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RESEARCH ARTICLE

An Application of Exponential-Lindley Distribution in Modelling Cancer Survival Data

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Abstract

Objective: To explore multiple myeloma cancer survival data using the Exponential-Lindley model and compare the performance with other related models. **Methods:** In this article both simulated and real data sets were used for comparison. The model parameters are estimated for real and simulated data sets using the maximum likelihood method. The real data sets consist of 48 Cancer patients with multiple myeloma with 25% of them was censored observations. We consider both type I and type II censoring mechanism with 5%, 10% and 20% censoring. The performance was compared using three measures Deviance, AIC and BIC. **Findings:** The performance of Exponential-Lindley model was compared with related models Exponential, Lindley, Power Lindley, Gamma, and Burr-XII models. Simulated and real data sets are applied in the underlying models. The Exponential-Lindley model outperformed compared to other models. **Novelty:** The new model is novel alternative for modelling censored survival data for different levels and types of censoring.

Keywords: Cancer survival; Censoring; Simulation; ExpG family; Exponential Lindley Model

1 Introduction

The main feature of survival data is that they are not symmetrically distributed and the end point of interest are not observed for some individuals leading to censoring situation. The presence of censoring makes this domain tough and survival techniques were developed to handle them. The percentage of censoring in the data affects the performance of the models by[1,2].

Non-parametric, parametric, and semi-parametric survival analysis models were used to fit the censored data. In a parametric approach, different statistical distributions, including exponential, Weibull, gamma, Lindley, and log-logistic, have been suggested to handle lifetime data.

New generalisations of Lindley's models have been developed in the past few years such as Alpha power transformed power Lindley distribution[3], modified Lindley[4] and a new class of Lindley distribution and its properties and applications are discussed in [5]. The Exponential-Lindley model [6] for complete data was introduced by Balogun. A new extension of Lindley's distribution and its application are discussed in [7]. The sine-modified Lindley model and its properties are presented in [8].

The key benefit of creating a new family of lifetime models is the improvement in flexibility and fit at the expense of one or more extra parameters. These new lifetime models are essential for handling failure data in a variety of sectors, including the life sciences, biological sciences, medicine, and industry.

The main objective of this work is to explore the utility of Exponential-Lindley models in the presence of Type I and Type II censoring. No major work has been made to model censored survival data using Exponential-Lindley (EL) distribution using different censoring schemes. In this paper type-I and type-II censoring were considered with different levels of censoring mechanisms.

The rest of the section is laid out as follows. In Section 2, the Exponential-Lindley model and its graphical representation, as well as the methodology applied are presented. Section 3 presents the findings and related discussion. In section 4, ends with the main conclusion.

2 Methodology

2.1 Exponential-Lindley model

Lindley distribution fundamental properties with application to lifetime data was proposed by Ghitany [9]. Extend the Lindley model by using the concept of an exponential generator of probability distribution to create a new model Exponential-Lindley was developed Balogun [4]. The two-parameter model was using an exponential generator (Exp-G family) with an additional shape parameter ($\lambda > 0$). Let us assume that $t_{1,}\, t_{2,\ldots,} t_{n,}$ follow the Exponential-Lindley distribution with the shape parameters $((\gamma,\ \lambda)$ whose probability density function and cumulative distribution function are presented below.

$$f\left(t_i,\gamma,\lambda\right)=\frac{\gamma^2\lambda\left(1+t_i\right)e^{-\gamma t_i}}{\left(\gamma+1\right)\left[\left(1+\frac{\gamma t_i}{\gamma+1}\right)e^{-\gamma t_i}\right]^2}*\exp\left\{-\lambda\left[\frac{1-\left(1+\frac{\gamma t_i}{\gamma+1}\right)e^{-\gamma t_i}}{\left(1+\frac{\gamma t_i}{\gamma+1}\right)e^{-\gamma t_i}}\right]\right\} \tag{2.1}$$

$$\mathrm{F}\left(t_i,\gamma,\lambda\right)=1-\exp\left\{-\lambda\left[\frac{1-\left(1+\frac{\gamma t_i}{\gamma+1}\right)e^{-\gamma t_i}}{\left(1+\frac{\gamma t_i}{\gamma+1}\right)e^{-\gamma t_i}}\right]\right\} \quad t_i>0 \text{ and } \gamma,\lambda>0 \tag{2.2}$$

The EL density function varies significantly depending on the values of shape parameters $\gamma,\ \lambda$ presented in Figure 1.

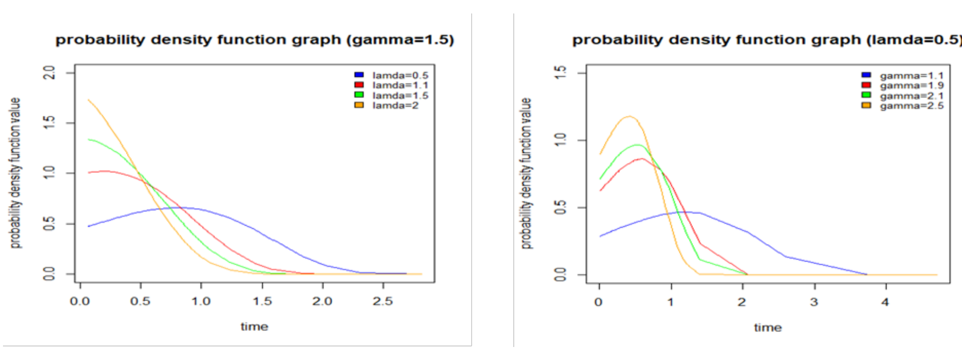

Fig 1. Density function for the Exponential-Lindley model

The Exponential-Lindley model is skewed and flexible, and its shape is determined by the parameters' values. The survival, hazard functions and Cumulative hazard are

$$\mathrm{S}\left(t_i,\gamma,\lambda\right)=\exp\left\{-\lambda\left[\frac{1-\left(1+\frac{\gamma t_i}{\gamma+1}\right)e^{-\gamma t_i}}{\left(1+\frac{\gamma t_i}{\gamma+1}\right)e^{-\gamma t_i}}\right]\right\} \tag{2.3}$$

$$h\left(t_i,\gamma,\lambda\right)=\frac{\gamma^2\lambda\left(1+t_i\right)e^{\gamma t_i}}{\left(\gamma+1\right)\left[\left(1+\frac{\gamma t_i}{\gamma+1}\right)\right]^2} \tag{2.4}$$

$$\mathrm{H}\left(t_i,\gamma,\lambda\right)=-\log\mathrm{S(t)}\quad t_i>0\text{ and }\gamma,\lambda>0 \tag{2.5}$$

2.2 Maximum likelihood method

Let us consider "d" to be the death among the n individuals who die at times t_1, $t_2\ldots t_d$ and that the remaining survival times of (n-d) individuals t_1^*, $t_2^*\ldots$ t_{n-d}^* are right-censored. Let δ_i be the indicator variable that takes the value zero when survival time is censored and unity when is uncensored. The likelihood of the sample data can be obtained by using

$$\mathrm{L}=\prod_{i=1}^{n}\left\{f\left(t_i\right)\right\}^{\delta i}\left\{S\left(t_i\right)\right\}^{1-\delta i} \tag{2.6}$$

The corresponding Log-likelihood for the equation (2.6) is given by

$$\mathrm{LL}=\sum_{i=1}^{n}\delta_i\ln f\left(t_i\right)+\sum_{i=1}^{n}\left(1-\delta_i\right)\ln S\left(t_i\right) \tag{2.7}$$

Using (2.1) and (2.3) in (2.6) to get the new likelihood function for the proposed model as,

$$\mathrm{L}=\prod_{i=1}^{n}\left\{\frac{\gamma^2\lambda\left(1+t_i\right)e^{-\gamma t_i}}{\left(\gamma+1\right)\left[\left(1+\frac{\gamma t_i}{\gamma+1}\right)e^{-\gamma t_i}\right]^2}*\exp\left\{-\lambda\left[\frac{1-\left(1+\frac{\gamma t_i}{\gamma+1}\right)e^{-\gamma t_i}}{\left(1+\frac{\gamma t_i}{\gamma+1}\right)e^{-\gamma t_i}}\right]\right\}\right\}^{\delta i}$$

$$\left\{\exp\left\{-\lambda\left[\frac{1-\left(1+\frac{\gamma t_i}{\gamma+1}\right)e^{-\gamma t_i}}{\left(1+\frac{\gamma t_i}{\gamma+1}\right)e^{-\gamma t_i}}\right]\right\}\right\}^{1-\delta i} \tag{2.8}$$

The Log-Likelihood function can be written as,

$$\mathrm{LL}=\sum_{i=1}^{n}\delta_i\left\{2n\log\gamma+n\log\lambda-n\log(\gamma+1)+\gamma t_i+\log\left(1+t_i\right)-2\log\left(1+\frac{\gamma t_i}{\gamma+1}\right)-\right.$$

$$\left.\lambda\sum_{i=1}^{n}\left[\frac{1-\left(1+\frac{\gamma t_i}{\gamma+1}\right)e^{-\gamma t_i}}{\left(1+\frac{\gamma t_i}{\gamma+1}\right)e^{-\gamma t_i}}\right]\right\}-\sum_{i=1}^{n}(1-\delta i)\lambda\left[\frac{1-\left(1+\frac{\gamma t_i}{\gamma+1}\right)e^{-\gamma t_i}}{\left(1+\frac{\gamma t_i}{\gamma+1}\right)e^{-\gamma t_i}}\right] \tag{2.9}$$

The non-linear equation can be solved using Newton Raphson's numerical methods.

2.3 Model selection

The model comparisons are evaluated using Deviance (-2LL), AIC (Akaike Information Criterion) and BIC (Bayesian Information Criterion) defined as

AIC = -2LL + 2k

BIC = -2LL +k (log N)

Where k is the number of parameters involved in the model and N is the sample size.

2.4 Right Censoring

The common type of censoring occurs when the survival time is "incomplete" on the right side of the follow-up period. Right censorship happens when a participant leaves the study before an event happens or when the study is over before the event has taken place. In this article, Type-I and Type-II right-censored observations were considered.

2.5 Estimation Procedure for Type I Censoring

The Type-I right censored sample has the form (T_i, δ_i), i = 1, 2…n and the respective log-likelihood function is defined by

$$\mathrm{LL} = \sum_{i=1}^{n} \delta_i \ln f(t_i) + \sum_{i=1}^{n} (1-\delta_i) \ln S(t_c) \tag{2.10}$$

The log-likelihood is given by for EL density

$$\mathrm{LL} = \sum_{i=1}^{n} \delta_i \ln \left[\frac{\gamma^2 \lambda (1+t_i) e^{-\gamma t_i}}{(\gamma+1)\left[\left(1+\frac{\gamma t_i}{\gamma+1}\right)e^{-\gamma t_i}\right]^2} * \exp\left\{ -\lambda \left[\frac{1-\left(1+\frac{\gamma t_i}{\gamma+1}\right)e^{-\gamma t_i}}{\left(1+\frac{\gamma t_i}{\gamma+1}\right)e^{-\gamma t_i}} \right] \right\} \right] +$$

$$\sum_{i=1}^{n} (1-\delta_i) \ln \left[\exp\left\{ -\lambda \left[\frac{1-\left(1+\frac{\gamma t_i}{\gamma+1}\right)e^{-\gamma t_i}}{\left(1+\frac{\gamma t_i}{\gamma+1}\right)e^{-\gamma t_i}} \right] \right\} \right] \tag{2.11}$$

2.6 Estimation Procedure for Type II Censoring

In this censoring scheme, the observations are recorded until a pre-specified s (s$\leq n$)failures are observed, and the remaining (n-s) observations are right censored. The non-censored observations are $t_1,\ t_{2,\ldots}\ t_s$ and the censored observations are $t_{s+1},\ t_{s+2,\ldots}\ t_n$ respectively.

Hence, Type II censoring data consists of the s smallest ordered observations $0 \leq\ t_{(1)} \leq t_{(2)} \leq \ldots\ \leq t_s$ having a common density function $f(t_i)$ and survival function $S(t_i)$ whose log-likelihood is given by

$$\mathrm{L} = \frac{n!}{(n-s)!} \mathrm{f}\left(t_{(1)}\right) f\left(t_{(2)}\right) \ldots f\left(t_{(s)}\right) \left[S\left(t_{(s)}\right)\right]^{n-s} \tag{2.12}$$

For EL density

$$\mathrm{LL} = \log \mathrm{n}! - \log(\mathrm{n}-\mathrm{s})! + \sum_{i=1}^{s} \log \left[\frac{\gamma^2 \lambda (1+t_i) e^{-\gamma t_i}}{(\gamma+1)\left[\left(1+\frac{\gamma t_i}{\gamma+1}\right)e^{-\gamma t_i}\right]^2} * \exp\left\{ -\lambda \left[\frac{1-\left(1+\frac{\gamma t_i}{\gamma+1}\right)e^{-\gamma t_i}}{\left(1+\frac{\gamma t_i}{\gamma+1}\right)e^{-\gamma t_i}} \right] \right\} \right]$$

$$+(\mathrm{n}-\mathrm{s}) \sum_{s=1}^{n} \log \exp \left\{ -\lambda \left[\frac{1-\left(1+\frac{\gamma t_i}{\gamma+1}\right)e^{-\gamma t_i}}{\left(1+\frac{\gamma t_i}{\gamma+1}\right)e^{-\gamma t_i}} \right] \right\} \tag{2.13}$$

3 Results and Discussion

3.1 Simulated Data

The Exponential-Lindley model is generated from the Exponential-G family of distributions and the Lindley distribution is a mixture of exponential and gamma distributions. A simulation study, was carried out the performance of the Exponential-Lindley model under Type-I and Type-II censoring schemes. As a result, the exponential distribution is used to generate the random samples $t_1, t_2 \ldots t_n$. To generate the Type-I censored data, determine the censored time limit t_c. The lifetime T_i is censored or uncensored. To calculate the MLE, the log-likelihood function was maximized using the optim function in the R language. The model validity is assessed by -2LL, AIC, and BIC. The results of simulated Type-I censored data with 5%, 10%, and 20% censoring percentages with the different sample sizes, respectively 100, 300, and 500 are presented in Table 1. To get the Type-II censored data, the sample $t_1, t_2 \ldots t_n$ generated from an exponential distribution and "s" is the number of failure observations that is pre-determined ($s \leq n$). The collected data will occur until s fails and the experiment ends. The remaining (n-s) units have survived and will become censored for observation. As a result, the time[(s+1): n] = time[s] condition follows. Table 2 shows the results of simulated Type-II censored data with 5%, 10%, and 20% censoring percentages for the different sample sizes of 100, 300, and 500.

Table 1 represents the estimated parameter values for the Exponential-Lindley, power Lindley, Gamma, Burr-XII, Lindley and Exponential models under Type-I simulated data. We observed that the estimated parameters are well-performed and consistent in the entire model. The proposed model performed much better with a low percentage of censored observations. Further, the Exponential-Lindley model has lower Deviance, AIC, and BIC values compared to other underlying models. Therefore, the Exponential-Lindley model is well performed based on the model selection procedure, which was stated in section 2.3.

Table 2 displays the estimated parameter values for the Exponential-Lindley, power Lindley, Gamma, Burr-XII, Lindley, and exponential models with Type II simulated data. We observed that the proposed model performed much better with a low percentage of censored observations. The Exponential-Lindley model has lower Deviance, AIC, and BIC values compared to other underlying models. Therefore, the Exponential-Lindley model is well performed based on the model selection procedure, which was stated in section 2.3.

Table 1. Parameter Estimation under Type-I censor

Sample Size	Censoring percentage	Model	Lambda	Alpha	Deviance	AIC	BIC
100	5%	Exponential-Lindley	0.6045	1.8203	**80**	**84**	**84**
		Power Lindley	2.2017	1.2425	95	99	99
		Gamma	0.4778	1.3013	101	105	105
		Burr-XII	2.4323	1.4370	111	115	115
		Lindley	2.1245	-	101	103	103
		Exponential	1.6094	-	105	107	107
	10%	Exponential-Lindley	0.6147	1.8027	**81**	**85**	**85**
		Power Lindley	2.1957	1.2399	96	100	100
		Gamma	0.4795	1.2986	101	105	105
		Burr-XII	2.4263	1.4349	112	116	116
		Lindley	2.1207	-	101	103	103
		Exponential	1.6060	-	106	108	108
	20%	Exponential-Lindley	0.6615	1.7210	**85**	**89**	**89**
		Power Lindley	2.1595	1.2251	99	103	103
		Gamma	0.4920	1.2824	104	108	108
		Burr-XII	2.3894	1.4225	115	119	119
		Lindley	2.0968	-	104	106	106
		Exponential	1.5850	-	108	110	110

Continued on next page

Table 1 continued

300	5%	Exponential-Lindley	0.4522	1.922	**276**	**280**	**281**
		Power Lindley	2.0088	1.3397	327	331	332
		Gamma	0.4724	1.4493	350	354	355
		Burr-XII	2.2218	1.5659	381	385	386
		Lindley	1.9549	-	357	359	360
		Exponential	1.4606	-	373	375	376
	10%	Exponential-Lindley	0.4740	1.8710	**283**	**287**	**288**
		Power Lindley	1.9909	1.3300	332	336	337
		Gamma	0.4790	1.4390	355	359	360
		Burr-XII	2.2038	1.5581	386	390	391
		Lindley	1.9436	-	362	364	365
		Exponential	1.4508	-	377	379	380
	20%	Exponential-Lindley	0.4889	1.8370	**289**	**293**	**294**
		Power Lindley	1.9781	1.3232	336	340	341
		Gamma	0.4839	1.4315	358	362	363
		Burr-XII	2.1908	1.5525	390	394	395
		Lindley	1.9354	-	365	367	368
		Exponential	1.4436	-	380	382	383
500	5%	Exponential-Lindley	0.4545	1.7698	**552**	**556**	**557**
		Power Lindley	1.8015	1.3635	631	635	636
		Gamma	0.4978	1.5139	671	675	676
		Burr-XII	2.001	1,6228	726	730	731
		Lindley	1.8008	-	688	690	691
		Exponential	1.3270	-	717	719	720
	10%	Exponential-Lindley	0.4626	1.7523	**556**	**560**	**561**
		Power Lindley	1.7961	1.3598	634	638	639
		Gamma	0.5003	1.5098	674	678	679
		Burr-XII	1.9953	1.6200	729	733	734
		Lindley	1.7971	-	691	693	694
		Exponential	1.3238		720	722	723
	20%	Exponential-Lindley	0.4852	1.7027	**570**	**574**	**575**
		Power Lindley	1.7779	1.3478	645	649	650
		Gamma	0.5091	1.4962	684	688	689
		Burr-XII	1.9766	1.6101	740	744	745
		Lindley	1.7843	-	700	702	703
		Exponential	1.3128	-	728	730	731

Table 2. Parameter Estimation under Type-II censor

Sample Size	Censoring percentage	Model	Lambda	Alpha	Deviance	AIC	BIC
		Exponential-Lindley	0.6546	1.7467	**82**	**86**	**86**

Continued on next page

5%

100

Table 2 continued

		Power Lindley	2.1882	1.2348	96	100	100
		Gamma	0.4821	1.2945	102	106	106
		Burr-XII	2.4203	1.4320	112	116	116
		Lindley	2.1165	-	102	104	104
		Exponential	1.6024	-	106	108	108
		Exponential-Lindley	0.6969	1.6847	**84**	**88**	**88**
	10%	Power Lindley	2.1679	1.2256	98	102	102
		Gamma	0.4891	1.2850	104	108	108
		Burr-XII	2.4002	1.4249	114	118	118
		Lindley	2.1034	-	104	106	106
		Exponential	1.5908	-	108	110	110
		Exponential-Lindley	0.7992	1.5391	**92**	**96**	**96**
	20%	Power Lindley	2.0975	1.1965	106	110	110
		Gamma	0.5158	1.2524	110	114	114
		Burr-XII	2.3275	1.401	120	124	124
		Lindley	2.0546	-	110	112	112
		Exponential	1.5479	-	112	114	114
		Exponential-Lindley	0.6995	1.5125	**326**	**330**	**331**
	5%	Power Lindley	1.8971	1.2708	364	368	369
		Gamma	0.5191	1.3792	382	386	387
		Burr-XII	2.1115	1.5148	414	418	419
		Lindley	1.8816	-	386	388	389
		Exponential	1.3969	-	400	402	403
300		Exponential-Lindley	0.7600	1.4425	**336**	**340**	**341**
	10%	Power Lindley	1.8773	1.2578	372	376	377
		Gamma	0.5288	1.3658	388	392	393
		Burr-XII	2.0917	1.5053	422	426	427
		Lindley	1.8674	-	392	394	395
		Exponential	1.3846	-	404	406	407
		Exponential-Lindley	0.8878	1.3008	**364**	**368**	**369**
	20%	Power Lindley	1.8124	1.2211	396	400	401
		Gamma	0.5633	1.3231	410	414	415
		Burr-XII	2.0235	1.4742	444	448	449
		Lindley	1.8176	-	410	412	413
		Exponential	1.3415	-	424	426	427
		Exponential-Lindley	0.5430	1.7043	**514**	**518**	**519**
	5%	Power Lindley	1.9084	1.4480	558	562	563
		Gamma	0.4119	1.7485	592	596	597
		Burr-XII	2.1407	1.7579	628	632	633
		Lindley	1.8721	-	642	644	645
		Exponential	1.3886	-	672	674	675
500		Exponential-Lindley	0.6127	1.5912	**542**	**546**	**547**
	10%						

Continued on next page

Table 2 continued

	Power Lindley	1.8700	1.4201	580	584	585
	Gamma	0.4262	1.7143	612	616	617
	Burr-XII	2.1008	1.7364	648	652	653
	Lindley	1.8492	-	658	660	661
	Exponential	1.3688	-	688	690	691
20%	Exponential-Lindley	0.7124	1.4268	**602**	**606**	**607**
	Power Lindley	1.7721	1.3616	640	644	645
	Gamma	0.4678	1.6294	666	670	671
	Burr-XII	1.9925	1.6821	708	712	713
	Lindley	1.7834	-	700	702	703
	Exponential	1.3120	-	728	730	731

3.2 Application to Multiple Myeloma data (Krall et al 1975)

The study was carried out at the Medical Centre of the University of West Virginia, USA. The data related to 48 patients with multiple myeloma, all of whom were aged between 50 and 80 years. Out of these 48 patients, 12 (25%) had censored survival times. The survival times were recorded in months. Other details can be seen in Krall et al. (1975). The results of the analysis were presented in Table 3.

Table 3 illustrates that the Exponential Lindley model appears to be more suited and appropriate for fitting censored survival data. As a result, it serves as the best substitute for mixture models like the Lindley and other models in the Exponential-G family. This model has an increasing hazard function, the same as the Lindley model. It is hence ideal for testing data on lifetime failures.

Table 3. The parameter estimates of the underlying model for the Multiple Myeloma Patient's survival data.

Model	Alpha	Lambda	Deviance
Exponential-Lindley	0.0149	10.6991	372
Power Lindley	0.1824	0.7608	400
Gamma	1.0790	21.6632	398
Burr-XII	0.0075	50.9551	444
Lindley	0.0823	-	408
Exponential	0.0428	-	399

Figures 2, 3 and 4 represent the probability density function, survival function, and hazard function of the underlying models involved in this article for multiple myeloma patient survival data.

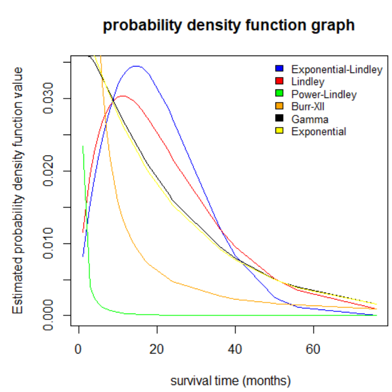

Fig 2. Estimated Probability Density Function

Further Figure 2, the proposed model has high skewed than the other models. In Figure 3, it is observed that the median survival time of the Exponential-Lindley model is 20 months. The other model produced less survival time. Figure 4, represents the hazard rate, which is increasing linearly.

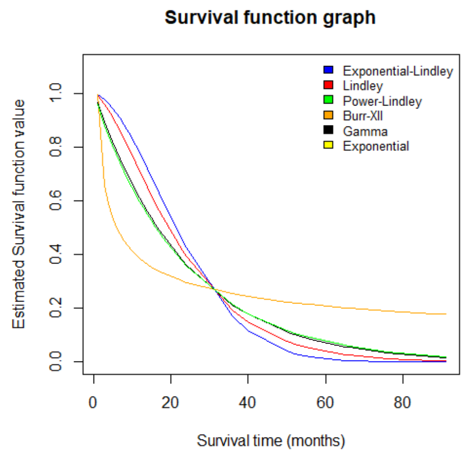

Fig 3. Estimated Survival Function

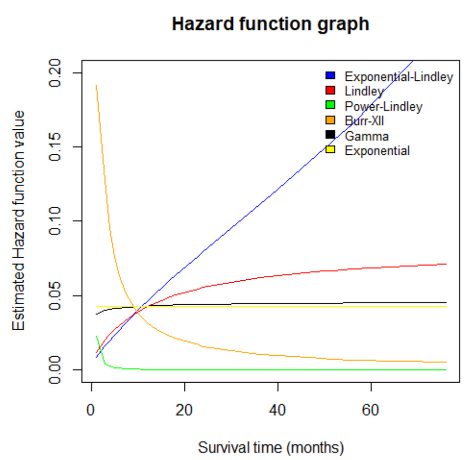

Fig 4. Estimated Hazard Rate Function

The study results are presented in tables Tables 1, 2 and 3, respectively. It is observed that the Exponential-Lindley model is better than the other models in simulated, and real right-censored data sets. The proposed model performed better with low levels of censoring percentages. When increasing the sample sizes the proposed model is more consistent and well performed. The median survival time of the Exponential Lindley model is larger than other models. Further, the Figures (Figures 2, 3 and 4) shows the Exponential-Lindley model is better than the other models. We conclude that the Exponential-Lindley model is well performed algebraically and graphically compared to other models in the simulated and real data sets.

4 Conclusion

This research article has concentrated on real and simulated (Type-I and Type-II) right-censored data sets with different censoring percentages. The result of the simulated Type-I and Type-II right-censored data revealed that the EL model performed better than the other models based on Deviance, AIC, and BIC values. The performance of the model improves with increase in sample size and decrease with censoring. As a result of the real data, the proposed model has a lower Deviance value than the other models. The Exponential-Lindley model better performed than the other underlying models. Further work is needed for other levels of censoring and sample sizes and also on parametric regression models.

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