INDIAN JOURNAL OF SCIENCE AND TECHNOLOGY

RESEARCH ARTICLE

Group S_4 Difference Cordial Labeling

A Lourdusamy[1], **E Veronisha**[2]*, **F Joy Beaula**[3]

1 Department of Mathematics, St Xavier's College, Palayamkottai -627002, India
2 REG.No:19211282092009, PG and Research Department of Mathematics, St Xavier's College (Autonomous), Palayamkottai-627002, Manonmaniam Sundaranar University,Abishekapatti-627012, India
3 REG.No:20211282092004, PG and Research Department of Mathematics, St Xavier's College (Autonomous), Palayamkottai-627002, Manonmaniam Sundaranar University,Abishekapatti-627012, India

OPEN ACCESS

Received: 17-03-2022

Accepted: 06-07-2022

Published: 20-08-2022

Citation: Lourdusamy A, Veronisha E, Beaula FJ (2022) Group S_4 Difference Cordial Labeling. Indian Journal of Science and Technology 15(32): 1561-1568. https://doi.org/10.17485/IJST/v15i32.614

* **Corresponding author**.

nishaedwin1705@gmail.com

Funding: None

Competing Interests: None

Published By Indian Society for Education and Environment (iSee)

ISSN
Print: 0974-6846
Electronic: 0974-5645

Abstract

Objective: To find the Group S_4 difference cordial labeling of some standard graphs. **Method:** Path, Cycle and some standard graphs are converted into Group S_4 difference cordial graphs by labeling the vertices with the elements of S_4 and the edges as the difference of the order of the elements labeled to the vertices of the graph. **Findings:** Group S_4 difference cordial labeling for path, cycle and some standard graphs. **Novelty**: Graph labeling can use for issues in Mobile Adhoc Networks such as connectivity, scalability, routing, modeling the network and simulation. In this paper we compute the new concept of Group S_4 difference cordial labeling and also we prove that some standard graphs are Group S_4 difference cordial graph.

AMS Subject Classification 2010: 05C78

Keywords: Difference cordial Labeling; Path; cycle; Bistar; Comb; Ladder; S4

1 Introduction

Graphs consider here are finite, simple, connected and undirected. Kala et. al defined group cordial prime labeling and also proved that many graphs satisfy group prime cordial labeling[(1–3)]. Lourdusamy et. al introduced the concept of group S_3 cordial remainder labeling[(4)]. Results on group S_3 cordial remainder labeling can be found in[(5–7)][(8)]. Group Q_8 difference cordial labeling[(9)] was introduced by Lourdusamy et al. In this paper we introduce the new labeling called group S_4 difference cordial labeling. Here we prove that path, cycle, bistar, comb, crown and ladder related graphs admit group S_4 difference cordial labeling.

2 S_4 Difference Cordial Labeling

S_4={e,a,b,c,d,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u,v,w,x} is the group of all permutations of 4 elements set, where

$e=(1), a=(12), b=(13), c=(14), d=(23), f=(24), g=(34), h=(12)(34),$

$i = (13)(24)$, $j = (14)(23)$,$k = (123)$,$l = (132)$,$m = (234)$,$n = (243)$,$o = (134)$,
$p = (143)$,$q = (124)$,$r = (142)$,$s = (1234)$, $t = (1243)$,$u = (1432)$, $v = (1324)$,
$w = (1342)$,$x = (1423)$.

It has one 1 order element {e}, nine 2 order elements {a,b,c,d,f,g,h,i,j}, eight 3 order elements {k,l,m,n,o,p,q,r} and six 4 order elements {s,t,u,v,w,x}.

Defini tion 2.1. Let $\propto : V(G) \to S_4$ be a function. For each edge xy assign the label 0 if $(o(\alpha(x)) - o(\alpha(y))| = 0\alpha$ is called group S_4 difference cordial labeling if $|v_\propto(\lambda) - v_\propto(\mu)| \leq 1$ and $|e_\propto(0) - e_\propto(1)| \leq 1$, where $v_\propto(\lambda)$ and $e_\propto(y)$ respectively denote the number of vertices labeled with an element λ and number of edges labeled with y(y=0,1). A graph G which admits group S_4 difference cordial labeling is called group S_4 difference cordial graph.

3 Main Results

Theorem 3.1. The path P_n is S_4 difference cordial graph.

Proof. Let $V(P_n) = \left(v_\beta : 1 \leq \beta \leq n\right\}$ and E$(P_n) = \left(v_\beta v_{\beta+1} : 1 \leq \beta \leq n-1\right\}$. Define $\propto : V(P_n) \to S_4$ be the vertex label as follows:

Let $n \equiv \gamma(mod24)$

$$\alpha(v_\beta) = \begin{cases} \text{a if } \beta \equiv 1 \pmod{24} \\ b \text{ if } \beta \equiv 2 \pmod{24} \\ \text{k if } \beta \equiv 3 \pmod{24} \\ \text{lif } \beta \equiv 4 \pmod{24} \\ \text{s if } \beta \equiv 5 \pmod{24} \\ \text{tif } \beta \equiv 6 \pmod{24} \\ \text{c if } \beta \equiv 7 \pmod{24} \\ d \text{ if } \beta \equiv 8 \pmod{24} \\ m \text{ if } \beta \equiv 9 \pmod{24} \\ n \text{ if } \beta \equiv 10 \pmod{24} \\ \text{u if } \beta \equiv 11 \pmod{24} \\ \text{vif } \beta \equiv 12 \pmod{24} \\ \text{f if } \beta \equiv 13 \pmod{24} \\ g \text{ if } \beta \equiv 14 \pmod{24} \\ \text{o if } \beta \equiv 15 \pmod{24} \\ p \text{ if } \beta \equiv 16 \pmod{24} \\ w \text{ if } \beta \equiv 17 \pmod{24} \\ x \text{ if } \beta \equiv 18 \pmod{24} \\ h \text{ if } \beta \equiv 19 \pmod{24} \\ \text{i if } \beta \equiv 20 \pmod{24} \\ \text{q if } \beta \equiv 21 \pmod{24} \\ r \text{ if } \beta \equiv 22 \pmod{24} \\ \text{e if } \beta \equiv 23 \pmod{24} \\ j \text{ if } \beta \equiv 0 \pmod{24} \end{cases}$$

If $n = 24z + r$, $z \geq 0$, $0 \leq \gamma \leq 23$. Here the elements of S_4 which are labelled to the first γ vertices from v_1 to v_γ are labeled $z+1$ times and the remaining $24-\gamma$ elements of S_4 are labeled z times.

So, $(v_\propto() - v_\propto(\mu)| \leq 1$.

Further

$$e_\propto(0) = \begin{cases} \left| \frac{24z+(\gamma-2)}{2} \right| + 1 \text{ if } \gamma \text{ is even } \gamma \neq 0 \\ \left| \frac{24z+(\gamma-2)}{2} \right| \text{ if } \gamma \text{ is odd} \end{cases}$$

$$e_\propto(1) = \begin{cases} \left\lfloor \frac{24z+(\gamma-2)}{2} \right\rfloor \text{ if } \gamma \text{ is even }, \gamma \neq 0 \\ \left\lceil \frac{24z+(\gamma-2)}{2} \right\rceil + 1 \text{ if } \gamma \text{ is odd} \end{cases}$$

This implies that $(e_\propto(0) - e_\propto(1)| \leq 1$. Hence $\propto$ is S_4 difference cordial graph.

Theorem 3.2. The cycle C_n is

Proo f. Let V$(C_n) = \left(v_\beta : 1 \leq \beta \leq n\right\}$ and E$(C_n) = \left(v_\beta v_{\beta+1} : 1 \leq \beta \leq n-1\right\} \cup (v_1 v_n\}$. Define $\propto : V(C_n) \to S_4$ be the vertex label as follows:

Let $n \equiv \gamma(mod 24)$

Case 1.$\gamma \neq 2,\ 0 \leq \gamma \leq 23.$

$$\alpha\left(v_\beta\right)=\begin{cases} \mathrm{e} \text{ if } \beta \equiv 1(\bmod 24) \\ \mathrm{a} \text{ if } \beta \equiv 2(\bmod 24) \\ b \text{ if } \beta \equiv 3(\bmod 24) \\ \mathrm{c} \text{ if } \beta \equiv 4(\bmod 24) \\ k \text{ if } \beta \equiv 5(\bmod 24) \\ \mathrm{l} \text{ if } \beta \equiv 6(\bmod 24) \\ s \text{ if } \beta \equiv 7(\bmod 24) \\ t \text{ if } \beta \equiv 8(\bmod 24) \\ d \text{ if } \beta \equiv 9(\bmod 24) \\ f \text{ if } \beta \equiv 10(\bmod 24) \\ m \text{ if } \beta \equiv 11(\bmod 24) \\ n \text{ if } \beta \equiv 12(\bmod 24) \\ u \text{ if } \beta \equiv 13(\bmod 24) \\ v \text{ if } \beta \equiv 14(\bmod 24) \\ g \text{ if } \beta \equiv 15(\bmod 24) \\ h \text{ if } \beta \equiv 16(\bmod 24) \\ \mathrm{o} \text{ if } \beta \equiv 17(\bmod 24) \\ p \text{ if } \beta \equiv 18(\bmod 24) \\ w \text{ if } \beta \equiv 19(\bmod 24) \\ x \text{ if } \beta \equiv 20(\bmod 24) \\ \mathrm{i} \text{ if } \beta \equiv 21(\bmod 24) \\ j \text{ if } \beta \equiv 22(\bmod 24) \\ q \text{ if } \beta \equiv 23(\bmod 24) \\ r \text{ if } \beta \equiv 0(\bmod 24) \end{cases}$$

If $n = 24z + r, z \geq 0,\ 0 \leq \gamma \leq 23$ *and* $\gamma \neq 2$. In view of the above labeling, we get

$$e_{\propto}(0)=\begin{cases} \left\lceil \frac{24z+\gamma}{2} \right\rceil + 1 \text{ if } \gamma \text{ is even} \\ \left\lceil \frac{24z+(\gamma-1)}{2} \right| \text{ if } \gamma \text{ is odd } \gamma \neq 1 \\ \frac{24z}{2}+1 \text{ if } \gamma = 1 \end{cases}$$

and

$$e_{\propto}(1)=\begin{cases} \left\lceil \frac{24z+\gamma}{2} \right| \text{ if } \gamma \text{ is even }, \gamma \neq 0 \\ \frac{24z+(\gamma-1)}{2} \mid +1 \text{ if } \gamma \text{ is odd } \gamma \neq 1 \\ \frac{24z}{2} \text{ if } \gamma = 1 \end{cases}$$

Case 2.$\gamma = 2$. Let $n = 24z + 2$. Starting from v_1 every consecutive 24 vertices are labeled same as Case 1. The remaining 2 vertices are labeled with a, e. In view of the above labeling, we get

$e_{\propto}(0) = e_{\propto}(1) = \frac{24z+\gamma}{2}$.

In the above two cases, $(e_{\propto}(0) - e_{\propto}(1)| \leq 1$. Hence the cycle C_n is S_4 difference cordial graph.

Theorem 3.3. The comb graph $P_n \odot K_1$ is S_4 difference cordial graph.

Proo f. Let the vertex set of $P_n \odot K_1$ be $\{v'_1, v'_2, \ldots.., v'_n, v''_1, v''_2, \ldots.., v''_n\}$ and E$(P_n \odot K_1) = \left(v'_\beta v'_{\beta+1} : 1 \leq \beta \leq n-1\right\} \cup \left(v'_\beta v''_\beta : 1 \leq \beta \leq n\right\}$. Let $\propto$: $V(P_n \odot K_1) \rightarrow S_4$be defined by,

$$\alpha\left(v'_\beta\right)=\begin{cases} a \text{ if } \beta\equiv 1(\bmod 12)\\ k \text{ if } \beta\equiv 2(\bmod 12)\\ s \text{ if } \beta\equiv 3(\bmod 12)\\ c \text{ if } \beta\equiv 4(\bmod 12)\\ m \text{ if } \beta\equiv 5(\bmod 12)\\ u \text{ if } \beta\equiv 6(\bmod 12)\\ f \text{ if } \beta\equiv 7(\bmod 12)\\ o \text{ if } \beta\equiv 8(\bmod 12)\\ w \text{ if } \beta\equiv 9(\bmod 12)\\ h \text{ if } \beta\equiv 10(\bmod 12)\\ j \text{ if } \beta\equiv 11(\bmod 12)\\ q \text{ if } \beta\equiv 0(\bmod 12)\end{cases}$$

$$\propto\left(v''_\beta\right)=\begin{cases} b \text{ if } \beta\equiv 1(\bmod 12)\\ l \text{ if } \beta\equiv 2(\bmod 12)\\ t \text{ if } \beta\equiv 3(\bmod 12)\\ d \text{ if } \beta\equiv 4(\bmod 12)\\ n \text{ if } \beta\equiv 5(\bmod 12)\\ v \text{ if } \beta\equiv 6(\bmod 12)\\ g \text{ if } \beta\equiv 7(\bmod 12)\\ p \text{ if } \beta\equiv 8(\bmod 12)\\ x \text{ if } \beta\equiv 9(\bmod 12)\\ \text{i if } \beta\equiv 10(\bmod 12)\\ e \text{ if } \beta\equiv 11(\bmod 12)\\ r \text{ if } \beta\equiv 0(\bmod 12)\end{cases}$$

Let $n=12z+\gamma$ where $z\geq 0$ and $0\leq\gamma\leq 11$. In view of the above defined labeling pattern, we observe that

$e_\propto(0)=12z+\gamma$, $e_\propto(1)=12z+\gamma-1$.

Hence comb graph is S_4 difference cordial graph.

Theorem 3.4. Bistar $B_{n,n}$ is S_4 difference cordial graph.

Proof. Let y and z be the apex vertices and $y_1, y_2, \ldots, y_n$, $z_1, z_2, \ldots, z_n$ be the pendent vertices. We define $\propto$: V($B_{n,n}$) $\to S_4$ as follows:

$\propto$ (y) = a, $\propto$ (z) = k;

for $0\leq\ \beta\leq n$,

$$\alpha\left(y_\beta\right)=\begin{cases} b \text{ if } \beta\equiv 1(\bmod 12)\\ c \text{ if } \beta\equiv 2(\bmod 12)\\ d \text{ if } \beta\equiv 3(\bmod 12)\\ f \text{ if } \beta\equiv 4(\bmod 12)\\ g \text{ if } \beta\equiv 5(\bmod 12)\\ h \text{ if } \beta\equiv 6(\bmod 12)\\ \text{i if } \beta\equiv 7(\bmod 12)\\ l \text{ if } \beta\equiv 8(\bmod 12)\\ n \text{ if } \beta\equiv 9(\bmod 12)\\ p \text{ if } \beta\equiv 10(\bmod 12)\\ j \text{ if } \beta\equiv 11(\bmod 12)\\ k \text{ if } \beta\equiv 0(\bmod 12)\end{cases}$$

and

$$\alpha\left(z_\beta\right)=\begin{cases} s \text{ if } \beta \equiv 1(\bmod 12) \\ t \text{ if } \beta \equiv 2(\bmod 12) \\ u \text{ if } \beta \equiv 3(\bmod 12) \\ v \text{ if } \beta \equiv 4(\bmod 12) \\ w \text{ if } \beta \equiv 5(\bmod 12) \\ x \text{ if } \beta \equiv 6(\bmod 12) \\ e \text{ if } \beta \equiv 7(\bmod 12) \\ m \text{ if } \beta \equiv 8(\bmod 12) \\ o \text{ if } \beta \equiv 9(\bmod 12) \\ q \text{ if } \beta \equiv 10(\bmod 12) \\ r \text{ if } \beta \equiv 11(\bmod 12) \\ a \text{ if } \beta \equiv 0(\bmod 12) \end{cases}$$

Clearly,

$$e_\propto(0)=\begin{cases} n+1 & \text{if } n \text{ is multiple of } 11 \\ n & \text{Otherwise} \end{cases}$$

$$e_\propto(1)=\begin{cases} n & \text{if } n \text{ is multiple of } 11 \\ n+1 & \text{Otherwise} \end{cases}$$

Hence $\propto$ is group S_4 difference cordial labeling.

Theorem 3.5. Ladder L_n is S_4 difference cordial graph.

Proof. Let V(L_n)= $\left(v'_{\beta,} v''_\beta : 1 \leq \beta \leq n\right\}$ and E(L_n)={ $v'_\beta v''_\beta : 1 \leq \beta \leq n$ }∪{$v'_\beta v'_{\beta+1}$, $v''_\beta v''_{\beta+1} : 1 \leq \beta \leq n-1$}.

Define $\propto$: V (L_n)→ S_4 as follows:

$$\propto\left(v'_\beta\right)=\begin{cases} k \text{ if } \beta \equiv 1(\bmod 12) \\ l \text{ if } \beta \equiv 2(\bmod 12) \\ \text{a if } \beta \equiv 3(\bmod 12) \\ s \text{ if } \beta \equiv 4(\bmod 12) \\ t \text{ if } \beta \equiv 5(\bmod 12) \\ u \text{ if } \beta \equiv 6(\bmod 12) \\ e \text{ if } \beta \equiv 7(\bmod 12) \\ j \text{ if } \beta \equiv 8(\bmod 12) \\ n \text{ if } \beta \equiv 9(\bmod 12) \\ o \text{ if } \beta \equiv 10(\bmod 12) \\ p \text{ if } \beta \equiv 11(\bmod 12) \\ r \text{ if } \beta \equiv 0(\bmod 12) \end{cases}$$

$$\alpha\left(v''_\beta\right)=\begin{cases} m \text{ if } \beta \equiv 1(\bmod 12) \\ b \text{ if } \beta \equiv 2(\bmod 12) \\ c \text{ if } \beta \equiv 3(\bmod 12) \\ d \text{ if } \beta \equiv 4(\bmod 12) \\ v \text{ if } \beta \equiv 5(\bmod 12) \\ f \text{ if } \beta \equiv 6(\bmod 12) \\ g \text{ if } \beta \equiv 7(\bmod 12) \\ h \text{ if } \beta \equiv 8(\bmod 12) \\ i \text{ if } \beta \equiv 9(\bmod 12) \\ q \text{ if } \beta \equiv 10(\bmod 12) \\ w \text{ if } \beta \equiv 11(\bmod 12) \\ x \text{ if } \beta \equiv 0(\bmod 12) \end{cases}$$

We observe that,

Let $s=\left(\frac{n}{6}\right\rfloor$

$$e_\propto(0)=\begin{cases} \left\lceil\frac{n-3}{2}\right| + n \text{ if } s \text{ is odd} \\ \left|\frac{n-1}{2}\right| + n \text{ if } s \text{ is even} \end{cases}$$

$$e_\propto(1)=\begin{cases} \left\lceil\frac{n-1}{2}\right| + n \text{ if } s \text{ is odd} \\ \left|\frac{n-3}{2}\right| + n \text{ if } s \text{ is even} \end{cases}$$

Hence ladder L_n is S_4 difference cordial graph.

Theorem 3.6. Slanting ladder SL_n is S_4 difference cordial graph.

Proof. Let V(SL_n)= $\left(v'_{\beta,} v''_{\beta} : 1 \leq \beta \leq n\right\}$ and E(SL_n)= $\{v'_{\beta} v'_{\beta+1},\ v''_{\beta} v''_{\beta+1}, v'_{\beta} v''_{\beta+1} : 1 \leq \beta \leq n-1\}$. Define $\propto$: V $(SL_n) \rightarrow$ S_4 as follows:

$$\propto\left(v'_{\beta}\right)=\begin{cases} k \text{ if } \beta \equiv 1(\bmod 12) \\ l \text{ if } \beta \equiv 2(\bmod 12) \\ n \text{ if } \beta \equiv 3(\bmod 12) \\ p \text{ if } \beta \equiv 4(\bmod 12) \\ q \text{ if } \beta \equiv 5(\bmod 12) \\ d \text{ if } \beta \equiv 6(\bmod 12) \\ s \text{ if } \beta \equiv 7(\bmod 12) \\ u \text{ if } \beta \equiv 8(\bmod 12) \\ v \text{ if } \beta \equiv 9(\bmod 12) \\ h \text{ if } \beta \equiv 10(\bmod 12) \\ i \text{ if } \beta \equiv 11(\bmod 12) \\ r \text{ if } \beta \equiv 0(\bmod 12) \end{cases}$$

$$\alpha\left(v''_{\beta}\right)=\begin{cases} g \text{ if } \beta \equiv 1(\bmod 12) \\ m \text{ if } \beta \equiv 2(\bmod 12) \\ a \text{ if } \beta \equiv 3(\bmod 12) \\ o \text{ if } \beta \equiv 4(\bmod 12) \\ b \text{ if } \beta \equiv 5(\bmod 12) \\ c \text{ if } \beta \equiv 6(\bmod 12) \\ f \quad \text{ if } \beta \equiv 7(\bmod 12) \\ t \text{ if } \beta \equiv 8(\bmod 12) \\ e \text{ if } \beta \equiv 9(\bmod 12) \\ w \text{ if } \beta \equiv 10(\bmod 12) \\ x \text{ if } \beta \equiv 11(\bmod 12) \\ j \text{ if } \beta \equiv 0(\bmod 12) \end{cases}$$

It is observed as

$$e_{\propto}(0)=\begin{cases} 3\left(\frac{n-1}{2}\right) & \text{if } n \text{ is odd} \\ 3\left[\frac{n-1}{2}\right|+1 \text{ or } 3\left[\frac{n-1}{2}\right]+2 & \text{if } n \text{ is even} \end{cases}$$

and

$$e_{\alpha}(1)=\begin{cases} 3\left(\frac{n-1}{2}\right) & \text{if } n \text{ is odd} \\ 3\left|\frac{n-1}{2}\right|+1 \text{ or } 3\left[\frac{n-1}{2}\right]+2 \text{ if } n \text{ is even} \end{cases}$$

Hence SL_n is S_4 difference cordial graph.

Theorem 3.7. The graph $L_n \odot K_1$ is S_4 difference cordial graph.

Proof. Let V($L_n \odot K_1$)= $\left(v^1_{\beta,} v^2_{\beta,}\ v^3_{\beta,} v^4_{\beta} : 1 \leq \beta \leq n\right\}$ and

E($L_n \odot K_1$)= $\{v^1_{\beta} v^2_{\beta,} v^2_{\beta} v^3_{\beta},\ v^3_{\beta} v^4_{\beta} : 1 \leq \beta \leq n\} \cup \{v^2_{\beta} v^2_{\beta+1},\ v^3_{\beta} v^3_{\beta+1} : 1 \leq \beta \leq n-1\}$.

Define $\propto$: V $(L_n \odot K_1) \rightarrow S_4$ as follows:

$$\propto\left(v^1_{\beta}\right)=\begin{cases} k \text{ if } \beta \equiv 1(\bmod 6) \\ m \text{ if } \beta \equiv 2(\bmod 6) \\ s \text{ if } \beta \equiv 3(\bmod 6) \\ g \text{ if } \beta \equiv 4(\bmod 6) \\ u \text{ if } \beta \equiv 5(\bmod 6) \\ w \text{ if } \beta \equiv 0(\bmod 6) \end{cases} \quad \propto\left(v^2_{\beta}\right)=\begin{cases} a & \text{if } \beta \equiv 1(\bmod 6) \\ n & \text{if } \beta \equiv 2(\bmod 6) \\ o & \text{if } \beta \equiv 3(\bmod 6) \\ q & \text{if } \beta \equiv 4(\bmod 6) \\ v & \text{if } \beta \equiv 5(\bmod 6) \\ x & \text{if } \beta \equiv 0(\bmod 6) \end{cases}$$

$$\alpha\left(v^3_{\beta}\right)=\begin{cases} b \text{ if } \beta \equiv 1(\bmod 6) \\ c \text{ if } \beta \equiv 2(\bmod 6) \\ p \text{ if } \beta \equiv 3(\bmod 6) \\ r \text{ if } \beta \equiv 4(\bmod 6) \\ h \text{ if } \beta \equiv 5(\bmod 6) \\ j \text{ if } \beta \equiv 0(\bmod 6) \end{cases} \quad \propto\left(v^4_{\beta}\right)=\begin{cases} l & \text{if } \beta \equiv 1(\bmod 6) \\ d & \text{if } \beta \equiv 2(\bmod 6) \\ f & \text{if } \beta \equiv 3(\bmod 6) \\ t & \text{if } \beta \equiv 4(\bmod 6) \\ \mathrm{i} & \text{if } \beta \equiv 5(\bmod 6) \\ e & \text{if } \beta \equiv 0(\bmod 6) \end{cases}$$

In this type of labeling pattern, we observe that

$$e_{\propto}(0) = \begin{cases} 2n + \left\lfloor \frac{n}{2} \right\rceil - 1 & \text{if } n \text{ is odd} \\ \frac{5n}{2} - 1 & \text{if } n \text{ is even} \end{cases}$$

$$e_{\propto}(1) = \begin{cases} 2n + \left\lfloor \frac{n}{2} \right\rfloor & \text{if } n \text{ is odd} \\ \frac{5n}{2} - 1 & \text{if } n \text{ is even} \end{cases}$$

Thus the graph $L_n \odot K_1$ is S_4 difference cordial graph.

Theorem 3.8. The graph $TL_n \odot K_1$ is S_4 difference cordial graph.

Proof. Let V($TL_n \odot K_1$)= $\left(v_{\beta,}^1 v_{\beta,}^2 \, v_{\beta,}^3 v_\beta^4 : 1 \leq \beta \leq n \right\}$ and E($TL_n \odot K_1$)= $\{v_\beta^1 v_{\beta,}^2 v_\beta^2 v_{\beta}^3, \, v_\beta^3 v_\beta^4 : 1 \leq \beta \leq n\} \cup \{v_\beta^2 v_{\beta+1}^2, \, v_\beta^3 v_{\beta+1}^3, v_\beta^2 v_{\beta+1}^3 \ : 1 \leq \beta \leq n-1\}$.

Define $\propto$: V $(TL_n \odot K_1) \to S_4$ as follows:

$$\propto \left(v_\beta^1 \right) = \begin{cases} k \text{ if } \beta \equiv 1(\bmod 6) \\ m \text{ if } \beta \equiv 2(\bmod 6) \\ t \text{ if } \beta \equiv 3(\bmod 6) \\ u \text{ if } \beta \equiv 4(\bmod 6) \\ g \text{ if } \beta \equiv 5(\bmod 6) \\ i \text{ if } \beta \equiv 0(\bmod 6) \end{cases} \quad \propto \left(v_\beta^2 \right) = \begin{cases} a & \text{if } \beta \equiv 1(\bmod 6) \\ n \text{ if } \beta \equiv 2(\bmod 6) \\ o \text{ if } \beta \equiv 3(\bmod 6) \\ f \text{ if } \beta \equiv 4(\bmod 6) \\ h \text{ if } \beta \equiv 5(\bmod 6) \\ j \text{ if } \beta \equiv 0(\bmod 6) \end{cases}$$

$$\alpha \left(v_\beta^3 \right) = \begin{cases} b \text{ if } \beta \equiv 1(\bmod 6) \\ c \text{ if } \beta \equiv 2(\bmod 6) \\ p \text{ if } \beta \equiv 3(\bmod 6) \\ q \text{ if } \beta \equiv 4(\bmod 6) \\ v \text{ if } \beta \equiv 5(\bmod 6) \\ x \text{ if } \beta \equiv 0(\bmod 6) \end{cases} \quad \alpha \left(v_\beta^4 \right) = \begin{cases} l & \text{if } \beta \equiv 1(\bmod 6) \\ s & \text{if } \beta \equiv 2(\bmod 6) \\ d & \text{if } \beta \equiv 3(\bmod 6) \\ r & \text{if } \beta \equiv 4(\bmod 6) \\ w & \text{if } \beta \equiv 5(\bmod 6) \\ e & \text{if } \beta \equiv 0(\bmod 6) \end{cases}$$

Let $n = 6z + \gamma$ where $z \geq 0$ and $0 \leq \gamma \leq 5$. We observe that,
$e_{\propto}(0) = 3(6z+\gamma) - 2$ and $e_{\propto}(0) = 3(6z+\gamma) - 1$.
Thus the graph $TL_n \odot K_1$ is S_4 difference cordial graph.

Theorem 3.8. The graph $P_n \odot 2K_1$ is S_4 difference cordial graph.

Proof. Let V($P_n \odot 2K_1$)= $\left(v_{\beta,}^1 v_{\beta,}^2 \, v_\beta^3 : 1 \leq \beta \leq n \right\}$ and E($P_n \odot 2K_1$)= $\{v_\beta^1 v_{\beta,}^2 v_\beta^2 v_\beta^3 : 1 \leq \beta \leq n\} \cup \{v_\beta^2 v_{\beta+1}^2 \ : 1 \leq \beta \leq n-1\}$.

Define $\propto$: V $(P_n \odot 2K_1) \to S_4$ as follows:

$$\propto \left(v_\beta^1 \right) = \begin{cases} a \quad \text{if } \beta \equiv 1(\bmod 8) \\ e \text{ if } \beta \equiv 2(\bmod 8) \\ k \text{ if } \beta \equiv 3(\bmod 8) \\ l \text{ if } \beta \equiv 4(\bmod 8) \\ m \text{ if } \beta \equiv 5(\bmod 8) \\ u \text{ if } \beta \equiv 6(\bmod 8) \\ o \text{ if } \beta \equiv 7(\bmod 8) \\ p \text{ if } \beta \equiv 0(\bmod 8) \end{cases} \quad \propto \left(v_\beta^2 \right) = \begin{cases} b & \text{if } \beta \equiv 1(\bmod 8) \\ c \text{ if } \beta \equiv 2(\bmod 8) & \\ f \text{ if } \beta \equiv 3(\bmod 8) & \\ g & \text{if } \beta \equiv 4(\bmod 8) \\ i \text{ if } \beta \equiv 5(\bmod 8) & \\ v & \text{if } \beta \equiv 6(\bmod 8) \\ x & \text{if } \beta \equiv 7(\bmod 8) \\ q & \text{if } \beta \equiv 0(\bmod 8) \end{cases}$$

$$\alpha \left(v_\beta^3 \right) = \begin{cases} s & \text{if } \beta \equiv 1(\bmod 8) \\ d & \text{if } \beta \equiv 2(\bmod 8) \\ l & \text{if } \beta \equiv 3(\bmod 8) \\ h & \text{if } \beta \equiv 4(\bmod 8) \\ n & \text{if } \beta \equiv 5(\bmod 8) \\ w & \text{if } \beta \equiv 6(\bmod 8) \\ j & \text{if } \beta \equiv 7(\bmod 8) \\ r & \text{if } \beta \equiv 0(\bmod 8) \end{cases}$$

Consider $n = 8z + \gamma$ where $z \geq 0$ and $0 \leq \gamma \leq 7$. In this type of labeling pattern,

$$e_{\propto}(0) = \begin{cases} n + \frac{n}{2} & \text{if } n \text{ is odd} \\ n + \left\lfloor \frac{n}{2} \right\rfloor & \text{if } n \text{ is even} \end{cases}$$

$$e_{\propto}(1) = \begin{cases} n + \frac{n}{2} - 1 & \text{if } n \text{ is odd} \\ n + \left| \frac{n}{2} \right| & \text{if } n \text{ is even} \end{cases}$$

Therefore $P_n \odot 2K_1$ is S_4 difference cordial graph.

4 Conclusion

The cordial labeling of graphs has been a topic of research for 35 years and it has many properties to be found. It is already settled that many graphs are group Q_8 difference cordial and group S_3 remainder cordial. Finally in this paper the new concept of group S_4 difference cordial labeling have been defined. We also proved that path, cycle, comb, bistar, crown and ladder related graphs admit group S_4 difference cordial labeling.

References

1) Kala R, Chandra B. Group S3 Cordial Prime Labeling of Graphs. *Malaya Journal of Mathematik*. 2019;1:403–407.
2) Kala R, Chandra B, Group D. Group S3 Cordial Prime Labeling of Some Graphs. *International Journal of Applied Engineering Research*. 2019;14:4203–4208. Available from: http://www.ripublication.com.
3) Kala R, Chandra B. Group S3 Cordial Prime Labeling of Some Splitting Graphs. *AIP Conference Proccedings*. 2020;p. 2271.
4) Lourdusamy A, Wency SJ, Patrick F. Group S3 Cordial Remainder Labeling. *International Journal of Recent Technology and Engineering (IJRTE)*. 2019;8(4):8276–8281.
5) Lourdusamy A, Wency SJ. Group S3 Cordial Remainder Labeling of Subdivision of Graphs. *Journal Applied Mathematics and Informatics*. 2020;38(3-4):221–238. Available from: https://doi.org/10.14317/jami.2020.221.
6) Lourdusamy A, Wency SJ. Several Results On Group S3 Cordial Remainder Labeling. *AIP Conference Proccedings*. 2020;2261(1). Available from: https://doi.org/10.1063/5.0016883.
7) Lourdusamy A, Wency SJ. Group S3 Cordial Remainder Labeling for Wheel and Snake Related Graphs. *Jordan Journal of Mathematics and Statistics*. 2021;14(2):267–286. Available from: https://doi.org/10.14317/jami.2021.223.
8) Lourdusamy A, Wency SJ. Group S3 Cordial Remainder Labeling for Path and Cycle Related Graphs. *Journal Applied Mathematics and Informatics*. 2021;39:223–237.
9) Lourdusamy A, Veronisha E, Docxupload. Q8 difference cordial labeling. *Journal of Mathematical and Computational Science*. 2021;p. 2999–3009. Available from: https://doi.org/10.28919/jmcs/5622.