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Glued Hypertree: Comparative Analysis and Distance-Based Topological Descriptors

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Abstract

Objectives : To introduce a new interconnection network, Glued hypertree, and to discuss and analyze its physicochemical properties using distance-based topological descriptors. A comparative analysis between the Glued tree and Glued hypertree is carried away in this paper. **Methods:** We compare glued hypertree with glued tree using some topological parameters. The approach to finding the topological indices is to partition the edge set using Djokovic Wrinkler relation and thus reduce it to quotient graphs. **Findings:** Distance-based topological indices of Glued hypertree were calculated and also we have analyzed how glued hypertree is better than glued tree. **Novelty:** We have evaluated and compared the various topological indices of Glued hypertree using a graphical representation.

Keywords: Glued Hypertree; Distance-based Indices; Messages Traffic Density; Average Distance

1 Introduction

A tree is an acyclic graph. In computer science, trees are used to represent data structures. The Tree is the fundamentally used theoretical model in various fields such as information theory, artificial intelligence, combinatorial optimization, operations research, and theory of electrical and design networks [1]. Various biological units such as DNA sequences can be characterized as the nodes of vertices and their mutations or interconnections with other species could be characterized as edges of a graph. Such rooted trees of biological interest are called phylogenetic trees or evolutionary trees [2]. The Binary tree is one of the common types of tree structure where at most two nodes (i.e., children) arise from each node, parent node. A complete binary tree is a binary tree with exactly two children from each parent node. Binary trees are used in data structures for storing and searching because they can be easily used, manipulated, and retrieved. Glued tree of dimension n is a structure formed by identifying the leaves of two complete binary trees of dimension n. Glued trees are introduced and studied in [3,4]. A topological network which is a combination of hypercube and complete binary tree results in a hypertree. The distance-based and degree-based topological indices of hypertree is studied in [2,5].

The topological indices of hypertree help in the QSAR study of dendrimers and the topological properties of dendrimeric metal-organic networks consisting of very heavy atoms. The biological and chemical applications of hypertree are referred to [2].

In the field of Chemical graph theory, topological indices are an area of research that provides the physicochemical properties of different chemical structures, especially for drug compounds [6,7]. In this Contemporary world, the idea of finding topological indices can be applied to the antiviral drugs of covid like Chloroquine, Hydroxychloroquine [8,9]. Since the QSPR (Quantitative Structure-Property Relationship) is more economically efficient compared to testing in labs which requires more time and money, computing topological indices play a significant role. Recent studies in chemical graph theory help in calculating various indices of interconnection networks that have different applications in various fields. The basic concept of topological indices was first introduced by Wiener to find the boiling point of alkanes. Different approaches are used to find the various topological indices like the cut method [10,11]. The technique is to divide the edges into convex components and thus reduce the graphs into quotient graphs and is applied in various recent papers [12,13]. In [14], they refined the technique further to partition the edge set, converting to a quotient graph and then shrinking it to reduced graphs. We implement this idea in this article to find various distance-based topological indices, which is elaborately discussed in Section 5.

Glued tree structure was introduced and is helpful in quantum walks. Also, the hyprtree structure has many applications such as in dendrimers and chemical compounds with heavy atoms. Being inspired by this, we developed a new interconnection network, Glued hypertree which can be more helpful in the transmission of signals in biological networks. By evaluating various distance-based topological descriptors, the physicochemical properties of the new network can be determined which can be used in shaping the properties of the same.

In this paper, we have introduced an interconnection network, Glued hypertree. Section 2 introduces the new network, followed by its properties in section 3. Then comparative analysis between the Glued tree and Glued hypertree is carried away in the following section. Later, we recall the basic concepts and theorems needed and applied the techniques to find the distance-based topological indices of Glued hypertree.

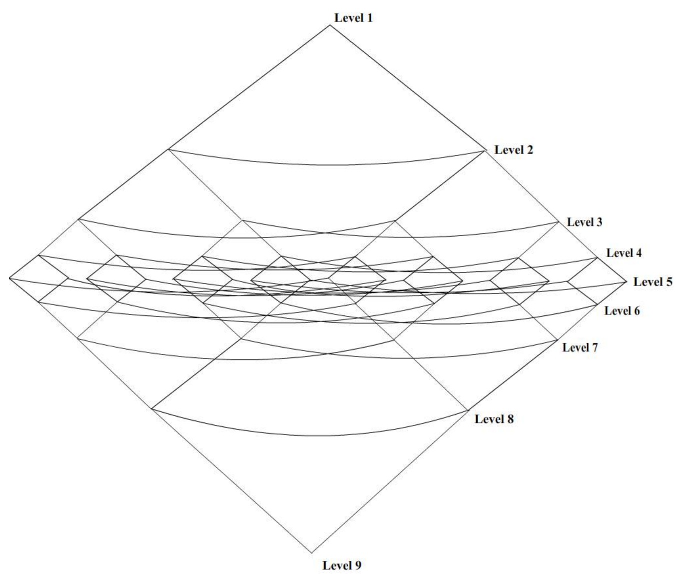

Fig 1. $GHT(5)$

2 Glued Hypertree

The basic skeleton for glued hypertree is a combination of hypertree, a complete binary tree, and a glued graph. Glued hypertree is formed by gluing two hypertrees of the same level. Refer to Figure 1.

Let us denote $2n-1$ level glued hypertree as $GHT(n),\ n \geq 2$. Level i and Level $2n-i$, $1 \leq i \leq n-1$, have the 2^i vertical edges connecting the nodes at Level $i+1$ and Level $2n-i-1$ respectively and 2^{i-1} horizontal edges connecting vertices with a difference of 2^{j-2}; $2 \leq j \leq n$ horizontal vertices from level 2 to level n and vice versa from level n to $2n-2$ as shown in Figure 1.

Glued hypertree has $3.2^{n-1}-2$ vertices and $3\left(2^{n-2}-2\right)+2^{n+1}$ edges. It is a non-planar graph. The diameter of an n-level glued hyper tree is $2(n-1)$. The edge connectivity of $GHT(n)=2$. Glued hypertree forms a hamiltonian path but is not a hamiltonian circuit and not pancyclic. Since it contains odd degree vertices, Glued hypertree is not eulerian.

3 Comparison study on Glued hypertree and Glued tree

In this section, we did a comparative analysis on glued tree and glued hypertree. We used some topological parameters like average vertex degree, network cost, network throughput, average distance, message traffic density to analyze glued tree and glued hypertree. The number of nodes and diameter of the glued tree and glued hypertree is $3.2^{n-1}-2$ and $2(n-1)$ respectively. The number of edges in a glued tree is $4.\ (2^{n-1}-1)$ and the number of edges in a glued hypertree is $3.\ \left(2^{n-2}-2\right)+2^{n+1}$.

3.1 Average vertex degree

The number of edges incident to a vertex is defined as vertex degree. The average vertex degree is the ratio of two times the number of edges to the number of vertices. The average vertex degree of the glued tree is $\frac{8(2^{n-1}-1)}{3.2^{n-1}-2}$ and the average vertex degree of glued hypertree is $\frac{6\left(2^{n-2}-2\right)+2^{n+2}}{3.2^{n-1}-2}$. The comparison graph for glued tree and glued hypertree is given in Figure 2.

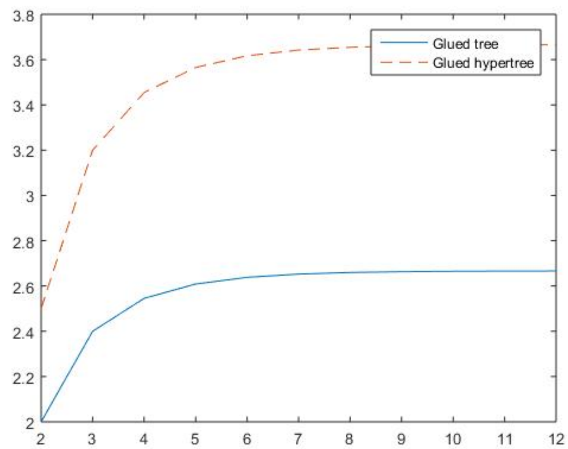

Fig 2. Average vertex degree of Glued tree and Glued hypertree

3.2 Network cost

The network cost of a graph is the product of diameter and vertex degree. The network cost of the glued tree is $\frac{16(2^{n-1}-1)(n-1)}{3.2^{n-1}-2}$ and network cost of glued hypertree is $\frac{2(n-1)(6\left(2^{n-2}-2\right)+2^{n+2})}{3.2^{n-1}-2}$. Figure 3 exhibits a comparison between glued tree and glued hypertree.

3.3 Network throughput

Network throughput is the ratio of total network bandwidth, proportional to the number of edges in the graph network to the diameter. For a glued tree of dimension n, network throughput is $\frac{2^{n-1}-1}{n-1}$ and for glued hypertree of dimension n, network throughput is $\frac{3\left(2^{n-2}-2\right)+2^{n+1}}{2(n-1)}$. Figure 4 gives the graphical representation of the glued tree and glued hypertree.

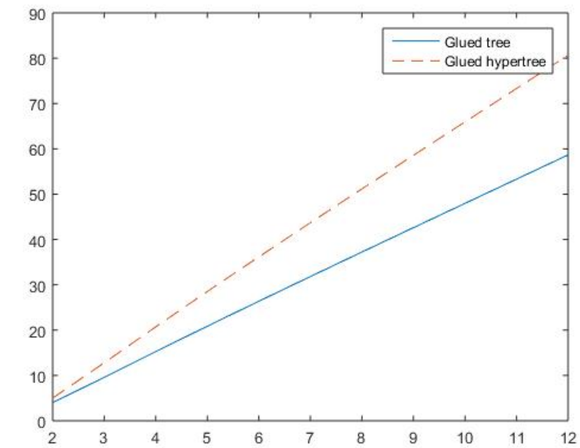

Fig 3. Network cost of Glued tree and Glued hypertree

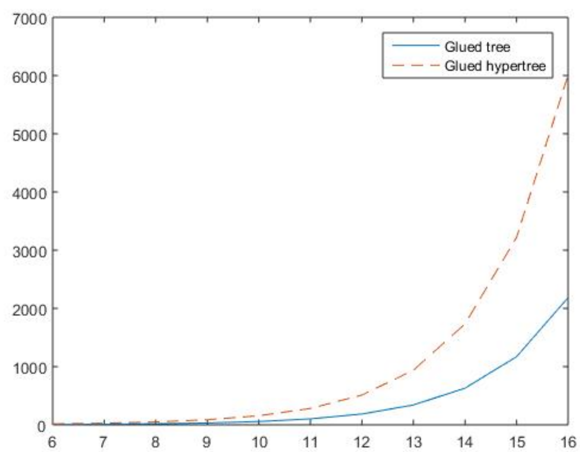

Fig 4. Network throughput of Glued tree and Glued hypertree

4 Topological indices and their terminology

Topological indices are used to characterize physicochemical properties of chemical structures such as boiling point, melting point, octanol partition coefficients, vapor pressures, etc. The graph G considered in the paper is a simple connected graph. The distance between two vertices, w and z, denoted by $d(w,z)$ is the number of edges in the shortest path connecting w and z. The degree of a vertex, w, is the number of edges incident to that vertex, w, denoted by $deg_G(w)$. Neighborhood of a vertex, z, is the set of vertices adjacent to z and is denoted by $N(z)$. For an edge $wz \in E(G)$, we define $N_w(wz|G) = \{u \in V(G) : d(w,u) < d(z,u)\}$ and $M_w(wz|G) = \{e \in E(G) : d(w,e) < d(z,e)\}$. The cardinality of $N_w(wz|G)$ and $M_w(wz|G)$ are denoted by $n_w(f)$ and $m_w(f)$, where $f = wz \in E(G)$. Distance-based topological indices and their definitions are given in Table 1. The vertex weight of a vertex u and the edge strength of an edge e is denoted by $w_v(u)$ and $s_e(e)$ respectively.

Definition 1 [12] "A strength-weighted graph $G_{sw} = (G, SW_V, SW_E)$ is a graph G together with a pair of strength-weighted functions (SW_V, SW_E) and defined as follows:

1. $SW_V = (w_v, s_v)$ where the vertex-weight function $w_v : V(G_{sw}) \to \mathbb{R}^+$ and the vertex-strength function $s_v : V(G_{sw}) \to \mathbb{R}^+$
2. $SW_E = (w_e, s_e)$, where the edge-weight function $w_e : E(G_{sw}) \to \mathbb{R}^+$ and the edge-strength function $s_e : E(G_{sw}) \to \mathbb{R}^+ \because$ "

Table 1. Distance-basedtopological indices

Topological indices	Mathematical expressions
Wiener [12]	$W(G)=\sum_{u,v\subseteq V(G)} w_v(u)\, w_v(v)\, d(u,v)$
Szeged [12]	$Sz(G)=\sum_{e=uv\in E(G)} s_e(e)\, n_u(e)\, n_v(e)$
Edge Szeged [12]	$Sz_e(G)=\sum_{e=uv\in E(G)} s_e(e)\, m_u(e)\, m_v(e)$
Edge vertex Szeged [12]	$Sz_{ev}(G)=\frac{1}{2} s_e(e)\, [n_u(e)\, m_v(e)+n_v(e)\, m_u(e)]$
Mostar	$Mo(G)=\sum_{e=uv\in E(G)} s_e(e)\, \lvert n_u(e)-n_v(e)\rvert$
Edge Mostar	$Mo_e(G)=\sum_{e=uv\in E(G)} s_e(e)\, \lvert m_u(e)-m_v(e)\rvert$
Padmakar Ivan [12]	$PI(G)=\sum_{e=uv\in E(G)} s_e(e)\, [m_u(e)+m_v(e)]$

Consider $w_v = w_e = s_e = 1,\ s_v\ = 0.$. Let $G_{sw} = (G,(w_v,s_v),s_e)$ be the strength weighted graph. Define degree of any vertex $v\in V(G_{sw})$ as $d_{G_{sw}}(u) = 2s_v(u)+\sum_{p\in N_{G_{sw}}(u)} s_e(up)$. For any edge $uv\in E(G_{sw})$, define

$n_v(e|G_{sw}) = \sum_{p\in N_v(e|G_{sw})} w_v(p)$

$m_v(e|G_{sw}) = \sum_{p\in N_v(e|G_{sw})} s_v(p)+\sum_{f\in M_v(e|G_{sw})} s_e(f)$.

For a graph G, the Djokovic-Winkler's relation on $E(G)$, Θ[10] is defined as follows if $d(a,c)+d(b,d)\neq d(a,d)+d(b,c)$, then $e=ab\in E(G)$ is Θ related with $f=cd\in E(G)$. The relation Θ is always reflexive and symmetric and its transitive closure Θ^* is an equivalence relation. The edges partitions into Θ^* classes and denote the Θ^* partition set of $E(G)$ be $\{E_i; 1\leq i\leq k\}$. For any $i\in[k]$, the quotient graph, G/E_i is a graph with its vertex set belonging to the components of $G-E_i$ and any two vertices x,y in G/E_i are adjacent if $xy\in E(G)$, where $x\in C_1$ and $y\in C_2$. A partition $X=\{X_1,X_2,...,X_r\}$ of $E(G)$ is coarser than $Y=\{Y_1,Y_2,...,Y_s\}$ if X_i is the union of one or more sets in Y.

Theorem 1[15] "Let (G,w) be a connected, weighted graph and let $\varepsilon=\{E_1,E_2,...,E_k\}$ be a partition of $E(G)$ coarser than Θ^*-partition. Then,

$W(G,w)=\sum_{i=1}^{k} W(G/E_i,w_i)$,

where $w_i: V(G/E_i)\to R^+$ is defined by $w_i(C)=\sum_{x\in C} w(x)$, for all connected components C of $G-E_i$. "

Theorem 2[14] "Let (G,w) be a connected, weighted graph, $a\in V(G)$ and $A=[a]_R$. Let (G',w') be defined with $G' = G-(A-a)$, $w'(a)=\sum_{x\in A} w(x)$ and $w'(x)=w(x)$ for any $x\notin A$. Then

$$W(G,w)=W(G',w')+\sum_{\{x,y\}\in\binom{A}{2}} 2w(x)w(y)."$$

Theorem 3[12] "For a connected strength-weighted graph $G_{sw}=(G,(w_v,s_v),s_e)$, let $E=E_1,E_2,...,E_k$ be a partition of $E(G)$ coarser than F. Let $X=W,Sz_v,Sz_e,Sz_{ev},PI,S$ and Gut. Then,

$X(G_{sw})=\sum_{i=1}^{k} X(G/E_i,(w_v^i,s_v^i),s_e^i)$,

where

• $w_v^i: V(G/E_i)\to R^+$ is defined by $w_v^i(C)=\sum_{x\in C} w_v(x), \forall\, C\in G/E_i$,

• $s_v^i: E(G/E_i)\to R^+$ is defined by $s_v^i(C)=\sum_{xy\in C} s_e(xy)+\sum_{x\in C} s_v(x),\ \forall\, C\in G/E_i$,

• $s_e^i: E(G/E_i)\to R^+$ is defined as the number of edges in E_i such that one end in C and the other end in D, for any two connected components C and D of G/E_i. "

We have used the same for calculating Mostar, edge Mostar, and total mostar indices.

Theorem 4. If $n\geq 2$, then

$$W(GHT(n))=\frac{108\left(2^{2n-4}\right)+24n-109\times 2^{2n}+108\times 2^n n-60\times 2^n+27\times 2^{2n}n+184}{12}$$

$$Sz(GHT(n))=\frac{509\times 2^{3n}}{448}-\frac{1249\times 2^{2n}}{24}+48\times 2^n n+14\times 2^n+9\times 2^{2n}n+\frac{860}{21}$$

$$Sz_e(GHT(n))=147\times 2^{(3n-6)}-133\times 2^{(2n-3)}+60\times 2^n-80+\frac{55\times 2^{3n}}{12}-\frac{4849\times 2^{2n}}{12}+408\times 2^n n+\frac{167\times 2^n}{3}+\frac{289\times 2^{2n}n}{4}+\frac{1120}{3}$$

$$Sz_{ev}(GHT(n))=\frac{3755\times 2^{3n}}{1344}-\frac{1775\times 2^{2n}}{12}+140\times 2^n n+\frac{118\times 2^n}{3}+\frac{51\times 2^{2n}n}{2}+\frac{2384}{21}$$

$$Mo(GHT(n)) = (5 \times 2^{2n} - 18 \times 2^{n+1} n + 93 \times 2^n - 152)/3$$

$$Mo_e\,(GHT\,(n)) = 9 \times 2^{2n-1} - 34 \times 2^n n + 92 \times 2^n - 160$$

$$PI\,(GHT\,(n)) = \frac{33 \times 2^{2n}}{8} - 19 \times 2^n + 24$$

Proof. First, we determine the Θ^* classes of $GHT(n)$. The graph $GHT(n)$ contains $(2^{n-1}-1)$ Θ^* classes. One set of Θ^* classes, say, F_{n-1} consists of horizontal edges and vertical edges between Level 1 and Level 2 and between Level $(2n-2)$ and Level $(2n-1)$. Each $(2^{n-1}-2)$ equivalence class consists of pair of vertical edges between the levels i and $(i+1)$ and between levels $(2n-i-1)$ and $(2n-i)$, where $2 \leq i \leq n-2$, left or right children of j, $2 \leq j \leq 2n-2$ level respectively, with a difference of 2^{i-1} along the horizontal vertices. In Figure 5, there are 3 Θ^* classes. One Θ^* class consists of horizontal edges and edges $\{(1,2),(1,3),(8,10),(9,10)\}$. Two Θ class contains $\{(2,4),(5,8),(3,6),(7,9)\}$ and $\{(2,5),(4,8),(3,7),(6,9)\}$ respectively. Thus the edges between Level i and $i+1$ have Θ^* relation with edges in the same level and edges between Level $(2n-i)$ and $(2n-i-1)$.

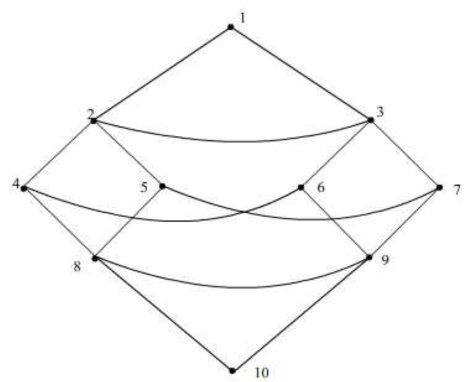

Fig 5. Glued hypertree of dimension 3

Consider a partition $\{E_1, E_2, ..., E_{n-1}\}$ coarser than the equivalence class. The classes E_i; $1 \leq i \leq n-2$ consists of vertical edges between $(i+1)^{th}$ and $(i+2)^{th}$ level and between $(2n-i-2)^{th}$ and $(2n-i-1)^{th}$ level respectively. Denote E_{n-1} as F_{n-1}.

In general, $GHT(n)/E_i$ is isomorphic to $K_{2,2^{n-i-1}}, 1 \leq i \leq n-2$. In $K_{2,2^{n-i-1}}, 1 \leq i \leq n-2$, two vertices of weight $2^{n-i}-1$ and edge weight $3(2^{n-i-1}-1)$ and other vertices with vertex and edge weight $3(2^i)-4$ and $11.2^{i-1}-10$ respectively, forms an equivalence class. The quotient graph $GHT(n)/E_{n-1}$ is isomorphic to diamond graph, with vertex and edge weights 1 and 0 for two opposite vertices and $3(2^{n-2})-2$ and $2(2^{n-1}-2)$ for the remaining adjacent vertices as shown in Figure 6.

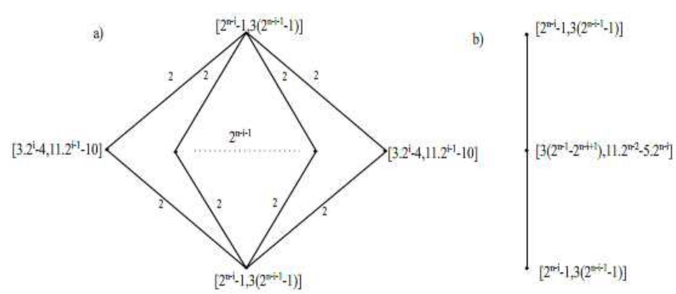

Fig 6. (a) Graph of $GHT(n)/E_i$, (b) reduced graph $(GHT(n)/E_i, w)$, $1 \leq i \leq n-2$.

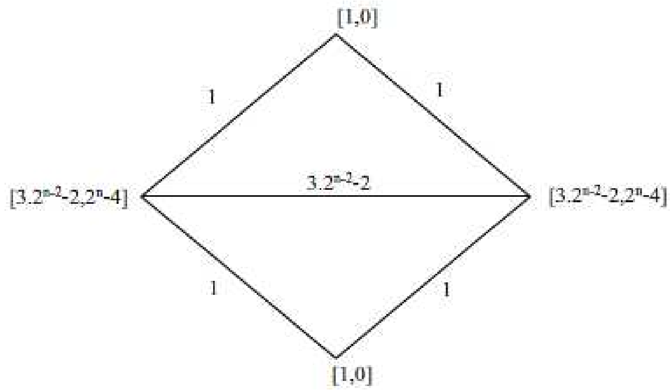

Fig 7. Graph of $GHT(n)/E_{n-1}$.

The Wiener index of $GHT(n)/E_i$ is

$$\begin{aligned} W(GHT(n)/E_i,w) &= W(P_3,w') + (2^{n-i-1})(2^{n-i-1}-1)(3(2^i)-4)(3(2^i)-4) \\ &= 2(2^{n-i}-1)(2^{n-i-1})(3(2^i)-4) + 2(2^{n-i}-1)^2 \end{aligned}$$

where w' assigns $2^{n-i}-1$ to two vertices and $2^{n-i-1}(3(2^i-4))$ to one vertex of P_3. Therefore,

$$\begin{aligned} W(GHT(n)) &= \textstyle\sum_{i=1}^{n-2}(W(GHT(n)/E_i,w) + 9(2^{2n-4}-2) \\ &= (2^{n-i+1}-2)(2^{n-i-1})(3(2^i)-4) + 2(2^{n-i}-1)^2 + 9\times 2^{2n-4} - 2 \\ &= \tfrac{24n-109\times 2^{2n}+108\times 2^n n-60\times 2^n+27\times 2^{2n}n+208}{12} + 9(2^{2n-4}) - 2 \\ &= 9(2^{2n-4}-2) + \textstyle\sum_{i=1}^{n-2}(2^{n-i-1}(2^{n-i-1}-1)(3\times 2^i-4)^2 + 2(2^{n-i}-1)^2 \\ &\quad + 2^{n-i}(2^{n-i}-1)(3\times 2^i-4)) \\ &= \tfrac{24n-109\times 2^{2n}+108\times 2^n n-60\times 2^n+27\times 2^{2n}n+108\times 2^{2n-4}+184}{12} \end{aligned}$$

The Szeged type indices of glued hypertree are calculated as follows:

$$\begin{aligned} Sz(GHT(n)) &= \textstyle\sum_{i=1}^{n-2}(Sz(GHT(n)/E_i,w) + (3\times 2^{n-2}-1)^3 + 4(3\times 2^{n-2}-1) \\ &= \textstyle\sum_{i=1}^{n-2}\{2^{n-i+1}\times(3\times 2^i+2^{n-i}-5)(3\times 2^{n-1}-3\times 2^i-2^{n-i}+3)\} \\ &\quad +4\times(3\times 2^{n-2}-1) + (3\times 2^{n-2}-1)^3 \\ &= \tfrac{1527\times 2^{3n}-69944\times 2^{2n}+64512\times 2^n n+18816\times 2^n+12096\times 2^{2n}n+55040}{1344} \end{aligned}$$

$$\begin{aligned} Sz_e(GHT(n)) &= \textstyle\sum_{i=1}^{n-2}(Sz_e(GHT(n)/E_i,w) + 147\times 2^{3n-6} - 133\times 2^{2n-3} + 60\times 2^n - 80 \\ &= \textstyle\sum_{i=1}^{n-2}\{2^{(}n-3i-1)(2^{i+4}-17\times 2^{2i-1}-5\times 2^{n-1})(34\times 2^{2i}-2^{i+5}+14\times 2^n \\ &\quad -17\times 2^{i+n})\} + 147\times 2^{3n-6} - 133\times 2^{2n-3} + 60\times 2^n - 80 \\ &= \tfrac{55\times 2^{3n}}{12} - \tfrac{4849\times 2^{2n}}{12} + 408\times 2^n n + \tfrac{167\times 2^n}{3} + \tfrac{289\times 2^{2n}n}{4} + \tfrac{1120}{3} + \\ &\quad 147\times 2^{3n-6} - 133\times 2^{2n-3} + 60\times 2^n - 80 \end{aligned}$$

$$\begin{aligned} Sz_{ev}(GHT(n)) &= \textstyle\sum_{i=1}^{n-2}(Sz_e(GHT(n)/E_i,w) + \tfrac{63\times 2^{3n}-480\times 2^{2n}+1728\times 2^n-2048}{64} \\ &= \tfrac{3755\times 2^{3n}}{1344} - \tfrac{1775\times 2^{2n}}{12} + 140\times 2^n + \tfrac{118\times 2^n}{3} + \tfrac{51\times 2^{2n}n}{2} + \tfrac{2384}{21} \end{aligned}$$

Mostar indices of glued hypertree are calculated as shown below

$$\begin{aligned} Mo(GHT(n)) &= \textstyle\sum_{i=1}^{n-2} Mo(GHT(n)/E_i,w) + 4(3\times 2^{n-2}-2) \\ &= \textstyle\sum_{i=1}^{n-2}\{2^{n-i+1}(3\times 2^{n-1}-6\times 2^i-2^{n-i+1}+8)\} + 4(3\times 2^{n-2}-2) \\ &= (5\times 2^{2n} - 18\times 2^{n+1}n + 93\times 2^n - 152)/3 \end{aligned}$$

$$\begin{aligned} Mo_e(GHT(n)) &= \textstyle\sum_{i=1}^{n-2} Mo_e(GHT(n)/E_i,w) + 4(6\times 2^{n-2}+2^n-8) \\ &= \textstyle\sum_{i=1}^{n-2}\left((2^{n-i-1}-1\right)(11.2^{i-1}+3.2^i-14) - 11.2^{i-1} - 3.2^i + 2^{n-i} + 10\} \\ &\quad 2^{n-i+1} + 4(6\times 2^{n-2}+2^n-8) \\ &= 9.2^{2n-1} - 34.2^n n + 92.2^n - 60 \end{aligned}$$

We calculated the Padmakar Ivan index of glued hypertree also.

$$\begin{aligned} PI(GHT(n)) &= \sum_{i=1}^{n-2} PI(G/E_i, w) + 12.2^{n-2} + 2\left(3.2^{n-2} - 2\right)^2 \\ &= (2^n - 4)\left(6.2^{n-1} - 4\right) + 2\left(3.2^{n-2} - 2\right)^2 \\ &= \frac{33.2^{2n}}{8} - 19.2^n + 24 \end{aligned}$$

Figure 8 and Figure 9 shows the representation of distance-based topological indices of $GHT(n)$.

Corollary 1 If $n \geq 2$, then

$$\mu(GHT(n)) = \frac{48n - 109\times 2^{2n+1} + 108\times 2^{n+1}n - 60\times 2^{n+1} + 27\times 2^{2n+1}n + 108\times 2^{2n-3} + 368}{12(3.2^{n-1}-2)(3.2^{n-1}-3)}$$

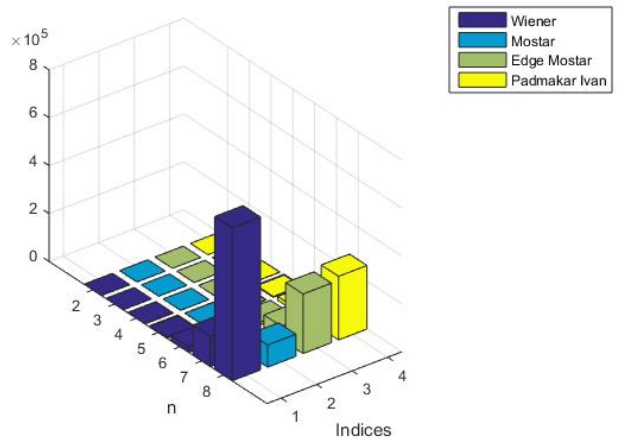

Fig 8. Graphical representation of numerical values of Wiener, Mostar, Padmakar Ivan indices for Glued hypertree

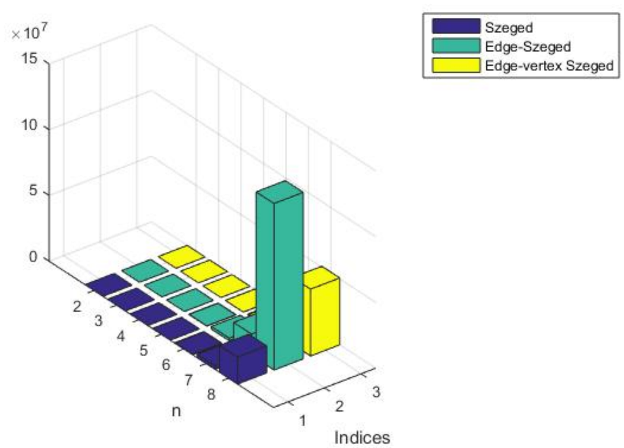

Fig 9. Graphical representation of numerical values of Szeged indices for Glued hypertree

4.1 Average distance

The average distance can be defined as the average sum of the distances between the pairs of vertices in a graph. Thus average distance can be derived from the Wiener index. The average distance of Glued tree and Glued Hypertree is $\frac{24n - 82.2^{2n} + 36.2^n n + 42.2^n + 27.2^{2n} n + 40}{6(3.2^{n-1}-2)(3.2^{n-1}-3)}$ and $\frac{24n - 109\times 2^{2n} + 108\times 2^n n - 60\times 2^n + 27\times 2^{2n} n + 108\times 2^{2n-4} + 184}{6(3.2^{n-1}-2)(3.2^{n-1}-3)}$ respectively. See Figure 10.

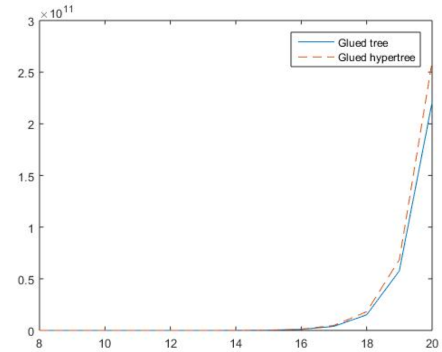

Fig 10. Average distance of Glued tree and Glued hypertree

4.2 Message Traffic Density

The message traffic density of a network is the ratio of the product of the average distance and number of nodes of the network to the number of links of the network. Message traffic density of Glued tree and Glued hypertree is $\frac{((3.2^n)/2-2)(4n-(41.2^{2n})/3+6.2^n n+7.2^n+(9.2^{2n}n)/2+20/3)}{2(2^n-2)(3.2^{n-1}-2)(3.2^{n-1}-3)}$ and $\frac{4((3.2^n)/2-2)(4n-2^{2n}/6+18.2^n n-10.2^n+(9.2^{2n}n)/2-124/3)}{(11.2^n-24)(3.2^{n-1}-2)(3.2^{n-1}-3)}$ respectively. Refer to Figure 11

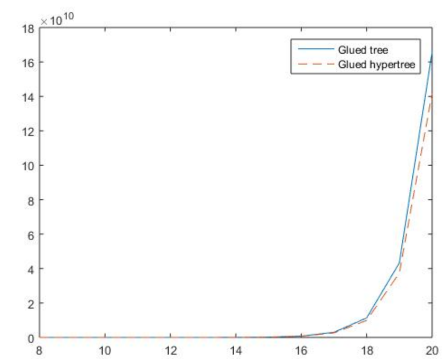

Fig 11. Message traffic density of Glued tree and Glued hypertree

5 Discussion

In the comparison of Glued hypertree with the Glued tree, Glued hypertree is a better interconnection network. The message traffic density of glued hypertree is less than that of higher internode communication performance. The network throughput is higher for glued hypertree, thus it maximizes the number of messages delivered per unit time through the network compared to glued tree. The distance-based indices give an overview of the topological properties of Glued hypertree. Thus this network can be used for future applications in the field of biology and chemistry like hypertrees[(16)].

6 Conclusion

In this article, we introduced an interconnection network, Glued hypertree which is a better interconnection network than glued tree and discussed its properties. Some distance-based topological indices of Glued hypertree are also studied, giving us an idea about its physiochemical properties. The physiochemical of the glued hypertree gives an insight into the application of glued hypertree in various fields of science like predicting biological activities of various heavy metal-based chemical compounds and in computer science. In the future, we can evaluate eccentricity-based topological indices and also degree-based indices using

M-polynomials of glued hypertree.

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