv) = \left[\sqrt[1]{(f(u)^3 - f(v)^3)^{\frac{1}{3}} \mid} \right]$$

Then the edge sets are,

$$f^*(x_i x_{i+1}) = \sqrt{\mid (81i^2 + 27i + 9)^{\frac{1}{3}} \mid}, \quad \text{for } 1 \leq i \leq n-1$$

$$f^*(x_i y_i) = \sqrt{\left| (27i^2 - 9i + 1)^{\frac{1}{3}} \right|}, \quad \text{for } 1 \leq i \leq n$$

$$f^*(x_i z_i) = \sqrt{\left| (27i^2 - 27i + 7)^{\frac{1}{3}} \right|}, \quad \text{for } 1 \leq i \leq n$$

Hence the graph G is a Super root cube of cube difference labeling.

Example 3.9

The graph obtained P_5 is given below.

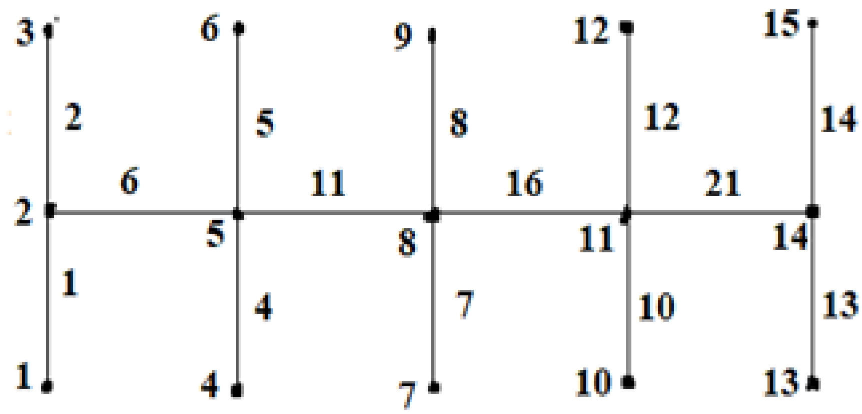

Fig 4.

4 Conclusion

In this article we discussed the concept of Super Root Cube of Cube Difference Labeling of Graphs are initiated and also some graphs are introduced and characterized. Then the relative results between path, cycle related graphs are discussed. Here all the edge values are distinct and the resulting edge values do not exceed the vertex value.

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