INDIAN JOURNAL OF SCIENCE AND TECHNOLOGY

RESEARCH ARTICLE

Lucas Antimagic Labeling of some Star Related Graphs

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OPEN ACCESS

Received: 15-09-2022
Accepted: 25-09-2022
Published: 10-12-2022

Citation: Sumathi P, Chandravadana N (2022) Lucas Antimagic Labeling of some Star Related Graphs. Indian Journal of Science and Technology 15(46): 2542-2547. https://doi.org/10.17485/IJST/v15i46.1862

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Funding: None

Competing Interests: None

Published By Indian Society for Education and Environment (iSee)

ISSN
Print: 0974-6846
Electronic: 0974-5645

Abstract

Objective: To identify a new family of Lucas antimagic graph. **Methods:** A (p,q) graph G is said to be a Lucas antimagic graph if there exists a bijection $f: E(G) \to \{L_1, L_2, \cdots L_q\}$ such that the induced injective function $f^* : V(G) \to \{1, 2, \ldots \sum L_q\}$ given by $f^*(u) = \sum_{e \in E(u)} f(e)$ are all distinct (where E(u) is the set of edges incident to u). **Findings:** In this paper the Lucas Antimagic Labeling of Subdivision of star, Shadow graph of star, Splitting graph of star, Subdivision of Bistar, Shadow graph of Bistar, Splitting graph of Bistar are found. **Novelty:** It involves the mathematical formulation for labeling the edges of a given graph which in turn gives rise to a new type of labeling called the Lucas antimagic labeling.

Keywords: Subdivision graph; Shadow graph; Splitting graph; Star; Bistar

1 Introduction

In this paper, graph $G(V,E)$ is considered as finite, simple and undirected with p vertices and q edges. A graph labeling is an assignment of integers to the vertices or edges. Labeled graphs are used in radar, circuit design, communication network, astronomy, cryptography etc. For detailed survey on graph labeling we refer to Gallian [1]. The notion of Antimagic labeling was introduced by N.Hartsfield and G.Ringel in the year 1990. Odd antimagic labeling was introduced by V.Vilfred and L.M.Florida. Here we introduce a new notion called Lucas antimagic labeling.

A (p,q) graph G is said to be a Lucas antimagic graph if there exists a bijection $f: E(G) \to \{L_1, L_2, \cdots L_q\}$ such that the induced injective function $f^* : V(G) \to \{1, 2, \ldots \sum L_q\}$ given by $f^*(u) = \sum_{e \in E(u)} f(e)$ are all distinct (where E(u) is the set of edges incident to u).

2 Methodology

Definition 2.1: Lucas number is defined by

$$\mathrm{L_n} = \begin{cases} 2 & \text{if n} = 1 \\ 1 & \text{if n} = 2 \\ L_{n-1} + L_{n-2} & \text{if n} > 2 \end{cases}$$

The first few Lucas numbers are 2,1,3,4,7,11,18,29,47,…

Definition 2.2: A (p,q) graph G is said to be a Lucas antimagic graph if there exists a bijection $f: E(G) \rightarrow \{L_1, L_2, \cdots L_q\}$ such that the induced injective function $f^* : V(G) \rightarrow \{1, 2, \ldots \sum L_q\}$ given by $f^*(u) = \sum_{e \in E(u)} f(e)$ are all distinct (where E(u) is the set of edges incident to u).

Definition 2.3:[2] The Subdivision graph is acquired from the graph G by including a new vertex between each pair of adjacent vertices of the graph G and it is denoted by S(G).

Definition 2.4:[3] The Shadow graph $D_2(G)$ of a connected graph G is formed by taking two copies of G say G' and G''. Join each vertex u' in G' to the neighbours of the corresponding vertex v' in G''.

Definition 2.5:[4] The Splitting graph $S'(G)$ of a graph G is acquired by including a new vertex v' corresponding to each vertex v of G such that $N(v) = N(v')$.

3 Results and Discussion

Theorem 3.1:

$S(K_{1,n})(n \geq 2)$ where $S(G)$ denotes subdivision of G is Lucas antimagic graph.

Proof:

Let $V(S(K_{1,n})) = \{u, u_i, v_i : 1 \leq i \leq n\}$

$E(S(K_{1,n})) = \{uu_i, u_iv_i : 1 \leq i \leq n\}$

Define a function $f: E(S(K_{1,n})) \rightarrow \{L_1, L_2, \ldots L_q\}$ by

$f(uu_i) = L_{n+i}, 1 \leq i \leq n, \quad f(u_iv_i) = L_{n+1-i}, 1 \leq i \leq n$

The induced function

$f^* : V(S(K_{1,n})) \rightarrow \{1, 2, \cdots \sum L_q\}$ is given by

$f^*(u) = \sum_{i=1}^{n} L_{n+i}$

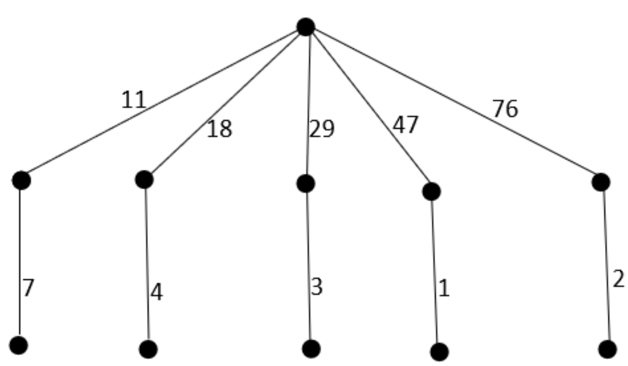
$f^*(u_i) = L_{n+1-i} + L_{n+i}, 1 \leq i \leq n$

$f^*(v_i) = L_{n+1-i}, 1 \leq i \leq n$

We observe that the vertices are all distinct.

Hence $S(K_{1,n})$ is Lucas antimagic graph.

Example 3.2: Subdivision graph $S(K_{1,5})$ and its Lucas antimagic labeling.



Theorem 3.3:

The shadow graph $D_2(K_{1,n})(n \geq 2)$ is Lucas antimagic Graph.

Proof:

Let $V(D_2(K_{1,n})) = \{u, v, u_i, v_i : 1 \leq i \leq n\}$

$E(D_2(K_{1,n})) = \{uu_i, vv_i, vu_i, uv_i : 1 \leq i \leq n\}$

Define a function $f: E(D_2(K_{1,n})) \rightarrow \{L_1, L_2, \ldots L_q\}$ by

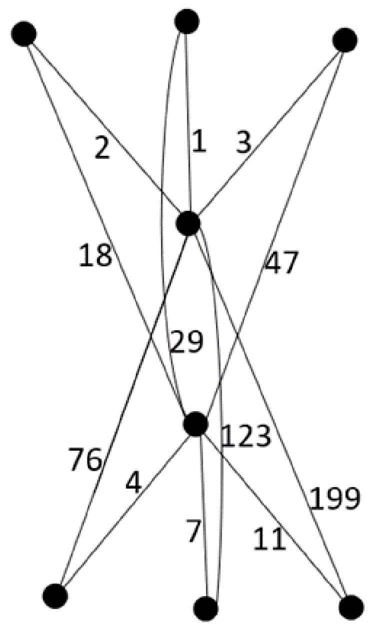
$f(uu_i) = L_i, 1 \leq i \leq n,$

$f(vv_i) = L_{n+i}, 1 \leq i \leq n$

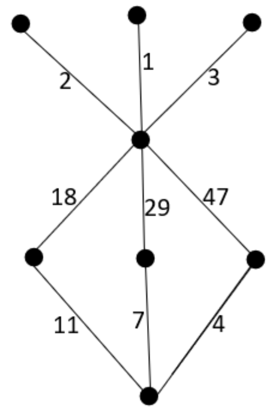
$f(vu_i) = L_{2n+i}, 1 \leq i \leq n$

$f(uv_i) = L_{3n+i}, 1 \leq i \leq n$

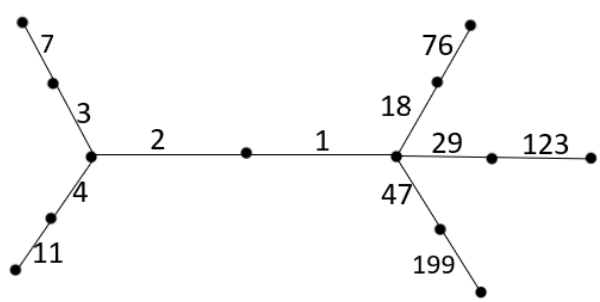
The induced function $f^* : V(D_2(K_{1,n})) \to \{1,2,\cdots \sum L_q\}$ is given by
$f^*(u) = \sum_{i=1}^{n} L_i + \sum_{i=1}^{n} L_{3n+i}$
$f^*(v) = \sum_{i=1}^{n} L_{n+i} + \sum_{i=1}^{n} L_{2n+i}$
$f^*(u_i) = L_i + L_{2n+i}\ ,1 \le i \le n$
$f^*(v_i) = L_{n+i} + L_{3n+i}\ ,1 \le i \le n$
We observe that the vertices are all distinct.
Hence $D_2(K_{1,n})$ is Lucas antimagic graph.
Example 3.4: Shadow graph $D_2(K_{1,3})$ and its Lucas antimagic labeling.



Theorem 3.5:
The splitting graph $S'(K_{1,n})(n \ge 2)$ is Lucas antimagic Graph.
Proof:
Let $V(S'(K_{1,n})) = \{u, v, u_i, v_i\ : 1 \le i \le n\}$
$E(S'(K_{1,n})) = \{uu_i,\ vv_i,\ uv_i\ :\ 1 \le i \le n\}$
Define a function $f : E(S'(K_{1,n})) \to \{L_1, L_2, \ldots L_q\}$ by
$f(uu_i) = L_i\ , 1 \le i \le n\ ,$
$f(uv_i) = L_{2n+i}\ ,\ 1 \le i \le n$
$f(vv_i) = L_{2n+1-i}\ ,\ 1 \le i \le n$
The induced function $f^* : V(S'(K_{1,n})) \to \{1,2,\cdots \sum L_q\}$ is given by
$f^*(u) = \sum_{i=1}^{n} L_i + \sum_{i=1}^{n} L_{2n+i}$
$f^*(v) = \sum_{i=1}^{n} L_{2n+1-i}$
$f^*(u_i) = L_i\ \ ,1 \le i \le n$
$f^*(v_i) = L_{2n+1-i} + L_{2n+i}\ ,1 \le i \le n$
We observe that the vertices are all distinct.
Hence $S'(K_{1,n})$ is Lucas antimagic graph.
Example 3.6: The Splitting graph $S'(K_{1,3})$ and its Lucas antimagic labeling.

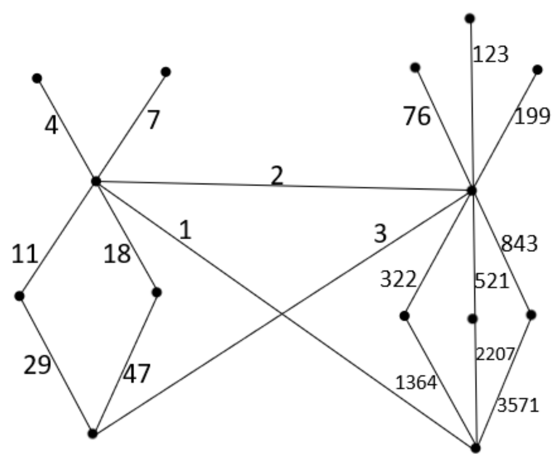


Theorem 3.7:
$S(B(m,n))(m \geq 2, n \geq 2)$ where $S(G)$ denotes subdivision of G is Lucas antimagic graph.
Proof:
Let $V(S(B(m,n))) = \{u, v, w, u_i, u_i' : 1 \leq i \leq m, w_j, w_j' : 1 \leq j \leq n\}$
$E(S(B(m,n))) = \{uv, vw, uu_i, u_iu_i' : 1 \leq i \leq m, ww_j, w_jw_j' : 1 \leq j \leq n\}$
Define a function $f: E(S(B(m,n))) \to \{L_1, L_2, \ldots L_q\}$ by
$f(uv) = L_1, \quad f(vw) = L_2$
$f(uu_i) = L_{2+i}, 1 \leq i \leq m, \quad f(u_iu_i') = L_{m+2+i}, 1 \leq i \leq m$
$f(ww_j) = L_{2m+2+j}, 1 \leq j \leq n, \quad f(w_jw_j') = L_{2m+n+2+j}, 1 \leq j \leq n$
For each edge label f, the induced vertex label f^* is defined by
$f^*(u) = L_1 + \sum_{i=1}^{m} L_{2+i}$
$f^*(v) = L_1 + L_2$
$f^*(w) = L_2 + \sum_{j=1}^{n} L_{2m+2+j}$
$f^*(u_i) = L_{2+i} + L_{m+2+i}, 1 \leq i \leq m$
$f^*(u_i') = L_{m+2+i}, 1 \leq i \leq m$
$f^*(w_j) = L_{2m+2+j} + L_{2m+n+2+j}, 1 \leq j \leq n$
$f^*(w_j') = L_{2m+n+2+j}, 1 \leq j \leq n$
We observe that the vertices are all distinct.
Hence $S(B(m,n))$ is Lucas antimagic graph.
Example 3.8: Subdivision graph $S(B(2,3))$ and its Lucas antimagic labeling.



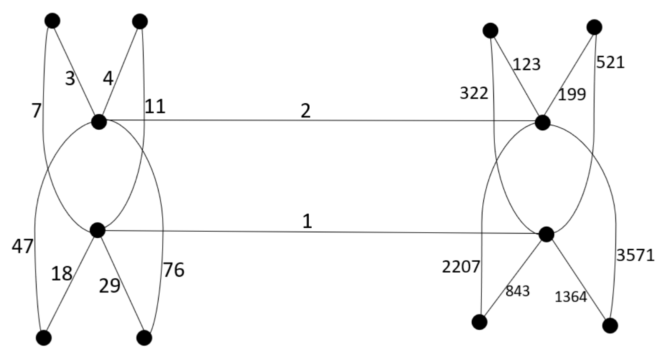
Theorem 3.9:
The Splitting graph $S'(B(m,n))(m \geq 2, n \geq 2)$ is Lucas antimagic graph.
Proof:
Let $V(S'(B(m,n))) = \{u_0, u_0', u_i, u_i' : 1 \leq i \leq m, v_0, v_0', v_j, v_j' : 1 \leq j \leq n\}$
$E(S'(B(m,n))) = \{u_0u_i', u_0u_i, u_0'u_i : 1 \leq i \leq m,$

$v_0v_j^{'}, v_0v_j, v_0^{'}v_j : 1 \le j \le n, u_0v_0, u_0v_0^{'}, u_0^{'}v_0\}$
Define a function $f: E(S^{'}(B(m,n))) \to \{L_1, L_2, \ldots L_q\}$ by
$f(u_0v_0) = L_1,\ f\left(u_0v_0^{'}\right) = L_2,\ f\left(u_0^{'}v_0\right) = L_3$
$f\left(u_0u_i^{'}\right) = L_{3+i},\ 1 \le i \le m, \qquad f(u_0u_i) = L_{m+3+i},\ 1 \le i \le m$
$f(u_0^{'}u_i) = L_{2m+3+i},\ 1 \le i \le m$
$f\left(v_0v_j^{'}\right) = L_{3m+3+j},\ 1 \le j \le n, \qquad f(v_0v_j) = L_{3m+n+3+j},\ 1 \le j \le n$
$f(v_0^{'}v_j) = L_{3m+2n+3+j},\ 1 \le j \le n$
For each edge label f, the induced vertex label $f^*$is defined by
$f^*(u_0) = L_1 + L_2 + \sum_{i=1}^{m} L_{3+i} + \sum_{i=1}^{m} L_{m+3+i}$
$f^*\left(u_0^{'}\right) = \sum_{i=1}^{m} L_{2m+3+i} + L_3$
$f^*(u_i) = L_{m+3+i} + L_{2m+3+i},\ 1 \le i \le m$
$f^*\left(u_i^{'}\right) = L_{3+i},\ 1 \le i \le m$
$f^*(v_0) = \sum_{j=1}^{n} L_{3m+3+j} + \sum_{j=1}^{n} L_{3m+n+3+j} + L_1 + L_3$
$f^*\left(v_0^{'}\right) = L_2 + \sum_{j=1}^{n} L_{3m+2n+3+j}$
$f^*(v_j) = L_{3m+n+3+j} + L_{3m+2n+3+j},\ 1 \le j \le n$
$f^*\left(v_j^{'}\right) = L_{3m+3+j},\ 1 \le j \le n$
We observe that the vertices are all distinct.
Hence $S^{'}(B(m,n))$ is Lucas antimagic graph.
Example 3.10: The Splitting graph $S^{'}(B(2,3))$ and its Lucas antimagic labeling.



Theorem 3.11:
The Shadow graph $D_2(B(m,n))(m \ge 2, n \ge 2)$ is Lucas antimagic graph.
Proof:
Let $V(D_2(B(m,n))) = \{u_0^{'}, u_0^{''}, u_i^{'}, u_i^{''} : 1 \le i \le m, v_0^{'}, v_0^{''}, v_j^{'}, v_j^{''} : 1 \le j \le n\}$
$E(D_2(B(m,n))) = \{u_0^{'}u_i^{'}, u_0^{''}u_i^{'}, u_0^{''}u_i^{''}, u_0^{'}u_i^{''} : \ 1 \le i \le m,$
$v_0^{'}v_j^{'}, v_0^{''}v_j^{'}, v_0^{''}v_j^{''}, v_0^{'}v_j^{''} : 1 \le j \le n, u_0^{'}v_0^{'}, u_0^{''}v_0^{''}\}$
Define a function $f: E(D_2(B(m,n))) \to \{L_1, L_2, \ldots L_q\}$ by
$f\left(u_0^{'}v_0^{'}\right) = L_1,\ f\left(u_0^{''}v_0^{''}\right) = L_2$
$f\left(u_0^{'}u_i^{'}\right) = L_{2+i},\ 1 \le i \le m,$
$f\left(u_0^{''}u_i^{'}\right) = L_{m+2+i},\ 1 \le i \le m, \qquad f(u_0^{''}u_i^{''}) = L_{2m+2+i},\ 1 \le i \le m$
$f(u_0^{'}u_i^{''}) = L_{3m+2+i},\ 1 \le i \le m$

$f\left(v_0^{'}v_j^{'}\right) = L_{4m+2+j}$, $1 \le j \le n$, $f(v_0^{''}v_j^{'}) = L_{4m+n+2+j}$, $1 \le j \le n$
$f(v_0^{''}v_j^{''}) = L_{4m+2n+2+j}$, $1 \le j \le n$
$f(v_0^{'}v_j^{''}) = L_{4m+3n+2+j}$, $1 \le j \le n$
For each edge label f, the induced vertex label $f^*$is defined by
$f^*\left(u_0^{'}\right) = L_1 + \sum_{i=1}^{m} L_{2+i} + \sum_{i=1}^{m} L_{3m+2+i}$
$f^*\left(u_i^{'}\right) = L_{2+i} + L_{m+2+i}$, $1 \le i \le m$
$f^*\left(u_i^{''}\right) = L_{2m+2+i} + L_{3m+2+i}$, $1 \le i \le m$
$f^*\left(u_0^{''}\right) = \sum_{i=1}^{m} L_{2m+2+i} + \sum_{i=1}^{m} L_{m+2+i} + L_2$
$f^*\left(v_0^{'}\right) = \sum_{j=1}^{n} L_{4m+2+j} + \sum_{j=1}^{n} L_{4m+3n+2+j} + L_1$
$f^*\left(v_j^{'}\right) = L_{4m+2+j} + L_{4m+n+2+j}$, $1 \le j \le n$
$f^*\left(v_j^{''}\right) = L_{4m+2n+2+j} + L_{4m+3n+2+j}$, $1 \le j \le n$
$f^*\left(v_0^{''}\right) = \sum_{j=1}^{n} L_{4m+2n+2+j} + \sum_{j=1}^{n} L_{4m+n+2+j} + L_2$
We observe that the vertices are all distinct.
Hence $D_2(B(m,n))$ is Lucas antimagic graph.
Example 3.12: The Shadow graph $D_2(B(2,2))$ is Lucas antimagic graph.



4 Conclusion

In this study, the concept of Lucas antimagic labeling is introduced and it is proved that various star related graphs are Lucas antimagic. Similar investigations are in process.

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