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RESEARCH ARTICLE

Mathematical Analysis of Transverse Displacement of Laminated Beam with Thermal Load

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Abstract

Objectives: This research emphasis on developing solution of transverse displacement for simply supported beam subjected to thermal load using hyperbolic shear deformation theory (HYSDT). Moreover, the significant transverse displacement analysis of single, three-layer orthotropic symmetric laminated beams exposed to sinusoidal thermal loads was taken under consideration with the aim to achieve exact solution for the above condition. **Methodology**: In this solution, we have used hyperbolic function in reference to thickness co-ordinate by considering shear deformation theory. The governing equations, boundary conditions were developed using principle of virtual work and thus exhibiting solution by Navier's theory. **Findin g:** It is observed that by considering single, triple layer transverse displacement for simply supported beam subjected to thermal loading as well as applying hyperbolic shear deformation theory (HYSDT) has predicted transverse displacement of 0.269, 0.210 and 0.562 for ply angles such as 0^0, 90^0, $0^0/90^0/0^0$ respectively with aspect ratio 20. In accordance with this we observed uniformity among the predicted values. **Novelty**: In this research we have introduced an advance function such as hyperbolic shear deformation theories (HYSDT) and transverse displacement for variable lengths and thickness of the beam was calculated by using MATLAB coding. Thus, achieving precise solution.

Keywords: Sinusoidal load; hyperbolic shear deformation theory; governing differential equations; MATLAB coding; thermal load

1 INTRODUCTION

In recent years, the concept of calculation of transverse deformation changed from a functionally graded material to composite materials that integrates the thermal components in the beam [1]. Where the Quasi-static and dynamical analyses of beam used to solve transverse deformation of beam using the differential quadrature method[2]. By taking into account[3] the dual mesh control domain methods were developed for solving deformation with approximate result. The nonlinear vibration of viscoelastic circular microplate based on softening and hardening nonlinear viscoelastic foundation under thermal loading[4] and[5]. The powerful analysis of sandwich nanoplates with a permeable practically evaluated center utilizing isogeometric examine[6]. To assess the related viable material properties, different homogeniza-tion plans including the advanced model[7]. The analysis of simply supported beam performed under approximate solution[8].

From the above findings, it is investigated that the existing solution for simply supported beam having uniformly distributed load with thermal load can be improved through, the developing a solution that can solve using von Kármán mathematical non linearity, supposition and Boltzmann constitutive connection, dual mesh control domain methods, Kirchhoff's theory, Reuss model, the Voigt model, the Mori-Tanaka model, Hashin-Shtrikman bound model Euler-Bernoulli equation, elementary beam theory (ETB), first order sheared formation theory (FSDT), higher order shear deformation theories (HSDT) and trigonometric shear deformation theory (TSDT). To achieve these possible improvements, the researcher developed a Navier solution.

2 PROPOSED WORK

This presents a detailed discussion of the methods, techniques, and tools the researchers used to satisfy the identified objectives. The hyperbolic shear deformation theory (HYSDT) gives exact solution for simply supported beam with thermal load using Navier equation. MATLAB code used for mathematical equation to find transvers displacement.

3 METHODOLOGY

Epoxy resins is the material which is used in present theory. Epoxy resins are mostly used in aerospace structures for high performance applications. The displacement field of the present composite laminated beam theory can be expressed as follows:

$$u(x,z) = u(x) - z\frac{dw}{dx} + \left[h\sinh\left(\frac{z}{h}\right) - \frac{4}{3}\frac{z^3}{h^2}\cosh\left(\frac{1}{2}\right)\right] f(x)$$

$$w(x,z) = w(x)$$

Where u is the displacement in the x direction and w is transverse displacement in the y direction of a point on the beam in mid plane. The temperature distribution across the thickness of laminated beam is assumed to be in the form as given below

$$T(x,z) = \frac{2}{h} \geq T_0(x)$$

In the above equation, T is the temperature change from a reference state which is a function of x and z. The thermal load T_0 is linearly varying across the thickness of laminated beam is a function of x.

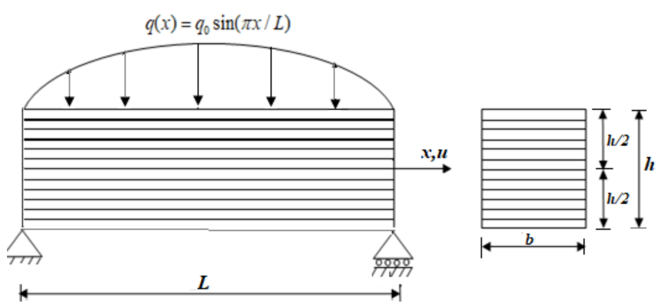

Fig 1. Simply supported beam with sinusoidally distributed load of single layer beam (0^0)

3.1 Non dimensional transverse displacement

$$\bar{w} = \frac{hw}{\alpha_L T L^2}$$

α_L Coefficient of thermal expansion to the fibre direction

For 0

$$\bar{w} = \left[\frac{1.6701\times10^5 \alpha_L L^2 T h^2 \pi\left(2003L^2+2.6950\times10^4 h^2\pi^2\right)}{\left(4.1728\times10^7 L^2 h^3 \pi^4+7.0043\times10^6 h^5 \pi^6\right)}\right]\left[\frac{h}{\alpha_L L^2 T}\sin\frac{\pi x}{L}\right]$$

For 90

$$\bar{w} = \left[\frac{1670.16\alpha_L L^2 T h^2 \pi^2 \left(2003L^2+2695h^2\pi^2\right)}{\left(1.6684\times10^6 L^2 h^3 \pi^4+2.0490\times10^6 h^5 \pi^6\right)}\right]\left[\frac{h}{\alpha_L L^2 T}\sin\frac{\pi x}{L}\right]$$

For $0^\circ/90^\circ/0^\circ$

$$\bar{w} = \left[\frac{2.1294\times10^4 \alpha_L L^2 T h^2 \pi^2 \left(5.6219\times10^{17} L^2+7.2006\times10^{18} h^2\pi^2\right)}{\left(2.8261\times10^{21} L^2 h^3 \pi^4+4.6571\times10^{21} h^5 \pi^6\right)}\right]\left[\frac{h}{\alpha_L L^2 T}\sin\frac{\pi x}{L}\right]$$

4 NUMERICAL RESULT

Table 1. Non-Dimensional TransverseDeflection at (x = L, z = 0.0) for Single Layer, Three Layers and Four Layers of Laminated Simply Supported Beam Subjected to Sinusoidally Distributed Load for Aspect Ratio (AS) 2 and 4.

Theory	Aspect Ratio (AS) 2			Aspect Ratio (AS) 4		
	0^0	90^0	$0^0/90^0/0^0$	0^0	90^0	$0^0/90^0/0^0$
Present HYSDT	4.912	0.851	9.951	1.711	0.369	3.468
TSDT [8]	4.160	0.734	9.045	1.460	0.340	3.022
HSDT [7]	5.055	0.849	10.562	1.730	0.369	3.533
FSDT [5]	0.226	0.261	0.462	0.208	0.217	0.437
ETB [3]	0.202	0.203	0.429	0.202	0.203	0.429

Table 2. Non-Dimensional Transverse Deflection at (x = L, z = 0.0) for Single Layer, Three Layers and Four Layers of Laminated Simply Supported Beam Subjected to Sinusoidally Distributed Load for Aspect Ratio (AS) 10 and 20.

Theory	Aspect Ratio (AS) 10			Aspect Ratio (AS) 20		
	0^0	90^0	$0^0/90^0/0^0$	0^0	90^0	$0^0/90^0/0^0$
Present HYSDT	0.465	0.230	0.965	0.269	0.210	0.562
TSDT [8]	0.420	0.225	0.869	0.258	0.210	0.540
HSDT [7]	4.466	0.230	0.960	0.269	0208	0.563
FSDT [5]	0.203	0.205	0.430	0.202	0.210	0.429
ETB [3]	0.202	0.203	0.429	0.202	0.203	0.429

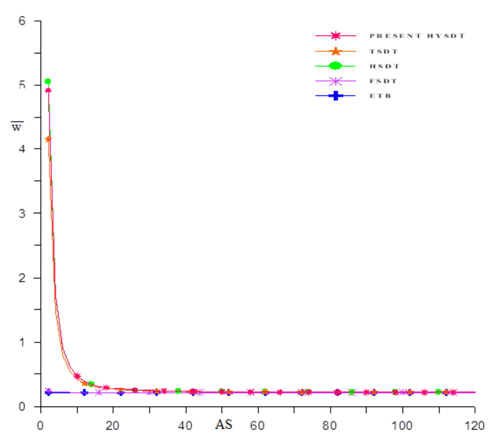

Fig 2. Variation of maximum non dimensional transverse displacement at (x = L, z = 0.0) for single layer with ply angle (0^0) of laminated simply supported beam subjected to sinusoidally distributed load for aspect ratio (AS)

5 RESULT ANALYSIS

The researcher performed analysis of simply supported beam with uniformly distributed load along with thermal load. The researcher has applied virtual work method and had developed governing differential equations in accordance with the hyperbolic shear deformation theory to satisfy the transverse displacement outcome. The Prime focus was to consider transverse displacement in order to satisfy the considerations of Navier's theory and thus exhibiting variable angle ply patterns by satisfying the needs of beam analysis. This systematic exposition of theory gives exact solution to simply supported beam analysis as compared to elementary beam theory (ETB), first order sheared formation theory (FSDT), higher order shear deformation theories (HSDT) and trigonometric shear deformation theory (TSDT).

6 DISCUSSION

In accordance with the result, we achieved for finding exact solution to the simply supported beam analysis by using Navier's theory, hyperbolic shear deformation theory (HYSDT) in comparison to first order sheared formation theory (FSDT), higher order shear deformation theories (HSDT) and trigonometric shear deformation theory (TSDT). We achieved exact solution of transverse displacement for different ply angles such as 00, 900 and 00/900/00. Convergence studies carried out for the transverse displacement of symmetric cross-ply beams showed significant variations among ETB, FSDT, HDST and TSDT with aspect ratio 2,4,10 and are tabulated in Tables 1 and 2. In these Tables the present outcomes are compared with the results of ETB, FSDT, HDST and TSDT and good agreement is observed. In Figure 1 the variations of transverse displacement with different aspect ratio and boundary conditions are given for HYSDT theory for single layer 00, 900 and three-layer symmetric 00/900/00 cross-ply beams, respectively. These results were calculated keeping x = L, z = 0.0 and it is interesting to note that transverse displacement of single and three-layer symmetric cross-ply beams for the Simply supported beam with boundary conditions converge with increasing aspect ratio (Figure 1). Hyperbolic shear deformation theory (HYSDT) represents exact solution with 5 % more accurate result than the ETB and FSDT.

7 CONCLUSIONS

The thermal response of single layer orthotropic, two-layer antisymmetric, and three-layer symmetric cross ply laminated beams subjected to pure thermal load for varied aspect ratios has been investigated using a computational model based on sinusoidal shear deformation theory. The transverse deformation, which is particularly critical in thick beams, is given special consideration. For laminated composite beams, a novel hyperbolic higher order shear deformation theory is provided. Because the theory accounts for proper transverse displacement distribution across the beam thickness, no shear correction factor is necessary. Classical beam theory, first-order shear deformation theory, and higher-order shear deformation theory are all used to compare the results. Under evenly distributed thermal load, the conclusions of the present theory match well with those of higher-order shear deformation theory. Except for ETB, when a symmetric cross-ply laminated beam is subjected to evenly distributed thermal load, transverse displacement changes as the aspect ratio changes from 2 to 20.

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