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Mixed Convection of Williamson Fluid along an Inclined Porous Microchannel with Chemical Reaction by taking Non-Constant Thermal Conductivity: An Entropy Analysis

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Abstract

Objective: Mixed convection of Williamson fluid along an inclined porous microchannel with the influence of first-order chemical reaction is considered. The thermal conductivity kept varying throughout the flow. Boundaries of the channel are maintained with slip and jump conditions. With these conditions in this present article we are aimed to analyse the velocity, heat transfer and entropy generation in the flow. **Method:** The non-linear equations which govern the flow are tackled utilizing the bvp4c technique which involves the finite difference method and improved by using the Lobatto III formula. **Findings:** Fluid flow, heat transfer and entropy production analysis are done for the various estimations of the parameters which are affecting the flow, and the study highlights that non constant thermal conductivity and mixed convection parameters have to be maintained at lower values for the efficient energy transfer in the model. Entropy production shows the dual trend for distinct estimations of Weinsenberg number. **Applications:** The obtained result in the present study helps industries to analyse the efficient energy transfer in their engineering designs and thermal equipment's. Also the flow of non-Newtonian fluid has applications in the field of blood flow, lubrication and in many engineering devices such as micro heat exchangers, micro mixers, micro cooling systems.

Keywords: Williamson fluid; Inclined microchannel; Variable thermal conductivity; Entropy production; Bejan number

1 Introduction

The flow in microchannel is an active area of study in the recent days due to its significance in the field of science and engineering. Fluid movement in microchannel involves the high rate of heat transfer due to its volume and surface ratio. Many of the mechanical and engineering equipment liberate heat at the time of their operations this leads to the loss in energy. To minimize the energy loss many micro cooling systems are introduced. Due to boundless applications the flow in microchannel has attracted many researchers and scientists to study in this area.

Makinde and Eegunjobi[1] looked into the movement of Casson fluid along a microchannel and analysed the irreversibility of the system. The flow of Williamson fluid flow along a plate which is moving is taken for study by Zaib et al.[2], the results of the study highlights that movement of the fluid is suppressed by Williamson parameter. A theoretical investigation is conducted to analyse the movement of Williamson liquid along a stretching sheet by Khan et al.[3], Kumar et al.[4]. Nanofluid flow along a microchannel and the entropy production from it is studied and analysed by Gireesha et al.[5] and Saima et al.[6]. Sahoo and Nandkeoiyar[7] reported a study of both forced and free convection of casson nanofluid with the effect of thermal radiation. Salahuddin and Malik[8] used Keller box method effectively to study the Williamson fluid flow through a cylinder. Jha and Aina[9] contribute their work to investigate the mixed convection through a microchannel by sucking and injecting the fluid. The work done by Hang and Qiang[10], Patil et al.[11] and Shobha et al.[12] are the good literature for the study of mixed convection through channels. Ibrahim and Anbessa[13] looked into the Williamson nano fluid flow along stretching sheet and results shows that enhancement in Williamson parameter leads to decelerate the fluid temperature. An analysis is made by Cimpean and Pop[14] to investigate the both free and forced flow of nano fluid along a porous filled channel, for the study they considered three nano fluid namely Al2O3-H2O,TiO2-H2O.Cu-H2O. Srinivasacharya and Bindu[15] looked into the micropolar fluid flow along an inclined channel and the flow problem is tackled using spectral quasi linearization technique results attained shows that angle of inclination and the Brinkman number should be maintained at lower values to minimize the entropy. Zaidi and Ahmad[16] made an investigation to analyse the flow along an inclined channel the channel is equipped with the slip and jump boundary, the attained results effectively discussed using the graphs.

A theoretical analysis of the Casson fluid flow along an inclined micro-annular channel is done by Idowu et al.[17]. Gireesha et al.[18] constructed a model which includes both inclined and horizontal convection of Williamson nano fluid and analysed the flow. Mohd et al.[19] considered the flow of ferro fluid along a boundary layer and reported that Fe3O4- H2O has lower heat transfer rate than the Fe3O4- kerosene ferrofluid. The contribution done by the authors Venkateswarlu et al.[20], Patil et al.[21], Xiaobio and Xuegong[22] are the good results for the study of fluid flow through vertical channel with the influence of chemical reaction. Abbas et al.[23] considered the Jeffrey fluid flow in a porous channel and analysed the impact of chemical reaction on it. Ziagta[24] contributed their work to analyse the both forced and free convection along a vertically placed channel by considering the chemical reaction effect. Umavathi et al.[25] looked into the study of free flow along a rectangular duct by maintaining non constant thermal conductivity.

Alireza[26] reported good results for the study of entropy generation. Makinde and Eegunjobi[27] reported the influence of convictive heating on the fluid flow. Anurag et al.[28] looked into the Casson fluid flow along microchannel. Krishna[29] and Renukadeve et al.[30] devoted their study to analyse the entropy generation in the flow with the influence of heat sink. Zahra et al.[31] contributed their work to study the bioconvection of Williamson nanofluid along a stretching sheet, and found that slip parameters influences the velocity, temperature of the flow. Ellahi et al.[32,33] looked into the entropy generation analysis and obtained interesting results for the boundary layer flow over a moving plate and flow of nanofluid through porous medium. An investigation is conducted by Shaheen et al.[34] to study the flow of hyperbolic nanofluid in a ciliated tube they found that addition of nano particles increases the thermal conductivity of the fluid. Bhatti et al[35] examined the convection of Williamson hydromagnetic nanofluid and reported the important results on the entropy generation in the flow.

We found good literature in the study of Williamson fluid flow along different channels with the influence of various parameters. Recently Gireesha et al.[22] studied Williamson fluid flow along the horizontal and inclined channels. They acknowledge that Weissenberg, Reynolds number amplifies the heat transfer rate.

But so far, there is no study is found to analyse the entropy generation in the mixed convection of Williamson fluid along the micro porous channel with the influence of chemical reaction, variable thermal conductivity. So in order to fill this research gap in this present article we aimed to analyse the entropy generation for the flow of Williamson fluid along the inclined micro porous channel with the influence of chemical reaction which includes activation energy in it and by taking thermal conductivity as a variable property.

2 Mathematical flow formulation

Convection of Williamson fluid is along an inclined microchannel with the inclination 'α' is taken for the study and the two plates of the microchannel is separated by distance 'a' and the temperature of the plates are at distinct temperatures T_h and T_a. The physical configuration of the considered flow model is displayed in figure 1.

Following are the assumptions we adopted for our present study,

- The flow is taken as viscous, laminar and steady.
- The thermal conductivity of the fluid taken as variable property whereas the density and viscosity of the fluid are taken as constant.
- The flow is carried out through the porous medium with the influence of magnetic field.

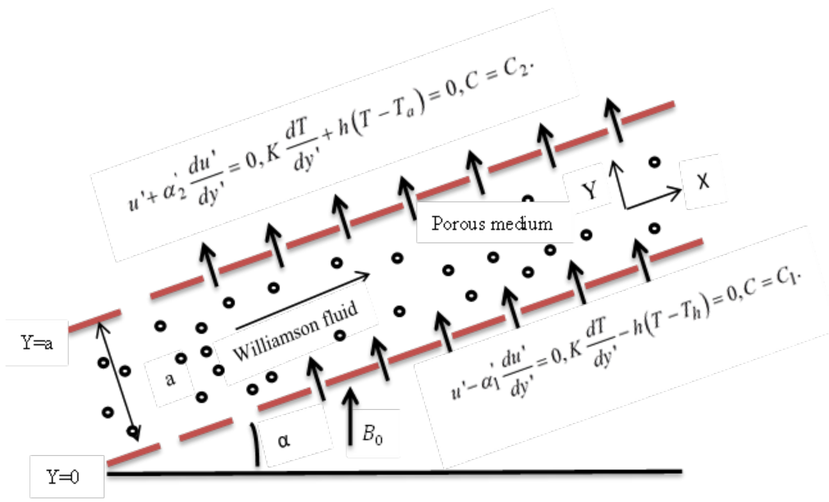

Fig 1. Flow configuration

Under the above mentioned conditions the momentum, temperature distribution and chemical reaction equations in dimensional form takes the form as below[5,18]:

$$-\frac{dp}{dx}+\mu\frac{d^2u'}{dy'^2}+\sqrt{2}\mu\Gamma\frac{du'}{dy'}\frac{d^2u'}{dy'^2}+\rho g\beta\left(T-T_a\right)\sin\alpha \\ +\rho g\beta\left(C-C_0\right)-\sigma B_0^2u'-\frac{\mu}{k}u'=0 \tag{1}$$

$$K\left(\frac{d^2T}{dy'^2}\right)+\mu\left(\frac{du'}{dy'}\right)^2+\frac{\mu}{k}u'^2+\sigma B_0^2u'^2\pm Q\left(T-T_a\right)=0 \tag{2}$$

$$D\left(\frac{d^2C}{dy'^2}\right)-\gamma(C-C_0)=0 \tag{3}$$

Employed boundary conditions for the considered flow model in dimensional form,

$$u'-\alpha_1'\frac{du'}{dy'}=0|_{y=0},u'+\alpha_2'\frac{du'}{dy'}=0|_{y=a} \tag{4}$$

$$K\frac{dT}{dy'}-h\left(T-T_h\right)=0|_{y'=0},K\frac{dT}{dy'}+h\left(T-T_a\right)=0|_{y'=a}, \tag{5}$$

$$C=C_1|_{y'=0},C=C_2|_{y'=a} \tag{6}$$

Thermal conductivity of the Williamson fluid is dependent on temperature and varied throughout the flow is denoted as:

$$K=k(1+\varepsilon\theta) \tag{7}$$

Where ε - variable thermal conductivity parameter.

Dimensionless variables used for the non-dimensionalization

$$y=\frac{y'}{a},\quad \theta=\frac{(T-T_a)}{(T_h-T_a)},\quad u=\frac{u'}{u_0} \tag{8}$$

Dimensionless form of the governing equations (1, 2) using (6) are

$$\left(1+We\frac{du}{dy}\right)\frac{d^2u}{dy^2}+\lambda\sin\alpha\theta+\lambda_C\phi-M^2u-\sigma^2u+P=0 \tag{9}$$

$$(1+\varepsilon\theta)\frac{d^2\theta}{dy^2}+Br\left(\frac{du}{dy}\right)^2+BrM^2u^2+Br\sigma^2u^2\pm\psi\theta=0 \tag{10}$$

$$\frac{d^2\phi}{dy^2}-\xi^2\phi=0 \tag{11}$$

Boundary conditions in dimensionless form,

$$u-\delta\frac{du}{dy}=0|_{y=0},u+\delta\frac{du}{dy}=0|_{y=1} \tag{12}$$

$$\frac{d\theta}{dy}-Bi(\theta-1)=0|_{y=0},\frac{d\theta}{dy}+Bi\,\theta=0|_{y=1} \tag{13}$$

$$\phi=-1|_{y=0},\phi=1|_{y=1}. \tag{14}$$

where Re = $\frac{\rho a u_0}{\mu_0}$ Reynolds number, Gr = $\frac{g\beta(T_h-T_a)a^3\rho^2}{\mu_0^2}$, Bi = $\frac{ah}{k}$ Biot number, $\theta_h=\frac{T_h}{T_a}$ characteristic temperature ratio, Br= E_cPr - Brinkman number, $Gr_c=\frac{g\beta(C_1-C_0)a^3\rho^2}{\mu_0^2}$, $\varepsilon=$ variable thermal conductivity parameter. $\lambda=\frac{Gr}{Re}$ mixed convection parameter, $\sigma^2=\frac{a^2}{K}$ porous parameter, $\lambda_C=\frac{Gr_c}{Re}$ concentration buoyancy parameter. ξ = concentration parameter. δ = slip parameter.

2.1 Analysis of thermodynamic second law

Entropy is the evaluation of the irreversibility in the model. To obtain the efficient flow model it necessary to minimize the entropy production in the model. Entropy analysis can be done second law of thermodynamics. In present considered flow model magnetic field, permeability, viscous dissipation and heat transfer are affecting the entropy production in the flow.

The equation of irreversibility takes the form as:

$$E_g=\frac{1}{T_a^2}K\left(\frac{dT}{dy'}\right)^2+\frac{\mu}{T_a}\left(\frac{du'}{dy'}\right)^2+\frac{\mu}{T_aK}u'^2+\frac{1}{T_a}\sigma B_0^2u'^2 \tag{15}$$

On non-dimensionalising, equation of irreversibility of the system reduces to:

$$Ns=\frac{E_g}{E_{g0}}=(1+\varepsilon\theta)\left(\frac{d\theta}{dy}\right)^2+\frac{Br}{T_P}\left(\frac{du}{dy}\right)^2+\frac{Br}{T_P}M^2u^2+\frac{Br}{T_P}\sigma^2u^2 \tag{16}$$

where $E_{g0}=\frac{k}{T_a^2}\frac{\triangle T^2}{a^2}$ – characteristic entropy generation.

Above equation can be written as,

$$Ns=N_h+N_v+N_P \tag{17}$$

N_h= Entropy due to transfer of heat, N_v=Entropy due to viscous dissipation, N_P= Entropy due to porous medium.

The ratio of entropy production due to heat transfer to the overall entropy production is denoted as Bejan number, it is defined as:

$$Be=\frac{N_h}{N_h+N_v+N_P} \tag{18}$$

3 Numerical procedure

The reduced system of nonlinear equations (9), (10) and (11) and the corresponding equations of the employed boundary conditions for the considered flow are (12), (13) and (14) to solve these equations we used the Bvp4c Technique which involves the finite difference method. By this, we transform the ODE, by assigning new variables.

Assuming, $u = y_1,\ u' = y_2,\ \ \theta = y_3,\ \theta' = y_4,$

Transforming the equations (9), (10) and (11) using the above assumptions we can write,

$$\begin{bmatrix} y_1' \\ y_2' \\ y_3' \\ y_4' \\ y_5' \\ y_6' \end{bmatrix} = \begin{bmatrix} y_2 \\ \sigma^2 y_1 + M^2 y_1 - \lambda \sin\alpha - \lambda_C y_5 - P/(1+We y_2) \\ y_4 \\ \frac{-Br y_2^2 - Br\sigma^2 y_1^2 - BrM^2 y_1^2 - \psi y_3}{1+\varepsilon y_3} \\ y_6 \\ \xi^2 \phi \end{bmatrix}$$

and the boundary conditions can be written as,

$y_1(0) = \delta y_2,\ y_1(a) = -\delta y_2,\ y_4(0) = Bi(\theta - 1),\ y_4(a) = -Bi\theta,\ y_6(0) = -1,\ y_4(a) = 1.$

4 Results and discussion

The considered flow model is solved numerically and the solutions for velocity, temperature, entropy generation and Bejan number are obtained and the influence of chemical reaction parameter, variable thermal conductivity parameter, Biot number, mixed convection parameter, concentration buoyancy parameter, and inclination are discussed using the graphs. The results calculated by maintaining constant values as: P=1, Br=0.7, λ=2, Bi=1, We =0.5, M=1, $\alpha=\pi/4$, σ =0.5, ϕ=1, ε=0.1, λ_C=2, ξ=0.5, δ=0.1.

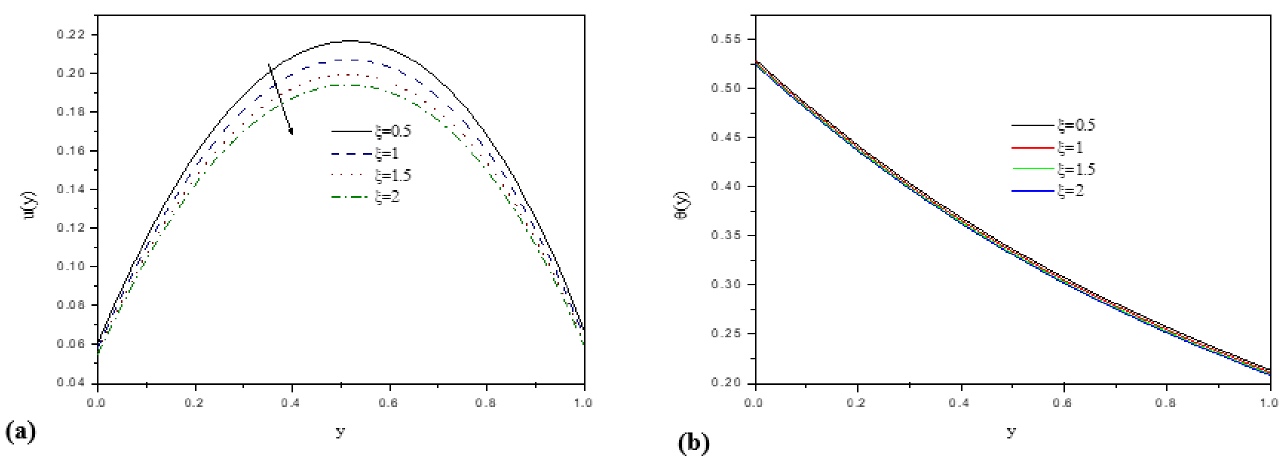

Fig 2. Impact of ξ on the velocity (a) and temperature (b)

Figure 2a is the velocity profile for the variation of chemical reaction parameter. With the increase in the concentration the movement of the fluid particles is reduced due to this we observe in the figure that with the accelerated values of ξ velocity of the fluid decelerates. In Figure 2b with magnification in chemical reaction parameter due to absorption of heat the temperature profile supresses. Variation of velocity and temperature with the influence of Br is displayed Figure 3a and Figure 3b. As the Brinkman number increases that leads to the viscous dissipation hence here it is noted that Brinkman number makes significant effect on the flow and hence here we observe that with the improved estimations of Brinkman number the temperature and velocity profile shows the enhanced nature.

Figure 4a to Figure 4c present the temperature profile, entropy production and Bejan number for the different values of ε. It is interestingly observed in Figure 4a that with the enhanced values of the ε the temperature profile accelerates and with the higher values of non-constant thermal conductivity parameter entropy production rises in the system it can be seen in Figure 4b. Entropy generation due to heat transfer is more compared to the overall entropy generation in the system hence

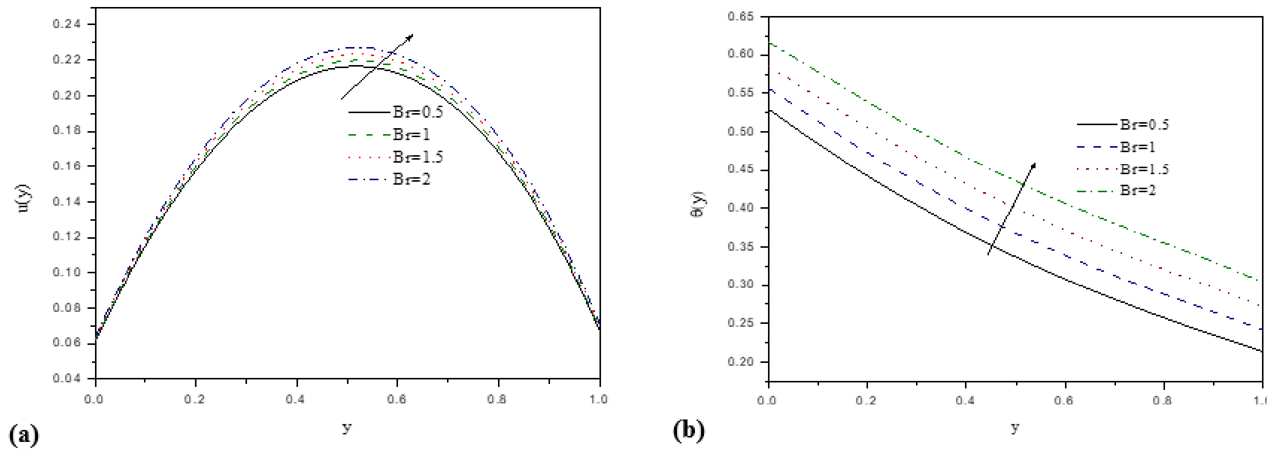

Fig 3. Impact of Br on the velocity (a) and temperature (b)

with the exaggerated values of the variable thermal conductivity parameter the Bejan number profile magnifies it is displayed in Figure 4c. To minimize the entropy production in the model the ε should be maintained at the lower values.

Figure 5 is the display of the velocity and temperature profile variation for the different estimations of concentration buoyancy parameter, λ_C is the ratio of the concentration Grashof number to the Reynolds number, increase in λ_C imply that reduction in the inertial force and the dominance of the buoyancy force which makes significant effect on the fluid flow hence the velocity and temperature of the fluid shows the accelerated nature.

Figure 6a and Figure 6b present the nature of the velocity, temperature of the fluid for the variation of mixed convection parameter. The λ is the ratio of the buoyancy force to the inertial force. As $\lambda \to 0$ lead to the dominance of the inertial force, this is responsible for the free convection in the channel. Higher estimations of λ imply the enhancement in the buoyancy force that drives the forced convection in the channel. In figure 6a it is noted that augmented values of the λ lead to the magnification in the velocity. λ enhances the temperature hence thermal profile of the fluid shows the accelerated nature it can be seen in Figure 6b. For the efficient energy transfer entropy production in the system should be lower, with an increase in λ entropy production is increasing in Figure 6c, to reduce the entropy λ must be maintained at lower values. In Figure 6d it is observed that overall entropy production in the system is much high than the entropy production due to heat transfer hence the Bejan number profile shows the declining nature with the increased values of mixed convection parameter.

Figure 7 is the variation of velocity, entropy production and Bejan number against the Weinsenberg number. Increment in We leads to the enhancement the viscosity and the thickness of the fluid due to this movement of fluid particle is reduced hence it reported in Figure 7a that initially velocity of the fluid supresses and as it moves near the another wall of the microchannel fluid velocity enhances. Dual trend is observed in Figure 7b as the Weinsenberg number accelerates the entropy production reduces and as the fluid moves towards another wall of the channel entropy production enhances and similar dual trend nature is observed in Figure 7c for the Bejan number.

Figure 8a and Figure 8b is the graphs of velocity and thermal profile for the variation of Biot number. The Biot number consists of the ratio of the resistance of the solid boundary to the boundary layer hence Bi makes a significant effect on the flow of Williamson fluid. Figure 8a is the velocity profile here we observe that the velocity profile shows the accelerated nature for the higher estimations of Bi. Figure 8b is the temperatures profile it showing the dual trend at the walls of the microchannel. At the lower wall temperature is an increasing function and as it moves towards another wall temperature starts decaying. Entropy production increases with the increment in Biot number hence for the efficient energy transfer Biot number should be kept small it can be noticed in Figure 8c. Due to the dominance of the entropy generation due to heat transfer, in Figure 8d we found that the Bejan number showing the enhanced trend with the higher estimations of Biot number.

The Velocity of the considered fluid shows the enhanced nature for the higher inclinations of the channel and similar nature is observed for the thermal field this is due to the increase in the applied force on the flowing fluid. In figure 9a we are noticing that entropy production at both the walls exaggerates with an increase in α. In figure 9b the Bejan number profile shows the diminishing nature this is due to the dominance of the total irreversibility with the irreversibility due to heat transfer. To minimize the entropy generation in the system inclination has to be maintained at a lower rate.

The comparison of numerical values of velocity with the published work of Gireesha et al. [18] and Makinde and Eegunjobi [27].

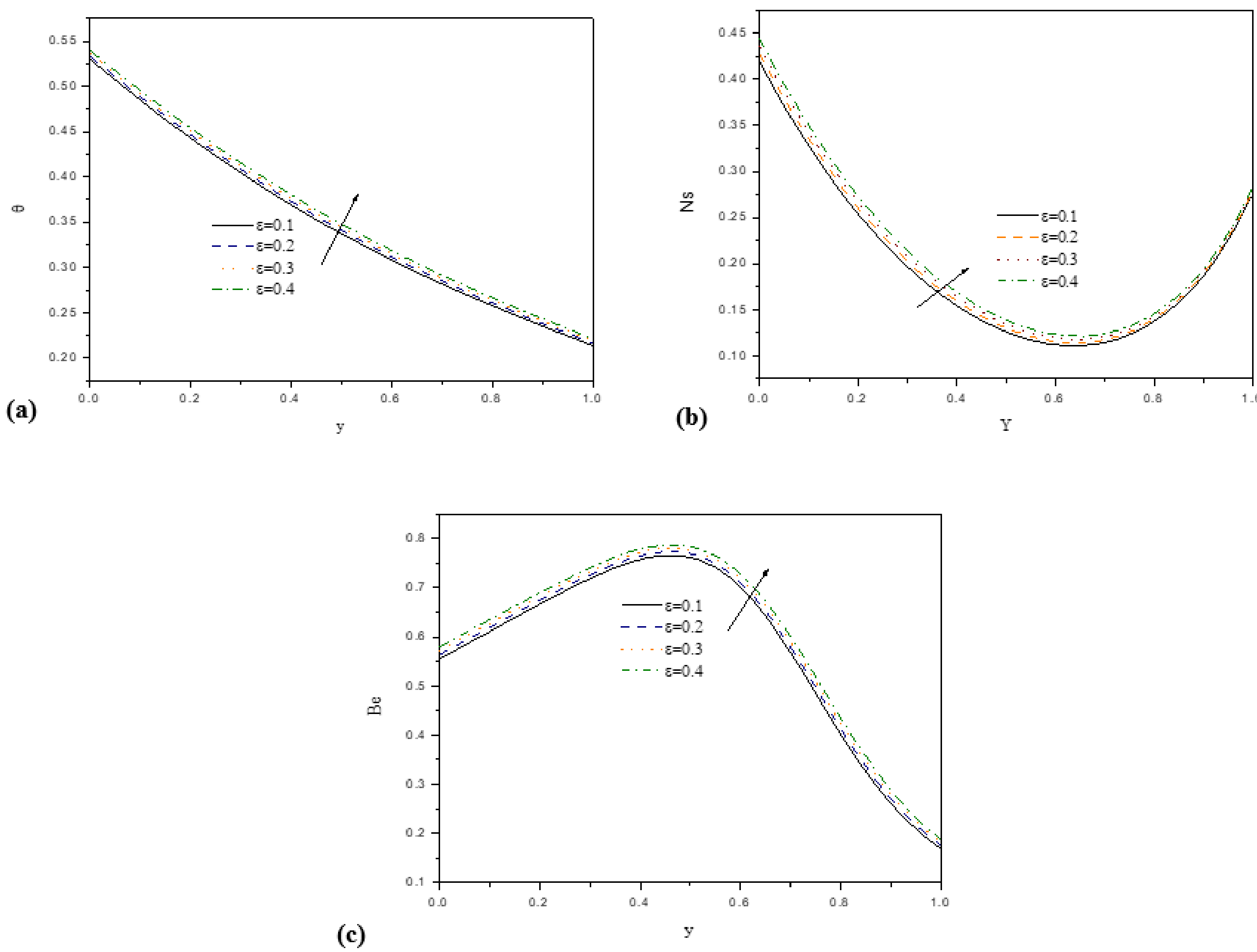

Fig 4. Impact of ε on the temperature (a), entropy generation (b) and Bejan number (c)

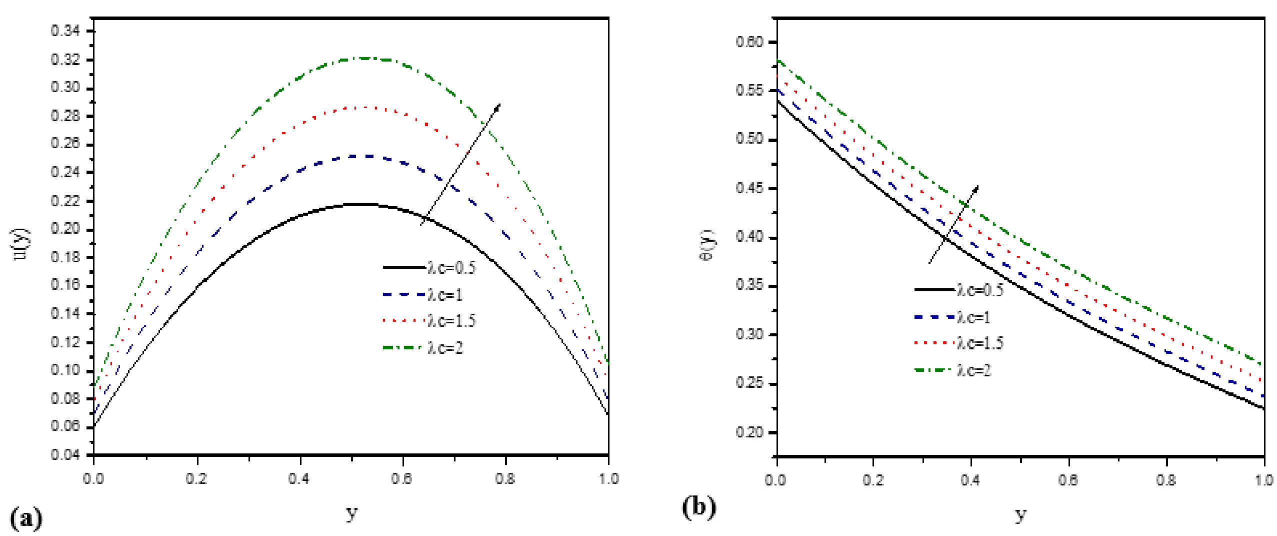

Fig 5. Impact of λ_C on the velocity (a) and temperature (b)

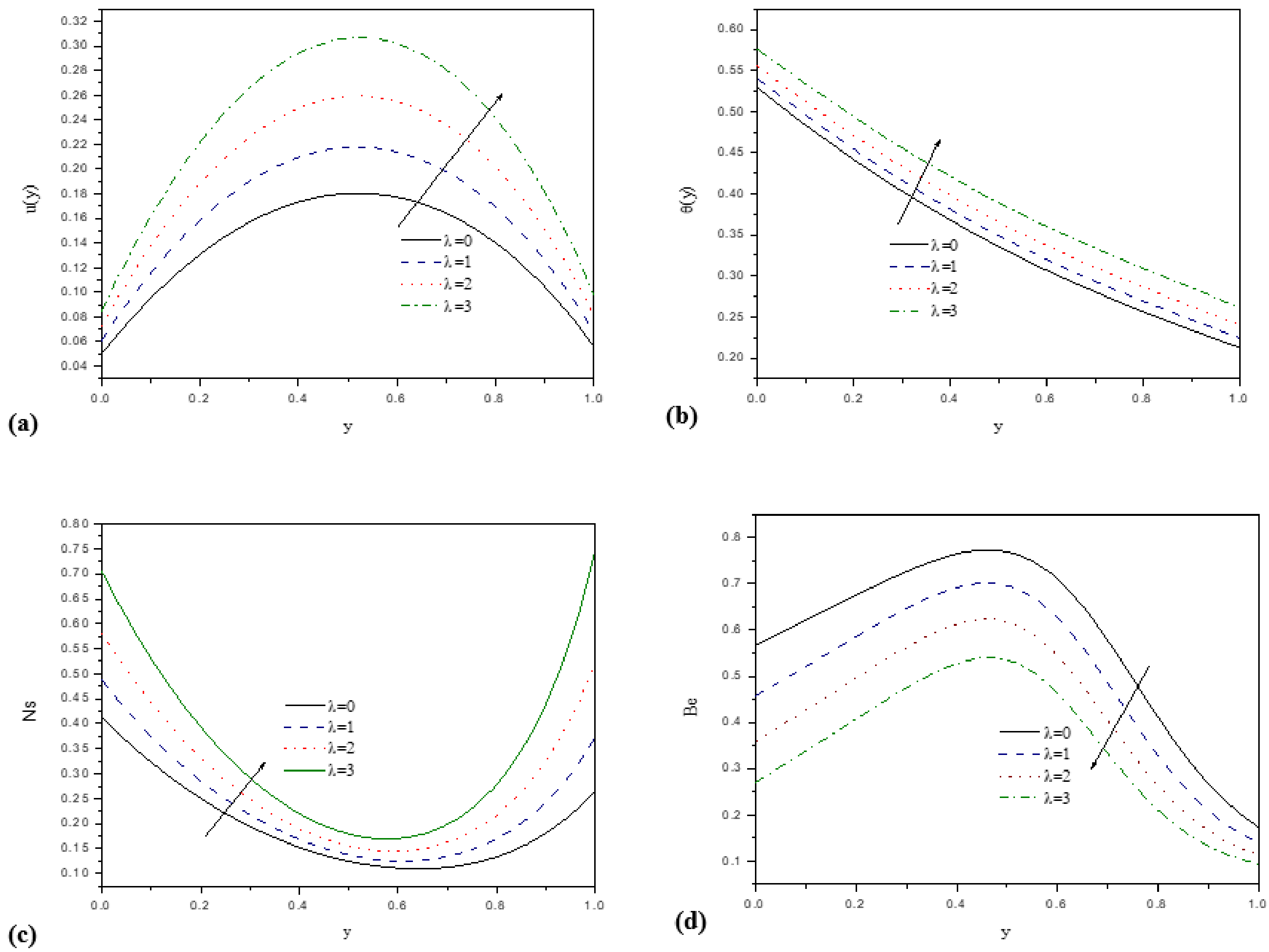

Fig 6. Impact of λ on the velocity (a), temperature (b), entropy generation (c) and Bejan number (d).

Table 1. The comparison ofnumerical values of velocity with the published work of Gireesha et al. [18] and Makinde and Eegunjobi [27]

y	Makinde and Eegunjobi [27]	Gireesha et al. [18]	Present study
0	0.0000000	0.0000000	0.000000
0.2	0.0711487	0.0711487	0.071149
0.4	0.0963903	0.1137694	0.113770
0.6	0.1215460	0.1215460	0.121550
0.8	0.0867637	0.0867637	0.086764
1	0.0000000	0.0000000	0.000000

the display of the numerical solution obtained for the velocity for the limiting case. The obtained results are in good agreement with theexisting studies Gireesha et al. [18] and Makinde and Eegunjobi [27]. The result is obtained by maintaining We= λ_C= λ=M=σ=ψ=0. P=1, Br=1.

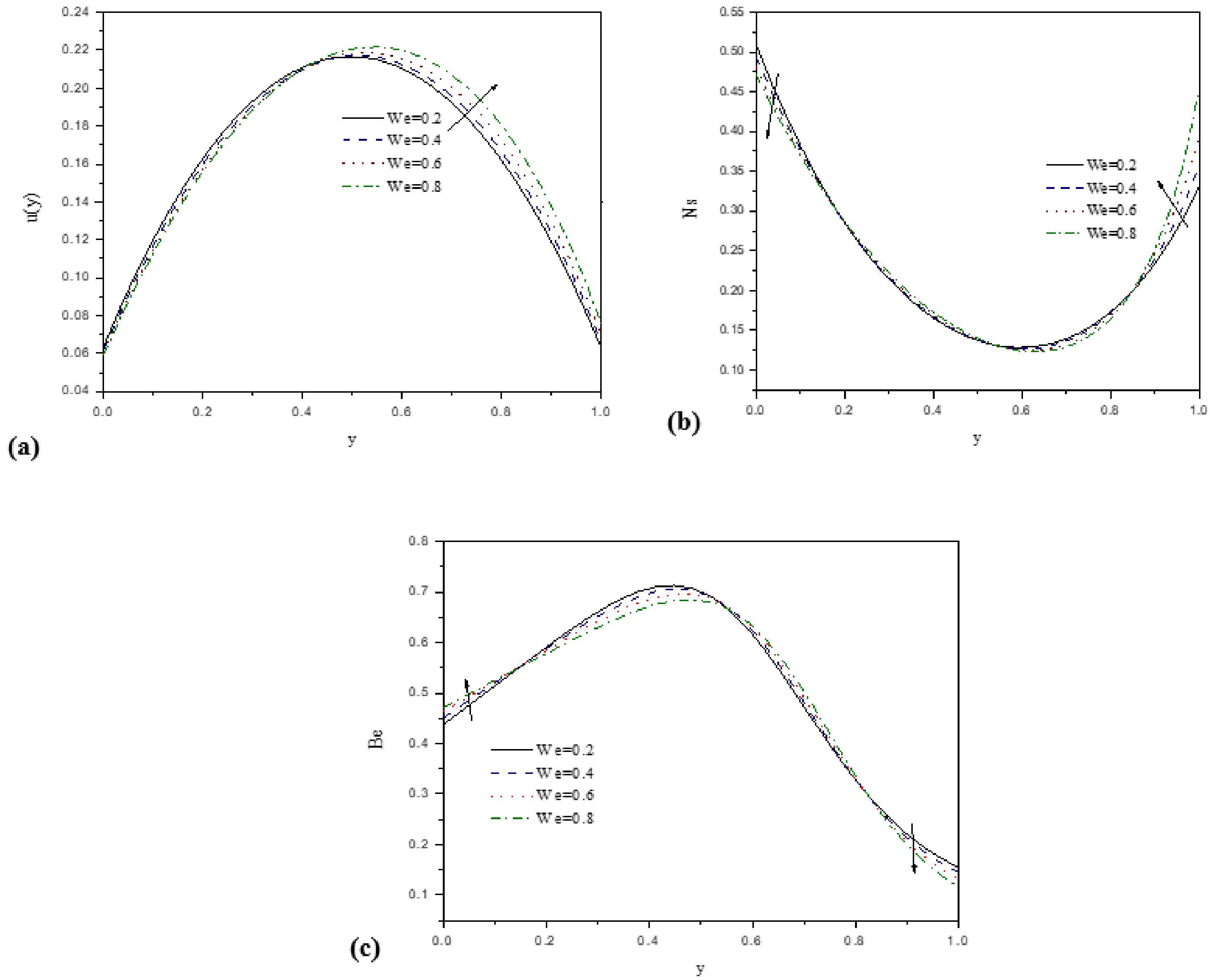

Fig 7. Impact of We on the velocity (a), entropy generation (b) and Bejan number (c)

5 Conclusion

The entropy generated in the mixed convection of Williamson fluid along an inclined porous microchannel with the influence of variable thermal conductivity and chemical reaction in the presence of magnetic field is studied. The major outcomes of the present study are, porous

- ξ reduces the temperature and velocity of the fluid.
- Concentration buoyancy parameter, Br, ε, We, Bi and λ improves the velocity and temperature of the fluid.
- Magnified values of ε, λ, Bi and inclination of the channel enhance the entropy production.
- Increase in We shows the dual trend for the entropy production and Bejan number.
- Due to the dominant of the entropy production of the system to the entropy production due to heat transfer, Bejan number shows the declining trend for the λ. channel inclination.
- Bejan number shows upward nature for Bi and variable thermal conductivity parameter.

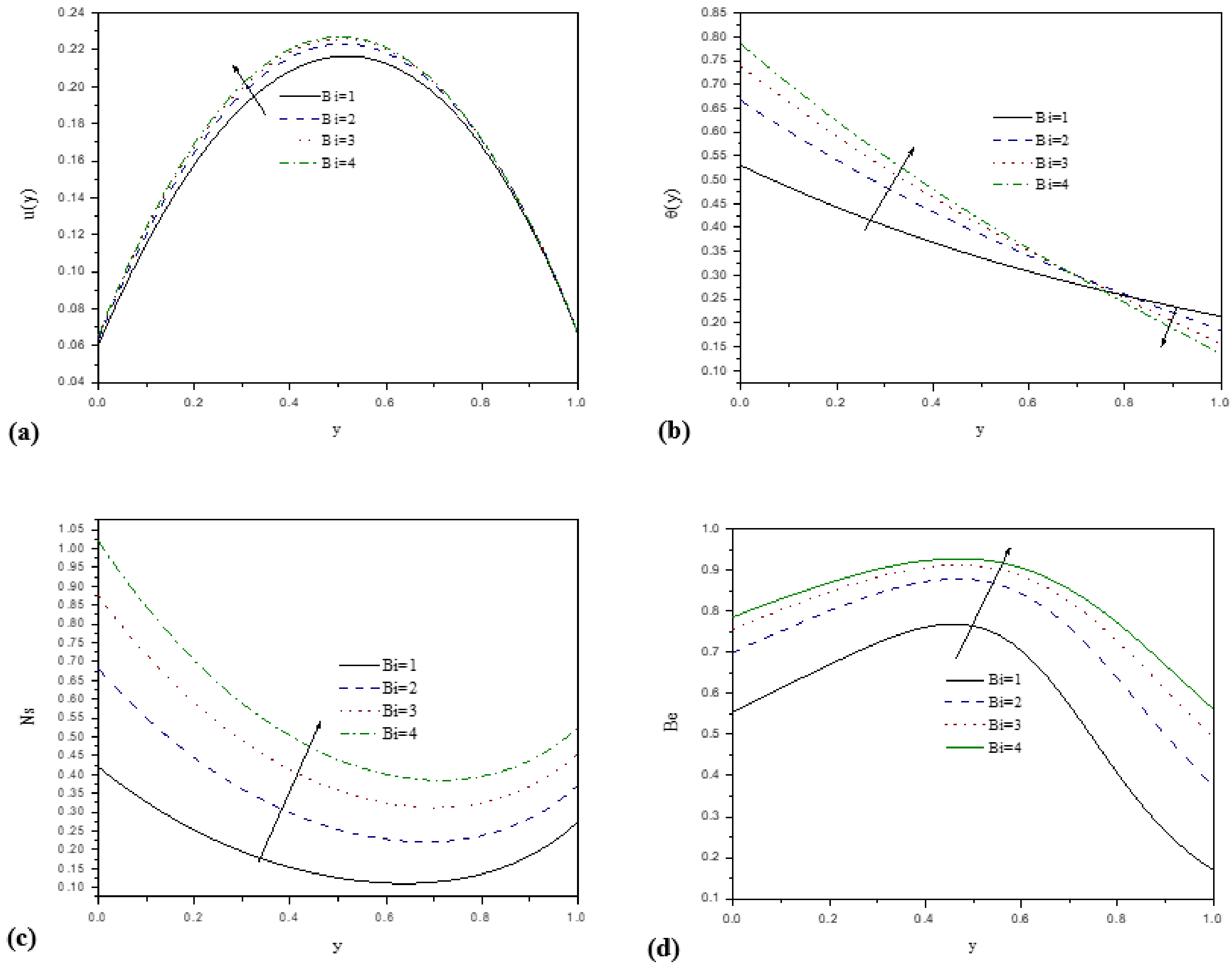

Fig 8. Impact of Bi on the on the velocity (a), temperature (a), entropy generation (b) and Bejan number (c)

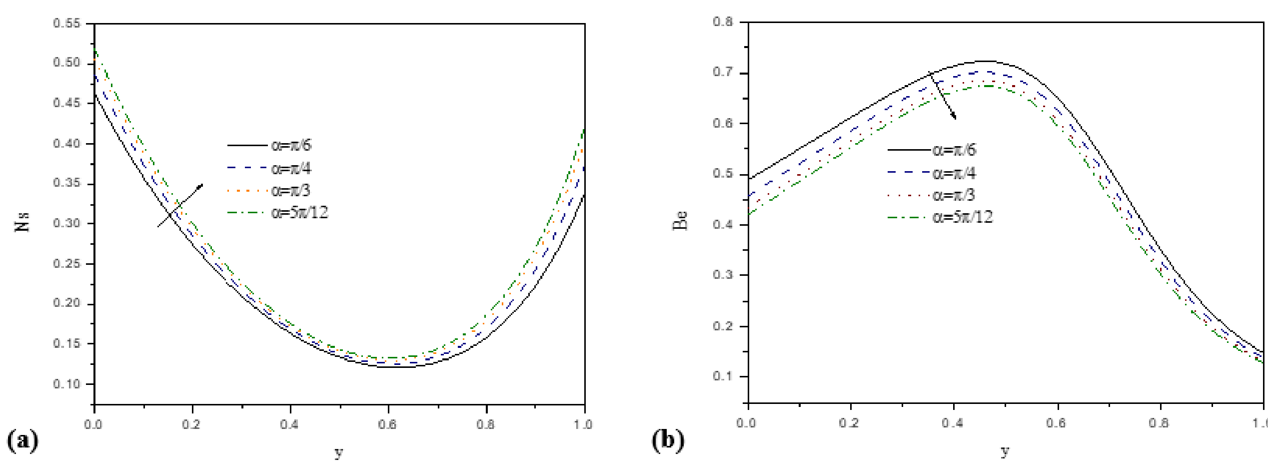

Fig 9. Impact of α on the on the entropy generation (a) and Bejan number (b)

6 Nomenclature

a - distance between the plates (m);
Bi - Biot number;
Br - Brinkman number
g - acceleration due to gravity (m/s^2)
Gr - Grashof number;
K - thermal conductivity;
M - Hartman number;
P - pressure (kg$m^{-1}s^{-2}$)
Re - Reynolds number;
T - temperature(K);
T_h - hot fluid temperature;
T_P - characteristic temperature ratio;
T_a - ambient temperature;
u - velocity (m/s);
We - Wessenberg number;
Greek symbols:
λ - mixed convection parameter;
λ_C - concentration buoyancy parameter;
θ - dimensionless temperature;
α - channel inclination;
ε - variable thermal conductivity parameter;
μ - dynamic viscosity $(kg\ m^{-1}s^{-1})$;
ξ - chemical reaction parameter;
ϕ - concentration parameter;
ψ - heat generation/absorption parameter;
σ - permeability parameter;

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