INDIAN JOURNAL OF SCIENCE AND TECHNOLOGY

RESEARCH ARTICLE

OPEN ACCESS

Received: 14-09-2022

Accepted: 08-10-2022

Published: 23-11-2022

Citation: Sumathi P, Geetha Ramani G (2022) Arithmetic Sequential Graceful Labeling on Star Related Graphs. Indian Journal of Science and Technology 15(44): 2356-2362. https://doi.org/10.17485/IJST/v15i44.1863

* **Corresponding author**.

geetharamani.v@gmail.com

Funding: None

Competing Interests: None

Published By Indian Society for Education and Environment (iSee)

ISSN
Print: 0974-6846
Electronic: 0974-5645

Arithmetic Sequential Graceful Labeling on Star Related Graphs

P Sumathi[1], G Geetha Ramani[2]*

1 Department of Mathematics, C. Kandaswami Naidu College for Men, Chennai, Tamil Nadu, India
2 Department of Mathematics, New Prince Shri Bhavani College of Engineering and Technology, Chennai, Tamil Nadu, India

Abstract

Objectives: To identify a new family of Arithmetic sequential graceful graphs. **Methods:** The methodology involves mathematical formulation for labeling of the vertices of a given graph and subsequently establishing that these formulations give rise to arithmetic sequential graceful labeling. **Findings:** In this study, we analyzed some star related graphs namely Star graph, U-star, t-star, and double star proved that these graphs possess Arithmetic sequential graceful labeling. **Novelty:** Here, we introduced a new labeling called Arithmetic sequential graceful labeling and we give Arithmetic sequential graceful labeling to some star related graphs.

Keywords: Star graph; t-star; U-star; Graceful labeling; Double star

1 Introduction

Graph labeling is one of the most exciting areas of research in graph theory. Labeling is the process of giving values to edges or vertices. According to a recent dynamic survey,[(1)] of graph labeling, numerous scholars contribute their work on various forms of graceful labeling, such as Skolem Graceful Labeling[(2)], Edge Even Graceful Labeling[(3,4)]. Odd Graceful Labeling[(5)] , and so on. Motivated by the idea of graceful labeling, we developed arithmetic sequential graceful labeling as an injective function of a simple, finite, connected, undirected, and non-trivial graph G as an injective function $f : V(G) \to \{a, a+d, a+2d, a+3d, \ldots, 2(a+qd)\}$ where a ≥ 0 and $d \geq 1$ such that for each edge $uv \in E(G) f^* : E(G) \to \{d, 2d, 3d, 4d, \ldots, qd\}$ defined by $f^*(uv) = (f(u) - f(v)|$ is a bijective. Also, we proved some star related graphs are arithmetic sequential graceful.

Definition 1.1: A star graph is the complete bipartite graph $K_{1,n}$, a tree with one internal node and n leaves.

Definition 1.2: For t number of star graphs, ,$t \geq 2$ namely $K_{1,n_1}, K_{1,n_2}, \ldots, K_{1,n_t}$ then the $t-$ star graph is constructed by joining the apex vertices of each K_{1,n_i} and $K_{1,n_{i+1}}$ to vertices $u_i : 1 \leq i \leq t-1, n_i \geq 1$.

Definition 1.3 : A U-star graph is formed by linking the leaves of two star graphs by a bridge.

Definition 1.4 : The double star graph is created by linking the centers of 2 star graphs, $K_{1,n}$ and $K_{1,m}$, with an edge.

2 Methodology

A simple, finite, connected, undirected, non-trivial graph G is arithmetic sequential graceful graph, if the function $f : V(G) \to \{a, a+d, a+2d, a+3d, \ldots, 2(a+qd)\}$ where a ≥ 0 and $d \geq 1$ is an injective function and for each edge $uv \in E(G)$, the induced function $f^* : E(G) \to \{d, 2d, 3d, 4d, \ldots, qd\}$ defined by $f^*(uv) = |f(u) - f(v)|$ is a bijective. The function, then the graph G is called arithmetic sequential graceful graph.

3 Results and Discussion

Theorem 3.1:

The graph $K_{1,\ n}$, $n \geq 1$ admits arithmetic sequential graceful labeling.

Proof: Consider the graph $G = K_{1,n}$, $n \geq 1$

Let $V(G) = \{v_0\} \cup \{v_{i:1 \leq i \leq n}\}$ and

$E(G) = \{v_0 v_i : 1 \leq i \leq n\}$

Here $|V| = n+1, (E| = n$.

We define a function $f : V \to \{a, a+d, a+2d, a+3d, \ldots, 2(a+qd)\}$

The vertex labelings are as follows,

$$f(v_0) = a + nd$$

$$f(v_i) = a + (i-1)d, \qquad 1 \leq i \leq n$$

From the function $f^* : E(G) \to \{d, 2d, 3d, 4d, \ldots, qd\}$, we get the edge labels of the graph $K_{1,n}$, $n \geq 1$ as follows

Table 1. Edge labels of the graph$K_{1,n}$, $n \geq 1$

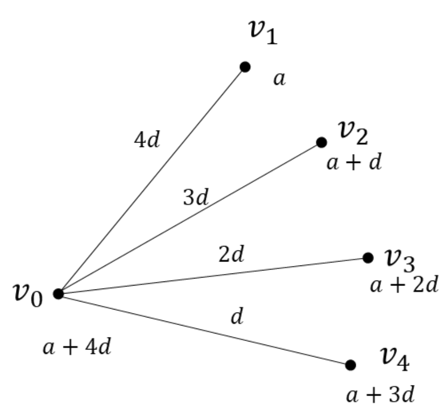
Nature of n	$n \geq 1$
$f^*(v_0 v_i)$	$\lvert(n-i+1)d\rvert$, $1 \leq i \leq n$

The function f is clearly injective and also Table 1 shows that

$f^* : E \to \{d, 2d, 3d, 4d, \ldots, qd\}$ is bijective. Hence f is arithmetic sequential graceful labeling and the graph $K_{1,n}$ is arithmetic sequential graceful graph.

Example 3.1.1:

The Star graph admits arithmetic sequential graceful labeling.



Theorem 3.2:

The double star admits arithmetic sequential graceful graph for $n, m \geq 1$.

Proof:

Let G be a double star. $V(G) = \{v_0, u_0\} \cup \{v_{i:1 \leq i \leq n}\} \cup \{u_{i:1 \leq i \leq m}\}$ and

$E(G) = \{v_0 u_0\} \cup \{v_0 v_{i:1 \leq i \leq n}\} \cup$

$\{u_0 u_i : 1 \leq i \leq m\}$
Here $|V| = n+m+2, |E| = n+m+1$
Define the function $f : V \to \{a, a+d, a+2d, a+3d, \ldots, 2(a+qd)\}$
The vertex labelings are as follows,

$$f(v_i) = a + (i-1)d, 1 \leq i \leq n$$

$$f(u_i) = a + (n+i)d, 1 \leq i \leq m$$

$$f(v_0) = a + (n+m+1)d$$

$$f(u_0) = a + nd$$

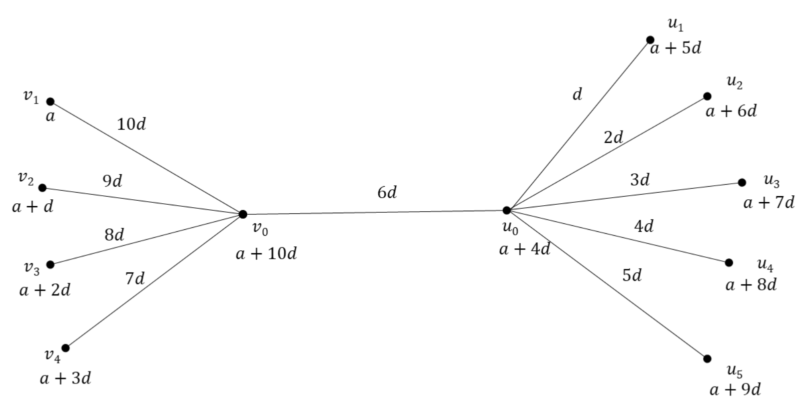
Table 2. Edge labels of the double star,$n,\ m \geq 1$.

Nature of n, m	$n, m \geq 1$
$f^*(u_0 v_0)$	$\|(m+1)d\|$
$f^*(v_0 v_i)$	$\|(n+m+2-i)d\|, 1 \leq i \leq n$
$f^*(u_0 u_i)$	$\|id\|, 1 \leq i \leq n$

The function f is clearly injective and also table 2 shows that $f^* : E \to \{d, 2d, 3d, 4d, \ldots, qd\}$ is bijective. Hence f is arithmetic sequential graceful labeling and the graph double star is arithmetic sequential graceful graph.

Example 3.2.1:

The double star graph admits arithmetic sequential graceful graphs.



Theorem 3.3:

The graph $t-$ star ,$t \geq 2$ admits arithmetic sequential graceful labeling.

Proof: Let G be a $t-$ star ,$t \geq 2$ graph.

Let $V(G) = \{v_{i:1 \leq i \leq m}\} \cup$
$\{u_j : 1 \leq j \leq m-1\} \cup \{v_{jk} : 1 \leq i \leq m; 1 \leq k \leq n_j\}$ and $E(G) = \{v_j v_{jk} : 1 \leq j \leq m; 1 \leq k \leq n_j\} \cup \{v_{iu_{i:1 \leq i \leq m-1}}\} \cup$
$\{u_{\ iv_{i+1}:1 \leq i \leq m-1\}}$
Here $|V| = \sum_{i=1}^{m} n_{i+2m-1},$

$$|E| = \sum_{i=1}^{m} n_{i+2m-2}$$

Define a function $f : V \to \{a, a+d, a+2d, a+3d, \ldots, 2(a+qd)\}$

The vertex labelings are as follows,

$$f(v_i) = a + (m-i)d,\ 1 \le i \le m$$

$$f(u_j) = a + \left|\sum_{i=1}^{j} n_i + j + m - 1\right., 1 \le j \le m-1$$

$$f(v_{1k}) = a + ([m+k] - 1)d, 1 \le k \le n_1$$

For $2 \le j \le m$,

$$f(v_{jk}) = a + \left[\sum_{i=1}^{j-1} n_{i+j+m-2+k}\right] d, 1 \le k \le n_j$$

From the function

$f^*: E(G) \to \{d, 2d, 3d, 4d, \ldots, qd\}$, we get the edge labels of the graph $t-$ star ,$t \ge 2$ as follows

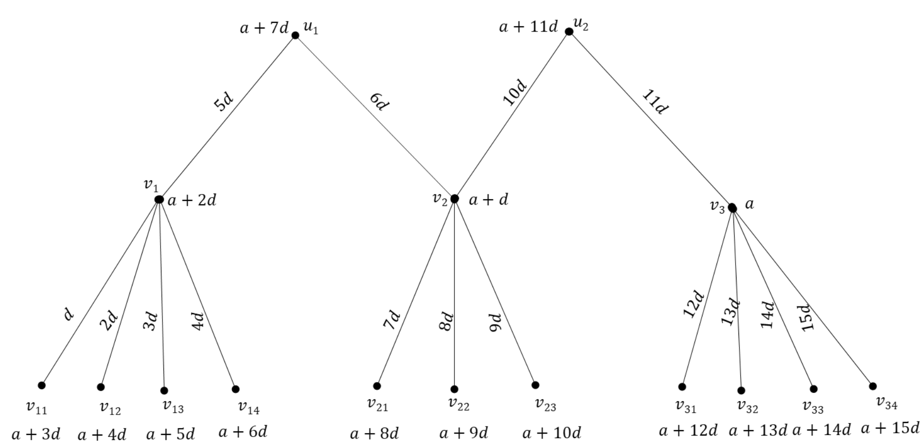
Table 3. Edge labels of the$t-$ star, $t \ge 2$

Nature of n	$n \ge 1$
$f^*(v_i u_j)$	$\left|\left[(m-i) - \left(\sum_{i=1}^{j} n_{i+j+m-1}\right)\right] d\right| 1 \le i \le m, 1 \le j \le m-1$
$f^*(v_i v_{1k})$	$\lvert(-i-k+1)d\rvert, 1 \le i \le m, 1 \le k \le n_1$
$f^*(v_i v_{jk})$	$\left|\left[(m-i) - \left(\sum_{i=1}^{j-i} n_i + j + m - 2 + k\right)\right] d\right| 1 \le i \le m, 1 \le k \le n_j$

The function f is clearly injective and also table 3 shows that $f^*: E \to \{d, 2d, 3d, 4d, \ldots, qd\}$ is bijective. Hence f is arithmetic sequential graceful labeling and the graph $t-star$ is arithmetic sequential graceful graph.

Example 3.3.1:

$3-star$ graph admits arithmetic sequential graceful labeling.



Theorem 3.4:

The $U-$ star graph admits arithmetic sequential graceful labeling. for $n \ge 1, n \in N$.

Proof: Let G be a $U-$ star graph. $V(G) = (w_1\} \cup (w_2\} \cup (v_0\} \cup (u_0\} \cup \{v_{i:1 \le i \le n}\} \cup$ $\{u_i : 1 \le i \le n\}$ and $E(G) = \{w_1 v_0, v_0 u_0, w_2 u_0\} \cup \{w_1 v_{i:1 \le i \le n}\} \cup \{w_2 u_{i:1 \le i \le n}\}$

Here $|v| = 2n+4, |E| = 2n+3$

Define the function $f: V \to \{a, a+d, a+2d, a+3d, \ldots, 2(a+qd)\}$

The vertex labelings are as follows,

Case (i): when $n = 1$

$$f(w_1) = a + d$$

$$f(w_2) = a$$

$$f(v_0) = a + 2d$$

$$f(u_0) = a + 5d$$

$$f(v_1) = a + 3d$$

$$f(u_1) = a + 4d$$

Case (ii): when $n = 2$

$$f(w_1) = a + d$$

$$f(w_2) = a$$

$$f(v_0) = a + 2d$$

$$f(u_0) = a + 6d$$

$$f(v_1) = a + 3d$$

$$f(v_2) = a + 4d$$

$$f(u_1) = a + 5d$$

$$f(u_2) = a + 7d$$

Case (ii): when $n \geq 3$

$$f(w_1) = a + d$$

$$f(w_2) = a$$

$$f(v_0) = a + nd$$

$$f(u_0) = a + (2n+2)d$$

$$f(v_i) = a + (i+1)d, 1 \le i \le n-2$$

$$f(v_{n-1}) = a + (n+1)d$$

$$f(v_n) = a + (n+2)d$$

$$f(u_i) = a + (n+2+i)d, 1 \le i \le n-1$$

$$f(u_n) = a + (2n+3)d$$

Table 4. Edge labels of the $U-$ star, $n \ge 1, n \in N$.

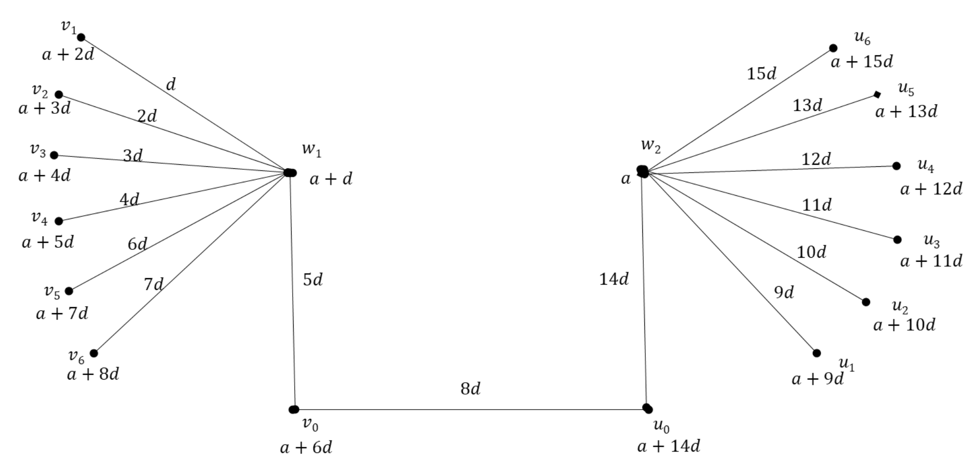
Nature of n	$n \ge 1$
$f^*(w_1 v_0)$	$\lvert(1-n)d\rvert, n \ge 1$
$f^*(v_0 u_0)$	$\lvert(n+2)d\rvert, n \ge 1$
$f^*(u_0 w_2)$	$\lvert(2n+2)d\rvert, n \ge 1$
$f^*(w_1 v_i)$	$\lvert id\rvert, 1 \le i \le n-2$
$f^*(w_2 u_i)$	$\lvert(n+2+i)d\rvert, 1 \le i \le n-1$

The function f is clearly injective and also table 4 shows that

$f^* : E \to \{d, 2d, 3d, 4d, \ldots, qd\}$ is bijective. Hence f is arithmetic sequential graceful labeling and the $U-$ star graph is an arithmetic sequential graceful graph.

Example 3.4.1:

The $U-$ star graph admits arithmetic sequential graceful labeling



4 Conclusion

We showed that the Star graph, $t-$ star, $U-$star, double star are arithmetic sequential graceful in this work. Analyzing arithmetic sequential graceful on other families of graphs is our future work.

References

1) Gallian JA. A dynamic survey of graph labeling. *Electronic Journal of combinatorics*. 2021;1. Available from: https://doi.org/10.37236/27.
2) Pasaribu M, Yundari Y, Ilyas M. Graceful Labeling and Skolem Graceful Labeling on the U-star Graph and It's Application in Cryptography. *Jambura Journal of Mathematics*. 2021;3(2):103–114. Available from: https://doi.org/10.34312/jjom.v3i2.9992.
3) Deen MRZE. Edge even graceful labeling of some graphs. *Journal of the Egyptian Mathematical Society*. 2019;27(1):1–5. Available from: https://doi.org/10.1155/2021/6643173.
4) Deen MRZE, Omar NA. Extending of Edge Even Graceful Labeling of Graphs to Strong-Edge Even Graceful Labeling. *Journal of Mathematics*. 2021;(6643173). Available from: https://doi.org/10.1155/2021/6643173.
5) Amri Z, Irvan I, Maryanti I, Sumardi H. Odd graceful labeling on the Ilalang graph (Sn, 3) it's variation. *Journal of Physics: Conference Series*. 2021;1731(1):012030.