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* **Corresponding author**.

derphung17@gmail.com

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Magnetized Bianchi Model with Time-dependent Deceleration Parameter

Derphungshar Basumatary[1]*, Mukunda Dewri[1]

1 Department of Mathematical Sciences, Bodoland University, Kokrajhar, 783370, BTR, Assam, India

Abstract

Objectives: To investigate the Bianchi Type-III cosmological model in the presence of a magnetic field using the time-dependent deceleration parameter in Sen-Dunn scalar-tensor theory. **Methods**: In this work, we approach a Bianchi type-III metric with the presence of a magnetic field in the energy-momentum tensor. The time-dependent deceleration parameter representing an accelerated expansion proposed by Banerjee and Das (Gen. Relativ. Gravit. 37:10, 2005)is applied to obtain the exact solution of the field equations. Also, to obtain a deterministic solution considering shear scalar proportional to scalar expansion resulting in$B=C^{\beta}$. Furthermore, taking the power law relation between the scalar field and the average scale factor, the model's following physical and geometrical properties are computed and discussed with the present observational data. **Findings:**The model is anisotropic, expanding, shearing, and non-rotating with increasing volume as time tends to infinity from zero volume at $t = 0$. The model approves an accelerating universe under the ΛCDM model. **Novelty:** The study provides a better comprehension of cosmic evolution within the Sen-Dunn scalar-tensor gravitational theory context. The model demonstrates its consistency with the cosmological observation.

Keywords: Bianchi type III metric; Electromagnetic field; Deceleration parameter; SenDunn theory; ΛCDM model

1 Introduction

The most prominent study made by supernovae (SNe), cosmic microwave background (CMB), and baryonic acoustic oscillations (BAO) in modern cosmology is the accelerated expansion of the universe. Still, the universe holds the greatest mysteries of its evolution, such as the accelerated expansion's origin and cause. Many cosmologists take many modified theories that alternate with Einstein's theory into account to understand the accelerated expansion of the universe. Researchers are paying attention to these alternative theories because it is thought to offer a natural gravitational alternative to dark energy, which explains the universe's dark energy and late-time cosmic acceleration. Dark energy is highly considered the main element for accelerated expansion. Many studies have been executed for dark energy models in different gravitational theories in both the isotropic and anisotropic backgrounds. Cosmologists employ isotropy as the most common and important type of spacetime to investigate how our universe (particularly DE) is currently behaving.However, the anisotropic models help us better understand the early stages of the universe's evolution. The finding of some anisotropy in the background radiation has now increased the importance of spatially homogenous and anisotropic Bianchi models. The two fluids, barotropic fluid and dark energy filled in Bianchi type-III universe were explored by Zia et al. [(1)] while considering the scale factor that results in a variable deceleration parameter. The EoS parameter emerges as the possible characteristic to study the nature of dark energy. In both interacting and non-interacting forms of the fluid,the equation of state for dark energy represents the phantom model at late times. For the non-interacting one, it begins in the quintessence region and moves through the ΛCDM model and eventually to thephantom model. Srivastava et al. [(2)] examined the anisotropic Bianchi type-III model with time-dependent deceleration parameter in general relativity; here, the EoS parameter for the new holographic dark energy (NHDE) falls under the k-essence region, which at late time coincides with the flat ΛCDM model.The Tsallis holographic dark energy model with time-dependent deceleration parameter in FRW universe addressed by Dixit et al. [(3)] effectively maintains the universe's acceleration. The EoS parameter, which corresponds to the suggested new holographic dark energy model, illustrates the change from quintessence to phantom. Further, the concept is also consistent with the flat ΛCDM model. In the Saez-Ballester modified theory of gravitation, Santhi and Sobhanbabu [(4)] also looked into the Tsallis holographic dark energy in Bianchi type III spacetime which coincides with the isotropic model discussed by Dixit et al. [(3)]. In the discussion of the anisotropic Bianchi Type-III dark energy cosmological model with a massive scalar meson field in general relativity, Raju et al. [(5)] noted that the EoS parameter of the model varies initially in the phantom region and then approaches the quintessence region as the model's physical parameters become independent of the scalar field at late times. In the case of modified holographic Ricci dark energy, Dixit et al. [(6)] explored the axially symmetric, spatially homogeneous anisotropic Bianchi type in Brans Dicke theory and noted that the EoS passes the phantom divide line and decreases with the increase in the scalar field. Many other authors investigated Bianchi type-III spacetime in different gravitational theories that provided many series of literature (Pawar and Sahare [(7)], Koronur [(8)], Bhardwaj and Rana [(9)], Baro et al. [(10)]).

In cosmology, the magnetic field plays a crucial role in characterizing the ionized behavior that conducts energy fluctuations during the universe's expansion. The existence of the magnetic field affects the expansion independent of its strength and may result in anisotropy in the accelerated expansion (Matravers and Tsagas [(11)]). Furthermore, the impact of the magnetic field is slightly more significant in the early epoch; its influence cannot be excluded at a later cosmic phase around the present epoch (Ray et al. [(12)]). Even though the skewness disappears in the absence of a magnetic field, it does not do so when a magnetic field is present, although the magnetic field diminishes along with time (Sharma et al. [(13)]). Thus, the cosmos is expanding faster due to the decrease in magnetic permeability (Hegazy and Rehman [(14)]).

The Bianchi type-III spacetime is studied abundantly because of the unique geometric characteristics that set it apart from the other Bianchi types. The present work is motivated by the studies on Bianchi type-III spacetime in various contexts. In this study, we try to firm the knowledge of the Bianchi universe, considering the scalar-tensor theory of gravitation in the presence of an electromagnetic field. In order to obtain the exact solutions of the field equation, the special form of time-dependent deceleration parameter generalized by Banerjee and Das [(15)] is imposed to derive the model. The exact solution of the model is obtained considering the shear scalar proportional to the expansion scalar and the power law relation of the scalar field with the average scale factor. In this study, the model for the Bianchi type-III spacetime with the magnetic field in the scalar-tensor theory is also accelerating with exponential expansion approaching the standard ΛCDM model. In section 2, we formulate the metric and derive the field equations with the presence of an electromagnetic field. In section 3, the work presents the solutions to the field equation by considering some plausible conditions. In sections 4 and 5, the model's cosmological parameters are presented. In sections 6 and 7, we interpret and provide the study's conclusion.

2 Metric and Field Equations

The metric for Bianchi type-III considered as

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{-2mx}dy^2 + C^2dz^2$$

where A, B, and C are the function of cosmic time t only.

The field equation given by Sen and Dunn [(16)] with gauge function (in natural units c = 1, $8\pi G = 1$) is

$$R_{ij} - \frac{1}{2}Rg_{ij} = \omega\varphi^{-2}\left(\varphi_{,i}\varphi_{,j} - \frac{1}{2}g_{ij}\varphi_{,k}\varphi^{,k}\right) - \varphi^{-2}T_{ij}$$

where $\omega = 3/2$, R_{ij} is the Ricci tensor, R is the Ricci scalar;$T_{ij}$is the energy-momentum tensor.

The energy-momentum tensor with the presence of an electromagnetic field is

$$T_{ij} = (\rho + p)\, u_i u_j + p g_{ij} + E_{ij}$$

Here $u^i = (0,0,0,1)$ is the four-velocity vector such that $g^{ij} u_i u_j = -1$. And taking

$$E_{ij} = \frac{1}{4\pi}[g^{hl} F_{ih} F_{jl} - \frac{1}{4} F_{hl} F^{hl} g_{ij}]$$

From the Maxwell equation,

$$\frac{\partial}{\partial x^j}\left(F^{ij}\sqrt{-g}\right) = 0$$

Eq. (5) leads to the result

$$F_{12} = K e^{-2mx}$$

where K and m are constants.

The field equation (2) with the equation (3), (4), and (6) for the equation (1) are

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{m^2}{A^2} = \frac{\omega}{2}\left(\frac{\dot{\varphi}}{\varphi}\right)^2 - \varphi^{-2}\left(p - \frac{K^2}{8\pi A^2 B^2}\right)$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = \frac{\omega}{2}\left(\frac{\dot{\varphi}}{\varphi}\right)^2 + \varphi^{-2}\left(p - \frac{K^2}{8\pi A^2 B^2}\right)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = \frac{\omega}{2}\left(\frac{\dot{\varphi}}{\varphi}\right)^2 + \varphi^{-2}\left(p - \frac{K^2}{8\pi A^2 B^2}\right)$$

$$\frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} - \frac{m^2}{A^2} = \varphi^{-2}\left(p + \frac{K^2}{8\pi A^2 B^2}\right) - \frac{\omega}{2}\left(\frac{\dot{\varphi}}{\varphi}\right)^2$$

$$m\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = 0$$

The average scale factor for Bianchi type-III spacetime is

$$a(t) = (ABC)^{\frac{1}{3}}$$

The spatial volume, Hubble parameter is defined as

$$V = ABC = R^3$$

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)$$

And deceleration parameter is defined as

$$q = -\frac{a\ddot{a}}{\dot{a}^2}$$

The expansion scalar, shear scalar, and the anisotropic parameter are

$$\theta = 3H$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{2}\left(\sum_{v=1}^{3} H_v^2 - \frac{1}{3}\theta^2\right)$$

$$A_m = \frac{1}{3}\sum_{v=1}^{3}\left(\frac{H_v - H}{H}\right)^2$$

where, H_v(v =x, y, z) the directional parameter, and for the metric (1), it is given as $H_x = \frac{\dot{A}}{A},\ H_y = \frac{\dot{B}}{B},\ H_y = \frac{\dot{C}}{C}$.

3 Solutions of the field equations

From equation (11), by considering m = 1, we have

$$A = nB$$

Where n is the integration constant. Here we consider $n = 1$ without the loss of generality. So the field equation (7)-(10) is transformed to

$$2\frac{\ddot{B}}{B} + \left(\frac{\dot{B}}{B}\right)^2 - \frac{m^2}{B^2} = \frac{\omega}{2}\left(\frac{\dot{\varphi}}{\varphi}\right)^2 - \varphi^{-2}\left(p - \frac{K^2}{8\pi A^2B^2}\right)$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = \frac{\omega}{2}\left(\frac{\dot{\varphi}}{\varphi}\right)^2 + \varphi^{-2}\left(p - \frac{K^2}{8\pi A^2B^2}\right)$$

$$\frac{\ddot{B}}{B} + 2\frac{\dot{B}\dot{C}}{BC} - \frac{m^2}{B^2} = \varphi^{-2}\left(p + \frac{K^2}{8\pi A^2B^2}\right) - \frac{\omega}{2}\left(\frac{\dot{\varphi}}{\varphi}\right)^2$$

Here, the nonlinear equation (20), (21), and (22) contains the unknown variables $B,\ C,\ p, \rho and \varphi$. So to obtain the exact solution of the field equations above, we consider some plausible conditions.

To obtain the model compatible with the cosmological observation, we consider the variable deceleration parameter q (Banerjee and Das [15]) given by

$$q = -1 + \frac{k}{1 + a^k}$$

where "a" is the scale factor, and $k\ (>\ 0)$ is constant. The importance of the deceleration parameter lies in the hypothesis that the present universe has a transitional phase of expansion. It was decelerating in the past and is accelerating at present. Many researchers generalized the deceleration parameter to study the dynamical behavior of the accelerating universe. Pradhan et al. [17] utilized the time-dependent deceleration parameter certifying the accelerating universe and observing the Type-III model of the singularity of the Bianchi type-I universe in the background of f(R, T) gravity. The concept of bilinear varying deceleration parameter (BVDP), which shows super-exponential expansion, is highly discussed by Mishra and Chand [18] to extract the solutions of field equation for the anisotropic Bianchi type-I universe in the Saez-Ballester theory of gravitation. Recently, a deceleration parameter which is a function of time as well as periodic known as periodic time-varying deceleration parameter (PTVDP), which oscillates periodically from 2 to -4, representing accelerating expansion, has been used to observe the cosmological properties of the early universe as well as the fate of the universe by (Alam and Singh [19], Garg et al. [20]).

From equation (23), we obtain the Hubble parameter as

$$H = \frac{\alpha e^{\alpha kt}}{e^{\alpha kt} - 1} = \alpha(1 + a^{-k})$$

where α is an integrating constant.

As Collins et al. [21] expressed that for the spatially homogeneous metric, the normal congruence to the homogeneous hypersurface satisfies the condition $\frac{\sigma}{\theta}$ = constant. This condition leads to

$$B = C^{\beta}$$

here, β is an arbitrary constant.

We consider the gauge function for the model as

$$\varphi = \varphi_0 a^f(t)$$

where, φ_0; and f being an ordinary constant.

From the equation (24) by integration, we have the scale factor as

$$a(t) = \left(e^{\alpha kt} - 1\right)^{\frac{1}{k}}$$

Here, α and k are positive constants.

Thus using equation (27) in equation (12), we get the following result as

$$A = B = \left(e^{\alpha kt} - 1\right)^{\frac{3\beta}{k(2\beta+1)}}$$

$$C = \left(e^{\alpha kt} - 1\right)^{\frac{3}{k(2\beta+1)}}$$

The model (1) with the equation (28) and (29) reduces to

$$ds^2 = -dt^2 + \left(e^{\alpha kt} - 1\right)^{\frac{3\beta}{k(2\beta+1)}} dx^2 + \left(e^{\alpha kt} - 1\right)^{\frac{3\beta}{k(2\beta+1)}} e^{-2mx} dy^{2} + \left(e^{\alpha kt} - 1\right)^{\frac{3}{k(2\beta+1)}} dz^2$$

4 Cosmological Parameters of the model

From the equation (20)-(22), we obtain the following energy density and pressure using equations (26), (28), and (29)

$$\rho = \varphi_0^2 \left(e^{\alpha kt} - 1\right)^{\frac{f}{k}} \left(\frac{9\left(1+3\beta-\beta^2\right)\alpha^2 e^{2\alpha kt}}{(2\beta+1)^2 (e^{\alpha kt}-1)^2} - \frac{3k\alpha^2 e^{\alpha kt}}{(2\beta+1)(e^{\alpha kt}-1)^2} + \frac{3\alpha^2 f^2 e^{2\alpha kt}}{4(e^{\alpha kt}-1)^2}\right] + \frac{K^2}{(e^{\alpha kt}-1)^{\frac{12\beta}{k(2\beta+1)}}}$$

$$-p = \frac{\varphi_0^2}{2} \left(e^{\alpha kt} - 1\right)^{\frac{f}{k}} \left(\frac{9\left(1+3\beta+4\beta^2\right)\alpha^2 e^{2\alpha kt}}{(2\beta+1)^2 (e^{\alpha kt}-1)^2} - \frac{3(3\beta+1)k\alpha^2 e^{\alpha kt}}{(2\beta+1)(e^{\alpha kt}-1)^2} - m^2\left(e^{\alpha kt} - 1\right)^{\frac{-6\beta}{(2\beta+1)k}} - \frac{3\alpha^2 f^2 e^{2\alpha kt}}{2(e^{\alpha kt}-1)^2}\right]$$

The spatial volume, expansion scalar, shear scalar, and anisotropic parameter for the model are as follows

$$V = \left(e^{\alpha kt} - 1\right)^{\frac{3}{k}}$$

$$\theta = \frac{3\alpha e^{\alpha kt}}{e^{\alpha kt} - 1}$$

$$\sigma = \frac{\sqrt{3}(\beta-1)\alpha e^{\alpha kt}}{(2\beta+1)(e^{\alpha kt}-1)}$$

$$A_m = \frac{2(\beta-1)^2}{(2\beta+1)^2} \tag{36}$$

Thus we also have

$$\frac{\sigma^2}{\theta^2} = \frac{(\beta-1)^2}{3(2\beta+1)^2} = \text{constant}(\neq 0,\ \text{for } \beta > 1) \tag{37}$$

Energy condition and Statefinder parameter

$$\rho + p = \frac{\varphi_0^2}{2}\left(e^{\alpha kt}-1\right)^{\frac{f}{k}}\left[\frac{9\left(1+5\beta-6\beta^2\right)\alpha^2 e^{2\alpha kt}}{(2\beta+1)^2\left(e^{\alpha kt}-1\right)^2} + \frac{3(3\beta-1)k\alpha^2 e^{\alpha kt}}{(2\beta+1)\left(e^{\alpha kt}-1\right)^2} + m^2\left(e^{\alpha kt}-1\right)^{\frac{-6\beta}{(2\beta+1)k}} + \frac{3\alpha^2 f^2 e^{2\alpha kt}}{2\left(e^{\alpha kt}-1\right)^2}\right] + \frac{K^2}{\left(e^{\alpha kt}-1\right)^{\frac{12\beta}{k(2\beta+1)}}} \tag{38}$$

$$\rho - p = \frac{\varphi_0^2}{2}\left(e^{\alpha kt}-1\right)^{\frac{f}{k}}\left(\frac{9(1+4\beta+3\beta^2)\alpha^2 e^{2\alpha kt}}{(2\beta+1)^2(e^{\alpha kt}-1)^2} + \frac{9(\beta-1)k\alpha^2 e^{\alpha kt}}{(2\beta+1)\left(e^{\alpha kt}-1\right)^2} - m^2\left(e^{\alpha kt}-1\right)^{\frac{-6\beta}{(2\beta+1)k}}\right] + \frac{K^2}{\left(e^{\alpha kt}-1\right)^{\frac{12\beta}{k(2\beta+1)}}}$$

$$\rho + 3p = \frac{\varphi_0^2}{2}\left(e^{\alpha kt}-1\right)^{\frac{f}{k}}\left(\frac{9\left(3\beta-14\beta^2-1\right)\alpha^2 e^{2\alpha kt}}{(2\beta+1)^2\left(e^{\alpha kt}-1\right)^2} + \frac{3(9\beta+1)k\alpha^2 e^{\alpha kt}}{(2\beta+1)\left(e^{\alpha kt}-1\right)^2} + m^2\left(e^{\alpha kt}-1\right)^{\frac{-6\beta}{(2\beta+1)k}} + \frac{6\alpha^2 f^2 e^{2\alpha kt}}{\left(e^{\alpha kt}-1\right)^2}\right] + \frac{K^2}{\left(e^{\alpha kt}-1\right)^{\frac{12\beta}{k(2\beta+1)}}} \tag{40}$$

From the following result, the energy condition identified that the Null Energy Condition (NEC), the Weak Energy Condition (WEC), and the Strong Energy Condition (SEC) are satisfied throughout the cosmic evolution of the model. The model's Dominant Energy Condition (DEC)l rapidly increases from negative phase to positive boundary after some cosmic time $t = 0.3$, and later it is satisfied for the stability of the universe's expansion.

The statefinder parameter {r,s} introduced by Shani et al. [22] is given as

$$r = \frac{\dddot{a}}{aH^3} = 1 + 3\frac{\dot{H}}{H^2} + \frac{\ddot{H}}{H^3}$$

$$s = \frac{(r-1)}{3\left(q-\frac{1}{2}\right)}$$

Here, H is the Hubble parameter, and q is the deceleration parameter. The cosmological diagnostic pair {r,s} allow us to determine the characteristic properties of the dark energy in a model-independent approach. With the equations (23) and (24), we can rewrite the diagnostic pair {r,s} as

$$r = 1 - \frac{3k}{e^{\alpha kt}} + \frac{k^2\left(e^{\alpha kt}\right)+1}{e^{2\alpha kt}}$$

$$s = \frac{2k\left(ke^{\alpha kt}+k-3e^{\alpha kt}\right)}{3e^{\alpha kt}\left(2k-3e^{\alpha kt}\right)}$$

We observe from the above result that when $t \to \infty$ we get the pair $(r,s\} \to \{1,0\}$, which explains that the model of the universe starts from the radiation era to the ΛCDM model.

5 Results and Discussion

The equation (30) represents the model for the Bianchi type-III cosmological model with the electromagnetic field in the framework of Sen-Dunn scalar-tensor theory. The physical and geometrical parameters for the model are discussed above:

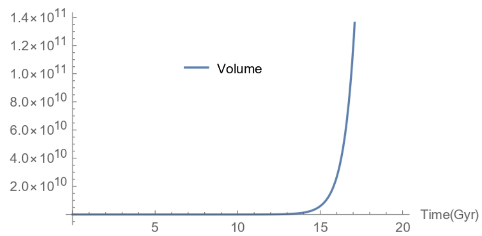

Fig 1. Volume Vs. Time: For$\alpha = 0.5,\ k = 1.75$

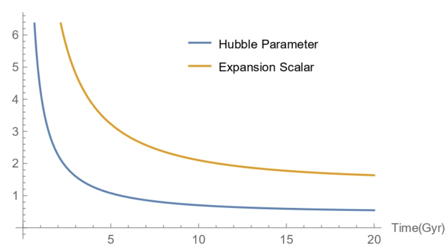

Fig 2. Hubble parameter, Expansion scalar Vs. Time:For$\alpha = 0.5,\ k = 1.75$

• The plotting of the figures are drawn against time (Gyr) by considering the values $\alpha = 0.5,\ \beta = 1.5,\ k = 1.75,\ f = -0.62, \varphi_0 = 1$. The graphical representation of the physical and kinematical parameters with this values signifies that the model obtained in this study are constrained with the recent cosmological observations.

• The model is expanding, shearing, and non-rotating, with the spatial volume increasing from the finite volume at $t = 0$ and expanding to an infinite volume as $t \to \infty$ is shown in Figure 1. The model's expansion scalar and Hubble parameter positively decrease with the increase in time, as shown in Figure 2, which shows that the universe's expansion rate is slower as time increases and expansion gradually ends for $t \to \infty$.

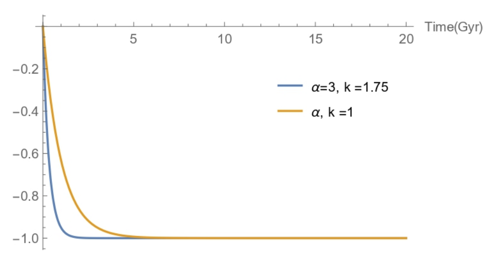

Fig 3. Deceleration Parameter Vs. Time

• The relation $\frac{\sigma^2}{\theta^2} = constant$ show that the model (30) does not tend to isotropy for $\beta \neq 1$ at late times. Its contrast to the Bianchi type I spacetime model considering a similar deceleration parameter proclaims isotropy for large values of "t" by Tiwari and Beesham [23].

• The model's energy density and the pressure given by equation (31) and (32)positively decreases.Also, it tends to be zero as $t \to \infty$; for a significant value of time t, as shown in Figure 4. A similar characteristic of the energy density, pressure, and Hubble parameter and expansion scalar is also observed in the string cosmological model for Bianchi type-III spacetime in Lyra geometry(Baro et al. [10]). The q bounds to the value $-1 \leq q < 0$from decelerating phase $q > 0$ to de-Sitter expansion

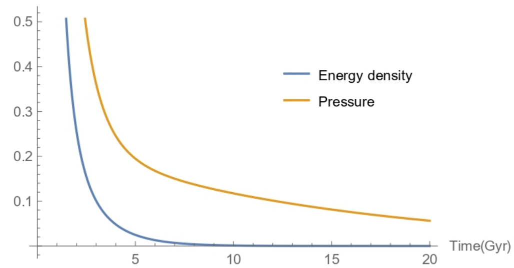

Fig 4. Energy density, Pressure Vs. Time: For$\alpha = 0.5,\ k = 1.75,\ \beta = 1.5,\ \ f = -0.62,\ \varphi_0 = 1$

or exponential expansion, which explains the transition from the early deceleration to the late time acceleration as shown in Figure 3 . The result of the deceleration parameter follows the recent cosmological data. Some model also exhibits super-exponential expansion at the early phase with $q < -1$ and, finally, at late times, tends to exponential expansion $q = -1$ (Santhi and Sobanbabu [4]).

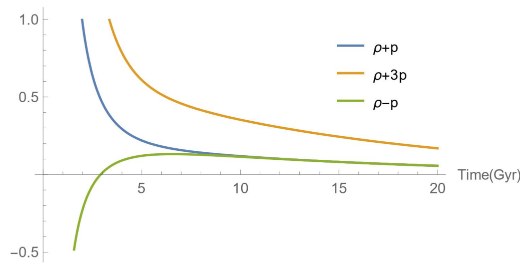

Fig 5. Energy condition Vs. Time:For $\alpha = 0.5,\ k = 1.75,\ \beta = 1.5,\ \ f = -0.62,\ \varphi_0 = 1$

• Initially, the dominant energy condition (DEC) gets violated for time $t < \ 0.3$, and then rapidly, it satisfies the energy condition as positively decreasing with time. The NEC, WEC, and SEC satisfy the energy condition throughout the time t (Figure 5). The energy condition depicts the early deceleration to the present acceleration of the universe.

• From the above result (43) and (44), the nature of the statefinder parameter is obtained as$\{r, s\} \rightarrow (1, 0)$ as $t \rightarrow \infty$, which shows that the universe approaches the ΛCDM model at late times, which matches the recent cosmological model of the universe. The same result was observed in the previous study considering a special form of scale factor for Bianchi type-VI$_0$ in the framework of Sen-Dunn theory (Basumatary and Dewri [24]).

6 Conclusion

This paper investigates the solutions of Bianchi type-III spacetime in the presence of an electromagnetic field in the Sen-Dunn scalar-tensor theory of gravitation. To deliver the solutions to the field equations, we consider the time-dependent deceleration parameter q, a signature flip property (Banerjee and Das [15]). The model (30) expands with zero volume at $t \ = \ 0$to infinite as time $t \rightarrow \infty$ and eventually, at late times, tends to the de-Sitter universe. For the significant value of t, the present model does not tend to isotropy, except for particular exceptional cases for $\beta = 1$. The model is expanding, shearing, non-rotating, and anisotropic throughout the evolution admitting initial singularity at $t \ = \ 0$, which shows a relevant characteristic of the model investigated by Pawar and Sahare [7]. The diagnostic pair {r,s} represents that the model of the universe tends to ΛCDM as time $t \rightarrow \infty$, which is consistent with the recent observational data. The study presents a toy piece of the Bianchi type-III cosmological model in the Sen-Dunn scalar-tensor theory of gravitation. The generated and given model represents an expanding and accelerating cosmological model, delivered by examining the above physical and geometrical factors. The model studied in this part is compatible with the recent cosmological observation.Therefore, more profound knowledge of the cosmic evolution of the universe will result from studying the Bianchi spacetime in the scalar-tensor theory of gravitation.

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