INDIAN JOURNAL OF SCIENCE AND TECHNOLOGY

RESEARCH ARTICLE

Mathematical Modeling to Assess the Impact of Covid-19 Transmission in Guyana, South America

Onika Bevny[1]*, G Kumar[2], A V Raja[3]

1 Department of Mathematics, Physics and Statistics, Faculty of Natural Sciences, University of Guyana, Georgetown, Guyana, South America
2 Assistant Professor of Statistics, SRM Arts and Science College, Kattankulathur, India
3 Senior Lecturer, Department of Mathematics, Physics and Statistics, Faculty of Natural Sciences, University of Guyana, Georgetown, Guyana, South America

OPEN ACCESS

Received: 18-09-2022

Accepted: 05-02-2023

Published: 04-03-2023

Citation: Bevny O, Kumar G, Raja AV (2023) Mathematical Modeling to Assess the Impact of Covid-19 Transmission in Guyana, South America. Indian Journal of Science and Technology 16(9): 622-631. https://doi.org/10.17485/IJST/v16i9.1889

* **Corresponding author**.

onika.bevny@gmail.com

Funding: None

Competing Interests: None

Published By Indian Society for Education and Environment (iSee)

ISSN
Print: 0974-6846
Electronic: 0974-5645

Abstract

Objective: This study aims to find the best mathematical model for modeling the Covid-19 data of Guyana. **Methods:** The 2-parameter, 3-parameter Weibull distribution, and the Transmuted Weibull Distribution was used to model the Covid-19 data of Guyana using cumulative deaths that occurred on a daily basis from March 12th, 2020 to November 30th, 2021. The Covid-19 data of Guyana was extracted from the 'ourworldindata' website. **Findings:** The transmuted Weibull distribution is the best model for modeling the Covid-19 data of Guyana since it had the lowest AIC value than the other models. **Novelty:** Several transmuted distributions were developed to model the Covid-19 data of France, the United Kingdom, and Canada. However, in this study, a different transmuted distribution was chosen to model the Covid-19 data of Guyana.

Keywords: Mathematical Modeling; Covid19; Cumulative Deaths; Transmuted Weibull Distribution and Simulation Study

1 Introduction

The world saw its first case of the Covid-19 virus in Wuhan, China in late 2019. This virus gives rise to a global pandemic from which the world is still recovering from. The virus contributed to a significant increase in deaths worldwide, destruction of families through emotional trauma, the loss of many jobs, a massive increase in the cost of living and major disruption in the education system. According to the World Health Organization (WHO), Corona virus disease (Covid-19) is an infectious disease caused by the SARS-CoV-2 virus.

The Corona virus disease (Covid-19) has led to a high mortality rate globally, triggering an unprecedented public health crisis. On March 11, 2020, the World Health Organization declared Covid-19 as a global pandemic[1]. Corona virus is a family of viruses that can cause illnesses such as the common cold and more severe diseases such as Middle East Respiratory Syndrome (MERS-Cov) and severe acute respiratory syndrome (SARS-Cov).

The Covid-19 disease has expanded worldwide, producing over 315 million cases and 5.5 million deaths reported by the World Health Organization (WHO). There have been about 46000 confirmed cases in Guyana as of January 14, 2022, with over 1070 deaths. The WHO has termed this current epidemic a global emergency and it is a public health responsibility on a massive scale.

The time between the commencement of symptomatic manifestation and death is approximately 2-8 weeks. In addition, most countries including Guyana have implemented 14 days quarantine upon testing positive. The Government of Guyana introduced a lockdown to suppress the rate of transmission of this deadly virus.

Mathematical modeling has been generating quantitative information for a while in epidemiology and providing useful guidelines for outbreak management and policy development [2]. The results of an article on 'Mathematical modeling of Covid-19 transmission dynamics in Uganda' indicated that the elimination of all imported cases at any given time would take approximately nine months for Uganda to get rid of the disease [3]. The implementation of a strict lockdown for a period of at least 21 days is expected to reduce the transmission of Covid-19 and a further extension of up to 42 days is required to significantly reduce the transmission of COVID-19 in India [4].

Transmuted distributions have an additional parameter that gives the flexibility of capturing any skewness in the data. As a result, a number of new distributions have been developed to model real-world data. Additionally, a number of researchers successfully fitted different Transmuted distributions to the Covid-19 data of various countries. The Odd lomax-G inverse Weibull distribution [5] was studied and applied to the Covid-19 data of France from January 1 to February 20,2021. A new distribution called the Odd Weibull inverse Top-Leone distribution [6] is used to model the Covid-19 data of United Kingdom and Canada. The New Generalized Lomax distribution was presented and applied to Covid-19 data [7].

In this study, the Transmuted Weibull distribution is used to model the Covid-19 data of Guyana from the period of March 1 2020 to November 30 2021. The model parameters are estimated by the method of maximum likelihood estimators by analyzing the Covid-19 cumulative deaths of Guyana. In addition, the properties and characterization of the model is included in the study.

2 Methodology

2.1 Transmuted Weibull Distribution

A random variable X is said to have a Weibull distribution with parameters $\eta > 0$ and $\sigma > 0$ if its probability density function (pdf) is given by

$$g(x) = \frac{\eta}{\sigma}\left(\frac{x}{\sigma}\right)^{\eta-1} exp\left(-\left(\frac{x}{\sigma}\right)^{\eta}\right) \;,\; x > 0 \tag{1}$$

The cumulative distribution function of X is given by

$$G(x) = \int_0^x g(x)\;\; dx$$

$$G(x) = \int_0^x \frac{\eta}{\sigma}\left(\frac{x}{\sigma}\right)^{\eta-1} exp\left(-\left(\frac{x}{\sigma}\right)^{\eta}\right) dx$$

$$G(x) = 1 - exp\left(-\left(\frac{x}{\sigma}\right)^{\eta}\right) \tag{2}$$

Using equation 2, we can obtain the CDF of the transmuted Weibull distribution.

$$F(x) = \;(1+\;\lambda)\;\left(1 - exp\left(-\frac{x^{\eta}}{\sigma}\right)\right)\;\lambda\;\left(1 - exp\left(-\frac{x^{\eta}}{\sigma}\right)\right)^2$$

$$\Longrightarrow F(x) = (1+\;\lambda)k - \;\lambda k^2$$

Where

$$k = \;1 - exp\left(-\frac{x^{\eta}}{\sigma}\right)$$

$$\Longrightarrow F\,(x) = k\,(1+\ \lambda - \lambda k)$$

$$\Longrightarrow F\,(x) = k\,(1+\ \lambda\ \ (1-k))$$

$$\Longrightarrow F\,(x) = \ \left(1-exp\left(-\left(\frac{x}{\sigma}\right)^{\eta}\right)\right)\ \left(1+\ \lambda exp\left(-\left(\frac{x}{\sigma}\right)^{\eta}\right)\right) \tag{3}$$

Differentiating equation 3 with respect to x, given by

$$f(x) = \frac{d}{dx}\,(F(X))$$

$$\Rightarrow f(x) = \frac{d}{dx}\left(\left(1-\exp\left(-\frac{x^{\eta}}{\sigma}\right)\right)\left(1+\lambda\exp\left(-\frac{x^{\eta}}{\sigma}\right)\right)\right)$$

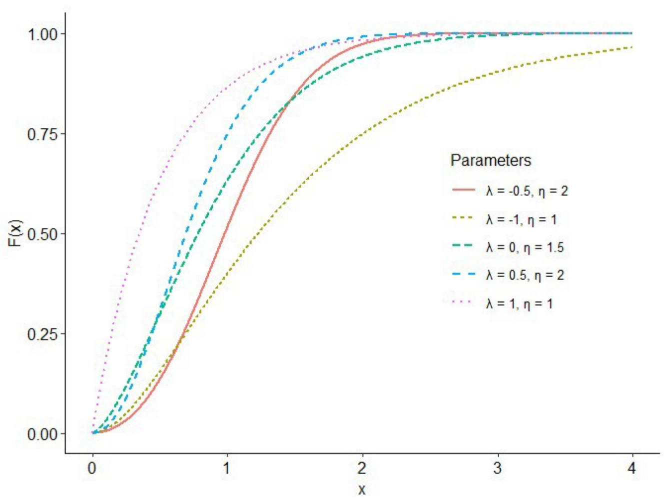

Fig 1. The CDF of Transmuted Weibull distribution for σ = 1 and different values of λ and η

After differentiating the above equation with respect to x, the pdf of the Transmuted Weibull distribution with parameters η, σ and λ is

$$f(x) = \frac{\eta}{\sigma}x^{\eta-1}\exp\left(\frac{-x^{\eta}}{\sigma}\right)\left(1-\lambda+2\lambda\exp\left(\frac{-x^{\eta}}{\sigma}\right)\right) \tag{4}$$

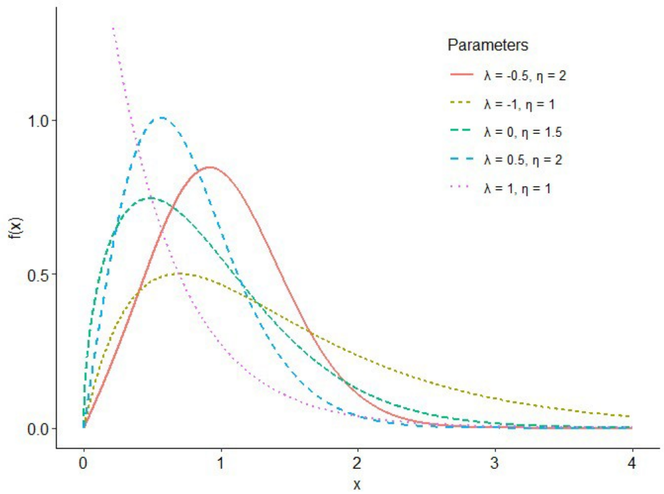

Fig 2. pdf of Transmuted Weibull distribution for σ = 1 and different values of λ and η

2.2 Properties of the model

2.2.1 Moments and quartiles

The expected value of the x^k is termed as k^{th} moment of origin of the random variable x which is given by

$$\mu_k^{'} = E(x)^k$$

Thus, the k^{th} moment of the transmuted Weibull distribution is given by

$$\mu_k' = \int_0^\infty x^k f(x;\eta,\sigma,\lambda)dx$$

$$\mu_k = \int_0^\infty x^k \frac{\eta}{\sigma} x^{\eta-1} \exp\left(\frac{-x^\eta}{\sigma}\right)\left(1-\lambda+2\lambda\exp\left(\frac{-x^\eta}{\sigma}\right)\right)dx$$

After solving the above equation,

$$\mu_k^{'} = \sigma^{\frac{k}{\eta}}\,\Gamma\left(\frac{k}{\eta}+1\right)\left(1-\ \lambda 2^{-\frac{k}{\eta}}\right) \tag{5}$$

2.2.2 Reliability Analysis

The reliability function of the transmuted Weibull distribution is

$$R(x) = \exp\left(-\left(\frac{x}{\sigma}\right)^\eta\right)\left(1-\lambda+\lambda\exp\left(-\left(\frac{x}{\sigma}\right)^\eta\right)\right) \tag{6}$$

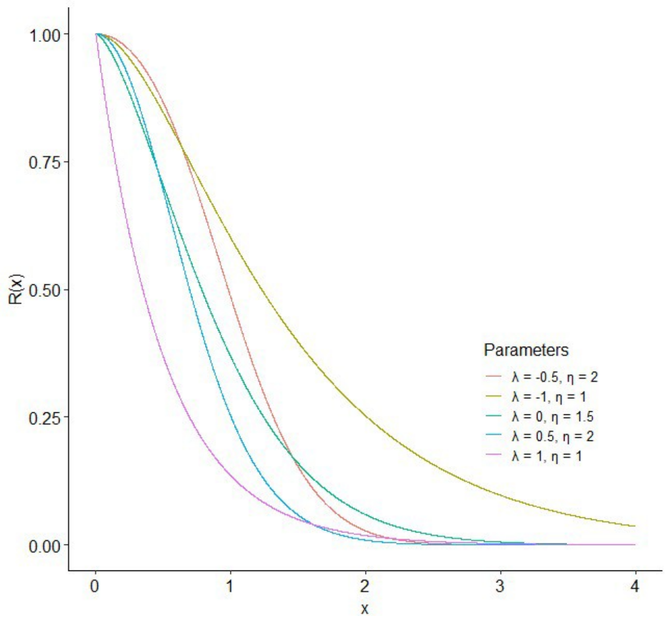

Fig 3. Reliability function of transmuted Weibull distributionf or σ=1 and different values of λ and η

The hazard rate function of the Transmuted Weibull distribution is given by

$$h(x) = \frac{\eta}{\sigma}\left(\frac{x}{\sigma}\right)^{\eta-1}\left(\frac{1-\lambda+2\lambda\exp\left(-\left(\frac{x}{\sigma}\right)^{\eta}\right)}{1-\lambda+\lambda\exp\left(-\left(\frac{x}{\sigma}\right)^{\eta}\right)}\right) \tag{7}$$

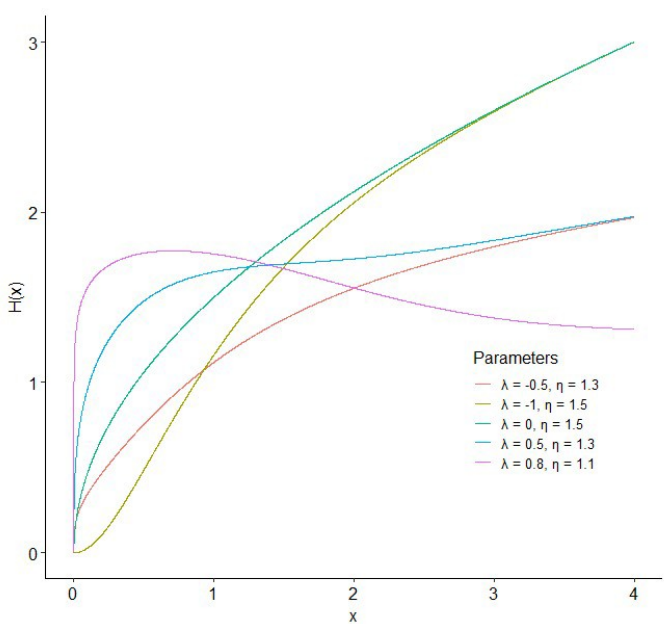

Fig 4. Hazard function of transmuted Weibull distribution for σ = 1 and different values of λ and η

2.2.3 Random Numbers Generator

By the method of inversion, random numbers from the Transmuted Weibull distribution can be generated as:

$$1-exp\left(-\left(\frac{x}{\sigma}\right)^{\eta}\right)\left(1-\ \lambda+\ \lambda exp\left(-\left(\frac{x}{\sigma}\right)^{\eta}\right)\right]=u$$

Where $u\sim U\left(0,1\right)$. After simple calculation it yields

$$X=\sigma\left(-\ln\left(1-\left(\frac{1+\lambda-\sqrt{(1+\lambda)^2-4\lambda U}}{2\lambda}\right)\right)\right)^{\frac{1}{\eta}} \tag{8}$$

2.3 Characterization of model

2.3.1 Mean

Setting $k=1$ in equation 5 leads to the mean of the Transmuted Weibull distribution.

$$E\left(X\right)=\sigma\Gamma^{\frac{1}{\eta}}\left(1+\frac{1}{\eta}\right)\left(1-\lambda+\ \lambda 2^{-\frac{1}{\eta}}\right) \tag{9}$$

2.3.2 Variance

The variance of the Transmuted Weibull distribution is given as:

$$\mathrm{Var}(X)=\sigma^2\left(\Gamma\left(1+\frac{2}{\eta}\right)\left(1-\lambda+\lambda 2^{-\frac{2}{\eta}}\right)-\Gamma^2\left(1+\frac{1}{\eta}\right)\left(1-\lambda+\lambda 2^{-\frac{1}{\eta}}\right)^2\right) \tag{10}$$

2.3.3 Standard deviation

The standard deviation of the Transmuted Weibull distribution is given as:

$$\sigma=\sigma^{\frac{1}{\eta}}\sqrt{\left(\Gamma\left(1+\frac{2}{\eta}\right)\left(1-\lambda+\lambda 2^{-\frac{2}{\eta}}\right)-\Gamma^2\left(1+\frac{1}{\eta}\right)\left(1-\lambda+\lambda 2^{-\frac{1}{\eta}}\right)^2\right)}$$

$$\Longrightarrow\ \sigma=\ \sigma^{\frac{1}{\eta}}\sqrt{\sigma^2-\ \sigma_1^2}$$

$$\sigma_k=\Gamma\left(\frac{k}{\eta}+1\right)\left(1-\lambda+\lambda 2^{-\frac{1}{\eta}}\right) \tag{11}$$

2.4 Estimation Procedure

The parameters of the Transmuted Weibull distribution are estimated using the maximum likelihood method.

Let $X_1,\ X_2,\ldots X_n$ be a random of size n from the Transmuted Weibull distribution with pdf from equation 4 and let $x_1,\ x_2,\ldots x_n$ be a realization of the random sample. The likelihood function $L=L\left(x_1,\ x_2,\ldots x_n;\ \eta,\sigma,\lambda\right)$ of this random sample is given by

$$L=\left(\frac{\eta}{\sigma}\right)^n\exp\left(-\sum_{i=1}^n\left(\frac{x_i}{\sigma}\right)^{\eta}\right)\prod_{i=1}^n\left(\left(\frac{x_i}{\sigma}\right)^{\eta-1}\times\left(1-\lambda+2\lambda\exp\left(-\left(\frac{x_i}{\sigma}\right)^{\eta}\right)\right)\right) \tag{12}$$

and the log-likelihood function $L = lnL(x_1,\, x_2, \ldots x_n;\ \eta, \sigma, \lambda)$ of this random sample is given by

$$L = \mathrm{n}\ln\frac{\eta}{\sigma} - \sum_{i=1}^{n} \ln\left(\frac{x_i}{\sigma}\right)^{\eta-1} + \sum_{i=1}^{n} \ln\left(1-\lambda+2\lambda exp\left(-\left(\frac{x_i}{\sigma}\right)^{\eta}\right)\right) \tag{13}$$

$$L = n\ln\eta - n\ln\sigma + (\eta-1)\textstyle\sum_{i=1}^{n}\ln(x_i) - \sum_{i=1}^{n}\left(\frac{x_i}{\sigma}\right)^{\eta} + \sum_{i=1}^{n}\ln\left(1-\lambda+2\lambda exp\left(-\left(\frac{x_i}{\sigma}\right)^{\eta}\right)\right) \tag{14}$$

Differentiating equation 14, with respect toη, σ and λ, respectively, and equating each derivative to zero we obtain the following equations:

$$\frac{\partial L}{\partial\eta} = \frac{n}{\eta} + \sum_{i=1}^{n}\left(1-\left(\frac{x_i}{\sigma}\right)^{\eta}\right)\ln\left(\frac{x_i}{\sigma}\right) - 2\lambda\sum_{i=1}^{n}\frac{ln\left(\frac{x_i}{\sigma}\right)\left(\frac{x_i}{\sigma}\right)^{\eta} exp\left(-\left(\frac{x_i}{\sigma}\right)^{\eta}\right)}{\left(1-\lambda+2\lambda exp\left(-\left(\frac{x_i}{\sigma}\right)^{\eta}\right)\right)} = 0 \tag{15}$$

$$\frac{\partial L}{\partial\sigma} = -\frac{n}{\eta} + \sum_{i=1}^{n}\left(1-\left(\frac{x_i}{\sigma}\right)^{\eta}\right) + \frac{2\lambda\eta}{\sigma}\sum_{i=1}^{n}\frac{\left(\frac{x_i}{\sigma}\right) exp\left(-\left(\frac{x_i}{\sigma}\right)^{\eta}\right)}{\left(1-\lambda+2\lambda exp\left(-\left(\frac{x_i}{\sigma}\right)^{\eta}\right)\right)} = 0 \tag{16}$$

$$\frac{\partial L}{\partial\lambda} = \sum_{i=1}^{n}\frac{2exp\left(-\left(\frac{x_i}{\sigma}\right)^{\eta}\right)-1}{\left(1-\lambda+2\lambda exp\left(-\left(\frac{x_i}{\sigma}\right)^{\eta}\right)\right)} = 0 \tag{17}$$

The maximum likelihood of the parameters are performed in the Mathematica software since it has a 'FindMaximum' function.

3 Results and Discussion

3.1 Simulation Study

A simulation study is carried out in order to test the efficiency of the estimation methods. The random sample sizes of n= 25, 75 and 175 are generated from the transmuted Weibull distribution with parameters η, λ and σ. The values of the parameters are chosen as η=0.6, λ =0.7 and σ= 0.9. For each combination of η, λ and σ and n there are 200 replications. The parameters are estimated using the maximum likelihood estimates. The values of the mean, Bias and RSME (Root mean square error) are obtained and the results are presented in Table 1.

Table 1. Mean, Bias and RSME of η, λ and σ based on 200 replications for the Transmuted Weibull distribution obtained using theMaximum Likelihood method

Sample Size (n)	Parameter Estimates	Mean	Bias	RSME
	$\widehat{\eta}$	0.6619	0.0619	0.0311
25	$\widehat{\lambda}$	0.6346	0.1154	0.2664
	$\widehat{\sigma}$	0.8486	0.0514	0.1161
	$\widehat{\eta}$	0.6845	0.0845	0.0376
75	$\widehat{\lambda}$	0.6031	0.1469	0.2063
	$\widehat{\sigma}$	0.7952	0.1048	0.1239
	$\widehat{\eta}$	0.7071	0.1071	0.0439
175	$\widehat{\lambda}$	0.5424	0.2076	0.1908
	$\widehat{\sigma}$	0.7502	0.1498	0.0872

From Table 1, we observe that as the sample sizes increase the BIAS and RSMEs of η ,λ and σ decrease steadily.

3.2 Data Analysis

The Covid-19 data is from Guyana and it covers a period of 628 days, from March12, 2020 to November 30, 2021.

The data is presented using the daily new deaths (ND), the daily cumulative deaths (CD) and the daily cumulative cases (CC) and calculated as follows:

$$x_i = \frac{ND_i}{CC_i - CD_i} \times 100, \quad CC_i - CD_i \neq 0 \tag{18}$$

Data points that have no new deaths are excluded from the data analysis.

Table 2. The first 100 data points

.0158	.0158	.0469	.6234	.6184	.5724	.0161	.0159	.0317	.0158
.9833	.9465	.9225	.4050	.8323	.7806	.7843	.7236	.3751	.7297
.7348	.6993	.6944	.8019	.6392	.9404	.6464	.9096	.9188	.6088
.5974	.2916	.2884	.5713	.2822	.5499	.5430	.8086	.5283	.2641
.5088	.8666	.2461	.4847	.4833	.4683	.6842	.2285	.2274	.4379
.2190	.2122	.2099	.4172	.2043	.5969	.1970	.3872	.1900	.1893
.3611	.1794	.1723	.1721	.3390	.1681	.1639	.3259	.3262	.3209
.1572	.3108	.1555	.2778	.2763	.1354	.1341	.1335	.2560	.1280
.1269	.1256	.2503	.2488	.2469	.1223	.1207	.2419	.3608	.2379
.2362	.1168	.1157	.2321	.1153	.1136	.1130	.1112	.1101	.1087

Table 3 gives the descriptive statistics of the Covid-19 data of Guyana. The sample size was reduced to 397 after excluding the data points which were zero. The mean of the Covid-19 data is 0.1997. The minimum value is 0.0156 while the maximum value is 0.9833. The value of skewness indicates that the distribution is highly skewed since it is greater than +1.

Table 3. Descriptive statistics of the Covid-19 data

N	Mean	Minimum	Maximum	Skewness
397	0.1997	0.0156	0.9833	1.979

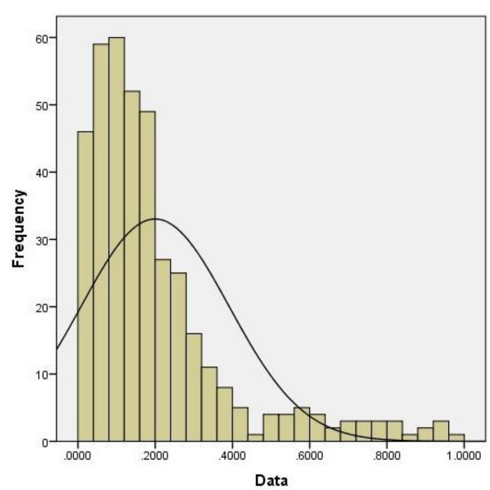

Fig 5. Histogram for the Covid-19 data of Guyana

Figure 5 shows the histogram of the Covid-19 data of Guyana. From the graph, the data is positively skewed since the tail on the right side is longer.

The Covid-19 data is fitted into the transmuted Weibull distribution, the 2-parameter Weibull distribution, with CDF:

$$F(x) = 1 - exp\left(-\left(\frac{x}{\sigma}\right)^{\eta}\right) \tag{19}$$

and the 3-parameter Weibull distribution with CDF:

$$F(x) = 1 - exp\left(-\left(\frac{x-\lambda}{\sigma}\right)^{\eta}\right) \tag{20}$$

.

The best mathematical model for fitting the Covid-19 data of Guyana is assessed using the Akaike Information Criterion (AIC) approach and is presented in Table 3. The AIC is given as

$$AIC = 2k - 2\,(log - likelihood) \tag{21}$$

where k is the number of parameters in the model. The model with the smallest AIC value is an indication that the model fits the Covid-19 data well.

Table 4. Estimated parameters, Maximum likelihood estimates and AIC

Distribution	Parameter estimates			Log-likelihood	AIC
	$\widehat{\eta}$	$\widehat{\lambda}$	$\widehat{\sigma}$		
2-parameter Weibull	1.145		0.210	248.742	-493.484
3-parameter Weibull	0.900	-0.100	0.700	-42.817	91.634
Transmuted Weibull	1.242	0.610	0.286	254.066	-502.132

In Table 4, the result shows that the Transmuted Weibull distribution has the lowest AIC value so it fits the data better than the 2-parameter and 3-parameter Weibull distribution.

4 Discussion

The results obtained using the Transmuted Weibull distribution, 2-parameter Weibull and 3-parameter Weibull distribution indicates that the transmuted Weibull distribution fit the Covid-19 data of Guyana well since it produce the lowest AIC value while the 3-parameter Weibull distribution had the highest AIC value.

Several Transmuted distributions are used to model the Covid-19 data of various countries. Furthermore, these distributions were used to model the Covid-19 data of France, United Kingdom and Canada. The goodness of fit of these distributions were assessed using the log-likelihood, Akaike's Information Criterion (AIC), Bayesian Information Criterion (BIC) and the Kolmogorov Smirnov test for the models. The fit of these distributions were compared with other competitive models.

To date, the transmuted distributions are considered to be the best model for modeling Covid-19 data because of its additional parameter which captures any skewness in the data. Also, the transmuted Weibull distribution was not used to model the Covid-19 data of any country that was reported in literature. The results produced better results since this particular transmuted distribution was fitted into the Covid-19 data of Guyana formed by the daily cumulative deaths from the period of March 12th, 2020 to November 30th, 2021.

5 Conclusion

Based on this study, the properties and characterization of the transmuted Weibull distribution was examined in detail. The transmuted Weibull distribution was successfully applied to model the Covid-19 data of Guyana. The best model for fitting the data was selected using the AIC approach. The results indicated that the 3-parameter Weibull had highest AIC value (91.634) while the transmuted Weibull distribution produces the lowest AIC value (-502.132). Therefore, based on the findings of this study, the deaths were high and as the vaccine progresses the number of deaths decreased and it was evident in the model. In Guyana, no work has been done using the cumulative deaths and the transmuted Weibull distribution.

References

1) Novel Coronavirus (2019-nCov): Situation Report -1. 2020. Available from: https://www.who.int/docs/default-source/wrindia/india-situation-report-1.pdf?sfvrsn=5ca2a672_0.
2) Wang J. Mathematical models for COVID-19: applications, limitations, and potentials. *Journal of Public Health and Emergency*. 2020;4:9. Available from: https://doi.org/10.21037/jphe-2020-05.
3) Mugisha JYT, Ssebuliba J, Nakakawa JN, Kikawa CR, Ssematimba A. Mathematical modeling of COVID-19 transmission dynamics in Uganda: Implications of complacency and early easing of lockdown. *PLOS ONE*. 2021;16(2):e0247456. Available from: https://doi.org/10.1371/journal.pone.0247456.
4) Ambikapathy B, Krishnamurthy K. Mathematical Modelling to Assess the Impact of Lockdown on COVID-19 Transmission in India: Model Development and Validation. *JMIR Public Health and Surveillance*. 2020;6(2):e19368. Available from: https://doi.org/10.2196/19368.
5) Almetwally EM. Application of COVID-19 Pandemic by Using Odd Lomax-G Inverse Weibull Distribution. *Mathematical Sciences Letters an International Journal*. 2021;10(2):47–57. Available from: https://www.naturalspublishing.com/files/published/56lj72t95ky92p.pdf.
6) Almetwally EM. The Odd Weibull Inverse Topp–Leone Distribution with Applications to COVID-19 Data. *Annals of Data Science*. 2022;9(1):121–140. Available from: https://doi.org/10.1007/s40745-021-00329-w.
7) Azm WSAE, Almetwally EM, Al-Aziz SN, El-Bagoury AA, Alharbi AH, Abo-Kasem R, et al. A New Transmuted Generalized Lomax Distribution: Properties and Applications to COVID-19 Data. *Computational Intelligence and Neuroscience*. 2021;2021:1–14. Available from: https://doi.org/10.1155/2021/5918511.