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RESEARCH ARTICLE

The Evaluation of the Rotating Metrics with $a \neq 0$ from the Mass Function $\hat{M}(u,r)$ with the reference to the Theory of General Relativity

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Abstract

Objectives: With the reference to the Einstein's theory of general relativity (1915), the evaluation of the rotating metrics such as Newman – Penrose Spin – Coefficients or NP Spin – Coefficients, the Ricci Scalars, the Weyl Scalars are designed with $a \neq 0$from the mass function or metric function $\hat{M}(u,r)$. **Methods:** The methods / analysis adapted are the theoretical and mathematical analysis on the Einstein's theory of general relativity. **Findings**: The Newman – Penrose Spin Coefficients (NP Spin – Coefficients), the Ricci Scalars, and the Weyl Scalars for the rotating metrics $\hat{M}(u,r)$ with $a \neq 0$ has been evaluated. Given by Wang and Wu (1999), the expanded form of the mass function or metric function with $a \neq 0$ has been used to evaluate the rotating metrics – NP Spin – Coefficients, the Ricci Scalars and the Weyl Scalars for $a \neq 0$. The outcome is that the evaluation of rotating metrics with $a \neq 0$ i.e. all the Newman – Penrose Spin Coefficients (NP Spin – Coefficients), the Ricci Scalars and the Weyl Scalars greatly simplifies the analysis of the theory of general relativity. **Novelty:** From the expended form of the mass function or metric function $\hat{M}(u,r)$ with $a \neq 0$ given by Wang and Wu (1999), all NP Spin – Coefficients, the Ricci Scalars, the Weyl Scalars has been derived for the option $a \neq 0$. This paper evaluates the rotating metrics with all NP Spin – Coefficients, the Ricci Scalars and the Weyl Scalars with which greatly simplifies the analysis of the theory of general relativity. Also it is new way of formulation of the theory of general relativity with $a \neq 0$.

Keywords: The Einstein's theory of general relativity; The mass function or metric function; The Newman – Penrose Spin - Coefficients (NP Spin – Coefficients); The Ricci Scalars; The Weyl Scalars.

1 Introduction

The Newman – Penrose Spin – Coefficients (the NP Spin – Coefficients), the Ricci Scalars, the Weyl Scalars for the rotating metrics with mass function $\hat{M}(u,r)$or metric function $\hat{M}(u,r)$ can be written as given below[1,2] :

The Newman – Penrose Spin – Coefficients or NP Spin – Coefficients with mass function or metric function $\hat{M}(u,r)$ are:

$$\begin{gathered} k^* = \sigma = \lambda = \varepsilon = 0 \\ \rho^* = -\frac{1}{\bar{R}}, \mu^* = -\frac{\Delta}{2\bar{R}R^2} \\ \alpha = \frac{(2ai - R\cos\theta)}{2\sqrt{2}\bar{R}\bar{R}\sin\theta}, \beta = \frac{\cot\theta}{2\sqrt{2}R} \\ \pi = \frac{ia\sin\theta}{\sqrt{2}\bar{R}\bar{R}}, \tau = -\frac{ia\sin\theta}{\sqrt{2}R^2} \\ \gamma = \frac{1}{\sqrt{2}\bar{R}R^2}\left[\left(r - \hat{M} - r\hat{M}_{,r}\right)\bar{R} - \Delta^*\right] \\ v = \frac{1}{\sqrt{2}\bar{R}R^2} i\, ar \cdot \sin\theta \hat{M}_{,u} \end{gathered} \tag{1}$$

The actual notations defined as the k^*, ρ^*, μ^*NP Spin – Coefficients k, ρ, μ. The notations defined ascan be respectively treated as energy density and null density in energy – momentum tensor for the rest of this paper. Now, the Ricci Scalars can be written as follows[3,4] :

$$\begin{gathered} \phi_{00} = \phi_{01} = \phi_{10} = \phi_{02} = \phi_{20} = 0 \\ \phi_{11} = \frac{1}{4R^2R^2}\left[4r^2\hat{M}_{,r} + R^2\left(-2\hat{M}_{,r} - r\hat{M}_{,rr}\right)\right] \\ \phi_{12} = \frac{1}{2\sqrt{2}R^2R^2}\left[ia\sin\theta\left\{R\hat{M}_{,u} - r\hat{M}_{,ru}\bar{R}\right\}\right] \\ \phi_{21} = \frac{-1}{2\sqrt{2}R^2R^2}\left[ia\sin\theta\left\{\bar{R}\hat{M}_{,u} - r\hat{M}_{,ru}R\right\}\right] \\ \phi_{22} = -\frac{1}{2R^2R^2}\left[2r^2\hat{M}_{,u} + a^2r\hat{M}_{,uu}\sin^2\theta\right\}\Bigg] \\ \Lambda^* = \frac{1}{12R^2}\left(2\hat{M}_{,r} + r\hat{M}_{,rr}\right) \end{gathered} \tag{2}$$

The Weyl Scalars can be written as follows:

$$\begin{gathered} \varphi_0 = \varphi_1 = 0 \\ \varphi_2 = \frac{1}{\bar{R}\bar{R}R^2}\left\{-R\hat{M} + \frac{\bar{R}}{6}\hat{M}_{,r}(4r + 2i\,a\cos\theta) - \frac{r}{6}\bar{R}\bar{R}\hat{M}_{,rr}\right\} \\ \varphi_3 = -\frac{ia\sin\theta}{2\sqrt{2}\bar{R}\bar{R}R^2}\left\{(4r + \bar{R})\hat{M}_{,u} + r\bar{R}\hat{M}_{,u\,r}\right\} \\ \varphi_4 = \frac{a^2r\sin^2\theta}{2\bar{R}\bar{R}R^2R^2}\left\{R^2\hat{M}_{,uu} - 2r\hat{M}_{,u}\right\} \end{gathered} \tag{3}$$

From all the above Newman – Penrose Spin – Coefficients or the NP Spin – Coefficients, we have found that, in general case, the rotating metrics possess actually a geodesic $(k^* = \varepsilon = 0)$ the shear free $(\sigma = 0)$, absolutely expanding $\theta \neq 0$and the non – zero twist $\left(\omega^{*2} \neq 0\right)$null vector l_a(Chandrasekhar, 1983)[5,6], where

$$\hat{\theta} \equiv -\frac{1}{2}(\rho + \bar{\rho}) = \frac{r}{R^2}, \omega^{*2} \equiv -\frac{1}{4}(\rho - \bar{\rho})^2 = -\frac{a^2\cos^2\theta}{R^2R^2} \tag{4}$$

And again, the energy momentum tensor for the rotating metric looks like as given below[7,8]:

$$T_{ab} = \mu l_{ab} l_b + 2\rho l_{(an_b)} + 2pm_{(a\bar{m}_b)} + 2\omega d_{(a\bar{m}_b)} + 2\bar{\omega} l_{(am_b)} \tag{5}$$

With the following Newman – Penrose Spin – Coefficients or NP Spin – Coefficients are as given below:

$$\begin{aligned} \mu &= -\frac{1}{KR^2\bar{R}^2}\left[2r^2\hat{M}_{,u} + a^2 r\sin^2\theta\hat{M}_{,uu}\right] \\ \rho &= \frac{2r^2}{KR^2\bar{R}^2}\hat{M}_{,r} \\ p &= -\frac{1}{K}\left[\frac{2a^2\cos^2\theta}{R^2\bar{R}^2}\hat{M}_{,r} + \frac{r}{\bar{R}^2}\hat{M}_{,rr}\right] \\ \omega &= -\frac{ia\sin\theta}{\sqrt{2}KR^2\bar{R}^2}\left[R\hat{M}_{,u} - r\bar{R}\hat{M}_{,ur}\right] \end{aligned} \tag{6}$$

All the above Newman – Penrose Spin – Coefficients or NP Spin – Coefficients have the relations with the Ricci Scalars as given below[9,10]

$$\begin{aligned} K\mu &= 2\phi_{22}, K\omega = -2\phi_{12} \\ K\rho &= 2\phi_{11} + 6\Lambda, Kp = 2\phi_{11} - 6\Lambda \end{aligned} \tag{7}$$

The result actually implies that when we obtain the Ricci Scalars $\phi_{11}, \phi_{12}, \phi_{22}$as in equation 2 for a given particular the space – time metric, we will be able to find μ, ρ and p which actually describe the energy momentum tensors.

The expanded form of rotating metrics given by Wang and Wu in 1999 for the rotating mass function $\hat{M}(u,r)$with the non – rotating solution $(a = 0)$ in the power of γ as given below:

$$\hat{M}(u,r) = \sum_{n=-\infty}^{\infty} q_n(u) r^n \tag{8}$$

Where $q_n(u)$ is an arbitrary function of μ. Wang and Wu considered the given above summation as in integral form when the spectrum index *'n'* is actually continuous in nature. Using the expression in equation 6, we can generate the rotating metrics for $a \neq 0$ as follows below by replacing the mass function or metric function $\hat{M}(u,r)$ of equation 8 with the help of arbitrary function $q_n(u)$. Thus we can rewrite equation 6 as given below[11,12]

$$\begin{aligned} \mu &= -\frac{1}{KR^2\bar{R}^2}\sum_{n=-\infty}^{\infty}\left[2q_n(u)_{,u} r^{n+2} + a^2\sin^2\theta q_n(u)_{,uu} r^{n+1}\right] \\ \rho &= \frac{2}{KR^2\bar{R}^2}\sum_{n=-\infty}^{\infty}(n+2)q_n(u) r^{n+1} \\ p &= -\frac{1}{K\bar{R}^2}\sum_{n=-\infty}^{\infty} nq_n(u) r^{n-1}\left[\frac{2a^2\cos^2\theta}{R^2} + (n-1)\right] \\ \omega &= -\frac{ia\sin\theta}{\sqrt{2}KR^2\bar{R}^2}\sum_{n=-\infty}^{\infty}\left[(R - n\bar{R})q_n(u)_{,u} r^n\right] \\ \gamma &= \frac{r}{\sqrt{2}\bar{R}R^2}\sum_{n=-\infty}^{\infty}\left[\left(1 - q_n(u) r^{n-1} - nq_n(u) r^{n-1}\right)\bar{R} - \Delta^*\right], \text{ where } \Delta^* = r^2 - 2r\sum_{n=-\infty}^{\infty} q_n(u) r^n + a^2 \\ \nu &= \frac{ia\sin\theta}{\sqrt{2}\bar{R}R^2}\sum_{n=-\infty}^{\infty} q_n(u)_{,u}{}^{nn+1} \\ \alpha &= \frac{(2ai - R\cos\theta)}{2\sqrt{2}\bar{R}\bar{R}\sin\theta}, \beta = \frac{ia\sin\theta}{\sqrt{2}\bar{R}R}, \tau = -\frac{ia\sin\theta}{\sqrt{2}R^2} \end{aligned} \tag{9}$$

The Ricci Scalars can be written as follows [13,14]

$$\begin{aligned}
\phi_{00} = \phi_{01} = \phi_{10} = \phi_{02} = \phi_{20} = 0 \\
\phi_{11} = \frac{1}{4R^2R^2}\sum_{n=-\infty}^{\infty} q_n(u)r^{n-1}\left[4nr^2 + R^2(-2n - n(n-1)\right] \\
\phi_{12} = \frac{ia\sin\theta}{2\sqrt{2}R^2R^2}\sum_{n=-\infty}^{\infty} q_n(u)_{,u}r^n(R - n\bar{R}) \\
\phi_{21} = \frac{-ia\sin\theta}{2\sqrt{2}R^2R^2}\sum_{n=-\infty}^{\infty} q_n(u)_{,u}r^n(R - n\bar{R})(\bar{R} - nR) \\
\phi_{22} = -\frac{1}{2R^2R^2}\sum_{n=-\infty}^{\infty} r^{n+1}\left[rq_n(u)_{,u} + a^2q_n(u)_{,u\,u}\sin^2\theta\right] \\
\Lambda^* = \frac{1}{12R^2}\sum_{n=-\infty}^{\infty} nq_n(u)r^{n-1}(n+1)
\end{aligned} \tag{10}$$

The Weyl Scalars can be written as follows [14,15] :

$$\begin{aligned}
\varphi_0 = \varphi_1 = 0 \\
\varphi_2 = \frac{1}{\bar{R}\bar{R}R^2}\sum_{n=-\infty}^{\infty} q_n(u)r^{n-1}\left\{-Rr + \frac{\bar{R}}{6}n(4r + 2i\,a\cos\theta) - \frac{1}{6}\bar{R}\bar{R}n(n-1)\right\} \\
\varphi_3 = -\frac{ia\sin\theta}{2\sqrt{2}\bar{R}\bar{R}R^2}\sum_{n=-\infty}^{\infty} q_n(u)_{,u}r^n\{(4r+\bar{R}) + n\bar{R}\} \\
\varphi_4 = \frac{a^2r\cdot\sin^2\theta}{2\bar{R}\bar{R}R^2R^2}\sum_{n=-\infty}^{\infty} r^n\left(q_n(u)_{,u\,u}\left(R^2 - 2rq_n(u)_{,u}\right)\right)
\end{aligned} \tag{11}$$

Hence, all the Newman – Penrose Spin – Coefficients or NP Spin – Coefficients are found above with the condition with $a \neq 0$ from the Mass Function $\hat{M}(u,r)$with reference to the Theory of General Relativity. This theoretical analysis is ideal for experimental set up for progress further in the theory of general relativity.

2 Conclusion

With given a start of the rotating metrics with mass function or metric function $\hat{M}(u,r)$, the Newman – Penrose Spin – Coefficients (NP Spin – Coefficients), the Ricci Scalars, the Weyl Scalars are formulated. From all of the Newman – Penrose Spin – Coefficients (NP Spin – Coefficients) we found that, in general case, the rotating metrics actually possess qualities like a geodesic $(k^* = \varepsilon = 0)$, the shear free $(\sigma = 0)$, absolutely expanding $(\hat{\theta} \neq 0)$ and a non – zero twist $\left(\omega^{*2} \neq 0\right)$ null vector l_a. Once we have found the Ricci Scalars, we can always find the energy momentum tensors from the Ricci Scalars. The expanded form of the rotating mass function or the metric function $\hat{M}(u,r)$ given by Wang and Wu in 1999 is a non – rotating solution $a \neq 0$ in the power of γ. With help of the expanded form of mass function or metric function $\hat{M}(u,r)$, we actually have generated the rotating metrics with $a \neq 0$ i.e. all the Newman – Penrose Spin – Coefficients (NP Spin Coefficients), the Ricci Scalars and the Weyl Scalars. The evaluation the rotating metrics with all NP Spin – Coefficients, the Ricci Scalars and Weyl Scalars with $a \neq 0$ which greatly simplifies the analysis of the theory of general relativity. Also it is new way of formulation of the theory of general relativity with $a \neq 0$.

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