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RESEARCH ARTICLE

Negative Valued Ideals of Fuzzy KM-Subalgebras

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Abstract

Objectives: A new notion of logical algebra named KM-algebras was introduced in 2019. KM-algebra is a generalization of some of the B-algebras, such as BCK, BCI, BCH, BE, BV-algebras, and d-algebras. Also, by fuzzification of KM-algebras, many exciting results have been analyzed. Now feel obligated to endure dreadful knowledge since no negative interpretation of information is provided. We believe it is also critical to provide mathematical tools to do this. To do this, we develop and utilize a new negative-valued function. **Methods:** An innovative concept that is Negative valued ideals of KM-algebras has been introduced, and some main results have been proved. The new notion of weak and strong p_{KM}-cut of the Negative valued KM-subalgebras has been introduced. And also, some of the interesting results have been proved. **Findings:** A very innovative idea of applying Negative valued ideals of KM-algebras, weak and strong p_{KM}-cut of the Negative valued KM-subalgebras in old age homes, and interpreting the outputs in R programming has been analyzed (N_{KM} stands for Negative Valued Function in KM-algebras). **Novelty:** With the result of this study, on the Negative valued functions of KM-Subalgebras, a very interesting concept of KMM-ideals in Negative valued KM-algebras has been developed. None other than Negative valued ideals of KM-algebras have analyzed more main results using weak and strong p_{KM}-cut of the Negative valued KM-subalgebras. Also, one of the main applications of Negative valued ideals of KM-algebras is, analyzing which Old age Home is better than the other using weak and strong p_{KM}-cut of the Negative valued KM-subalgebras. 2010 AMS Classification: 03G25.

Keywords: KM-algebras, N_{KM}-Structure of Fuzzy KM-algebras, N_{KM} - Subalgebras of Fuzzy KM-algebras, N_{KM} – ideal of KM-algebras, Weak and Strong P_{KM} - cut of N_{KM} - Structure of Fuzzy KM-algebras.

1 Introduction

The algebra of logic was invented in the middle of the nineteenth century. In broad terms, algebraic logic is the branch of mathematics that examines logics by connecting them with classes of algebras, logical matrices, and other algebra-related mathematical structures and then relating the qualities of the logics to the properties of the associated algebra.

BCK-algebra was introduced in 1966 by the mathematician Imai. Y & Iséki. K [(1)], and it has since been used in a number of fields in Mathematics. BCI-algebras was suggested by Iseki. K [(2)] in 1966 as a generalization of BCK-algebras. Many researchers built algebraic structures that are generalizations of BCK/BCI-algebras after the advent of BCK/BCI-algebras. To date, many types of logical algebras have been developed by the generalization of BCK/BCI algebras.

We've always used probability to cope with uncertainty. The sort of uncertainty that probability deals with is called "stochastic probability." The kind of uncertainty we deal with when we use fuzzy notions, on the other hand, is separate; there is no stochastic ambiguity. What choices do we have for dealing with ambiguity? The notion of membership was developed by Lotfi Zadeh [(3)]. In comparison, he established a fuzzy probability. Most of the generalization of the crisp set has been made on the unit interval [0,1]. A (crisp) set M in a universe K may be described in the form of its characteristic function $\lambda_K : K \to \{0,1\}$, which yields the value 1 for items belonging to the set K and the value 0 for elements excluded from the set K. Because a (crisp) set M in a universe K may be specified in the form of its characteristic function, which yields the value 1 for elements belonging, this is compatible with the asymmetry observation. To put it another way, the crisp set was generalized to fuzzy sets by spreading positive information that fit the crisp point 1 into the interval [0,1].

Because no negative interpretation of information is supplied, we now feel obligated to cope with terrible information. We feel it is also vital to supply mathematical tools in order to do this. To do this, we develop and use a new function called a negative-valued function. The most important achievement of this article is that it enables one to deal with both positive and negative data at the same time by combining concepts from this article with previously well-known positive data. Kalaiarasi et al. [(4)] introduced fuzzy sets in KM-algebras. The ideal hypothesis plays a vital role in the ongoing enhancement of KM-algebras (K stands for Kalaiarasi, and M represents for ManiMozhi). As a result, we were convinced to pursue these ideas for KM-algebras. Kalaiarasi et al. introduced a new notion of KM-algebra in 2021, which is a highly fascinating fuzzy product KM-algebra concept [(5)].

In 2019 [(6)], negative-valued functions on p-ideals of BCI algebras have been investigated. However, so many negative-valued functions on BCK/BCI algebras were introduced in 2009 [(7)], but none has analyzed the strong and weak cuts in Negative valued functions other than N_{KM}-structure. Also, the only logical algebra to give a detailed explanation about the negative valued ideals with an interesting application to old age homes is the one and only negative valued ideals in KM-algebras.

As a consequence, the following is how the article is structured. In section 2, some preliminaries of KM-algebras and KMM-ideals have been given. In section 3, a new notion of negative valued ideals has been analyzed using some examples, and important results have been proved. Also, a new idea, Strong and Weak p_{KM}-cut of N_{KM}-structure has been introduced with an example. Last but not least, one of the practical applications for negative valued fuzzy KM-algebras is given, and the result is interpreted with R-programming.

2 Methodology

Some Preliminary definitions for KM-algebra, KMM-ideals, Negative valued function of KM-algebra, Negative valued Subalgebra of KM-algebras have been given in this section, and some examples explaining these concepts have been given.

2.1 Definition [(5)]

Let K, a nonempty set, together with a binary operation '•' and a constant '0'. Then K= (K, •, 0) is called a KM-algebra [(5)] satisfying the following conditions:

(KM1)k•m = 0, if k = m

(KM2) k•m = k, if m = 0

(KM3) (k•m)•n = n• (k•m)

(KM4) k•m = 0 and m•k = 0 imply k = m, for all $k,m,n \in K$.

2.1.1 Example [(5)]

Let $M = \{vm_{00}, vm_{01}, vm_{02}\}$ be a set with the binary operation $' \cdot '$ and a constant $'vm_{00}'$with the following table.

Here in this example, the constant '0' in the definition given is ' vm_{00} '.

(KM1) $\mathrm{k} \cdot \mathrm{m} = vm_{00}$, if k = m

Here vm_{00} appears in the diagonal from upper left to lower right. The first Condition, KM1 is true in all three cases.

(KM2) k•m= k, if $\mathrm{m} = vm_{00}$

From the first row of table 1, KM2 is true

(KM3) (k•m)•n= n• (k• m), for all $k,m,n \in K$.

Table 1. Binary Operation '•'

•	vm_{00}	vm_{01}	vm_{02}
vm_{00}	vm_{00}	vm_{01}	vm_{02}
vm_{01}	vm_{01}	vm_{00}	vm_{02}
vm_{02}	vm_{02}	vm_{02}	vm_{00}

KM4)k •m=vm_{00} and m •k=vm_{00} imply k=m, for all $k,m,n \in K$.
Condition KM4 is true.
Therefore $(\mathrm{M}, \cdot, vm_{00})$ is a KM-algebra.

2.1.2 Example[5]

Let K be the set of all non-negative integers. For any k, m in K, let • be the binary operation defined by k · m = $\begin{cases} 0 & \text{if } k \le m \\ k+m & \text{otherwise} \end{cases}$.

In this example, 0 is the zero elements.
(KM1) k • m=0 if m=k.
(KM2) k • m=k if m=0.
(KM3) (k • m) • n = (k+m) • n = (k+m)+n = n+(k+m) = n • (k • m)
(KM4) k • m=0 and m • k=0 ⇒ k+m=0 and m+k=0
⇒ k+m=m+k
⇒ k=m.
Therefore, (K, •,0) is a KM-algebra.

2.2 Definition[5]

Let K, a KM-algebra, and I_d a subset of K which is nonempty. Then we call I_d the KMM- ideal of K if the following conditions are satisfied:
(i) $0 \in I_d$
(ii) $k \bullet m \in I_d$ and $m \in I_d$ imply $k \in I_d$ for all $k, m \in K$

2.2.1 Example[5]

For example, in 2.1.1, the KMM-ideals of M $\{m_{00}, m_{01}\}, \{m_{00}, m_{01}, m_{02}\}$ is.

2.3 Definition

Let K denote a KM-algebra and ς be a negative valued function from K into [-1,0], simply called an N_{KM}-function on K. Then we define $\mathfrak{K}(k,[-1,0]) = \{(K,\varsigma)/\varsigma : K \to [-1,0]\}$, where (K,ς) is an N_{KM}-Structure of K.

2.4 Definition

Let $(K,\bullet,0)$ be a KM-algebra with • as a binary operation and 0 as the null element in K. (K,ς) is said to be the N_{KM}-Subalgebra of K if the following assertion is satisfied by ς.
$\varsigma(k \cdot m) \le \bigvee\{\varsigma(k), \varsigma(m)\}$, forallk, $m \in K$

2.4.1 Example

In this Pandemic Situation, Educational Institutions suffer a lot to boost the students to gain basic and in-depth knowledge in their subjects, mainly the faculty members. The educator's job gets critical with respect to ensuring that the students stay drawn in and don't free their inspiration. Let us consider the students who are not attentive(vm_{00}), attentive(vm_{01}), most attentive(vm_{02}) in the online classroom during this pandemic condition.Let K={ vm_{00},v m_{01},v m_{02}} be the KM-algebra with the binary operation '•'having zero element 'vm_{00}'. Consider the Cayley table of Example 2.1.1 and Table 1.

Define an N_{KM} -function ς by
$k \begin{pmatrix} vm_{00} & vm_{01} & vm_{02} \\ -0.09 & -0.08 & -0.1 \end{pmatrix}$.

Clearly(K,ς) is an N_{KM}-Subalgebra of K.

3 Result and Discussion

3.1 Negative Valued ideals of KM-algebras

In this section, a new notion of negative valued ideals of KM-algebras has been analyzed using some examples. Also, some results on negative valued ideals of KM-algebras have been proved.

3.2 Definition

Let $(K,\ .,0)$ be a KM-algebra, then N_{KM} – structure (K,ς) is said to be N_{KM} – ideal of K if the following assertions are satisfied.

i) $\varsigma(0) \leq \varsigma(k)$&

ii) $\varsigma(\mathrm{k}) \leq \mathrm{V}\{\varsigma(\mathrm{k}\cdot\mathrm{m}),\varsigma(\mathrm{m})\} \quad \forall \mathrm{k},\mathrm{m} \in \mathrm{K}$

3.2.1 Example

Let $M = (vm_{00}, vm_{01}, vm_{02}\}$ be a set with the binary operation '•' and a constant 'vm_{00}' with the Table 1$(M,\ \bullet, vm_{00})$ is a KM-algebra.

Define an N_{KM}-function ς by

$$\begin{array}{c} M \\ \varsigma \end{array} \begin{pmatrix} vm_{00} & vm_{01} & vm_{02} \\ -0.09 & -0.08 & -0.01 \end{pmatrix}$$

&$\varsigma(vm_{00}) = -0.09 \leq -0.08 = \varsigma(vm_{01})$

Also, $\varsigma(vm_{00}) = -0.09 \leq -0.01 = \varsigma(vm_{02})$

Table 2. Condition $V\{c(k{\cdot}m),c(m)\}$

$\varsigma(k)$	$\varsigma(k\bullet m)$	$\varsigma(m)$	$(\varsigma(k\bullet m),\varsigma(m)\}$
$\varsigma(vm_{00}) = -0.09$	$\varsigma(vm_{00}\bullet vm_{00}) = \varsigma(vm_{00}) = -0.09$	-0.09	-0.09
$\varsigma(vm_{00}) = -0.09$	$\varsigma(vm_{00}\bullet vm_{01}) = \varsigma(vm_{01}) = -0.08$	-0.08	-0.08
$\varsigma(vm_{00}) = -0.09$	$\varsigma(vm_{00}\bullet vm_{02}) = \varsigma(vm_{02}) = -0.01$	-0.01	-0.01
$\varsigma(vm_{01}) = -0.08$	$\varsigma(vm_{01}\bullet vm_{00}) = \varsigma(vm_{01}) = -0.08$	-0.09	-0.08
$\varsigma(vm_{01}) = -0.08$	$\varsigma(vm_{01}\bullet vm_{01}) = \varsigma(vm_{00}) = -0.09$	-0.08	-0.08
$\varsigma(vm_{01}) = -0.08$	$\varsigma(vm_{01}\bullet vm_{02}) = \varsigma(vm_{02}) = -0.01$	-0.01	-0.01
$\varsigma(vm_{02}) = -0.01$	$\varsigma(vm_{02}\bullet vm_{00}) = \varsigma(vm_{02}) = -0.01$	-0.09	-0.01
$\varsigma(vm_{02}) = -0.01$	$\varsigma(vm_{02}\bullet vm_{01}) = \varsigma(vm_{02}) = -0.01$	-0.08	-0.01
$\varsigma(vm_{02}) = -0.01$	$\varsigma(vm_{02}\bullet vm_{02}) = \varsigma(vm_{00}) = -0.09$	-0.01	-0.01

Therefore, in all the above, $\varsigma(\mathrm{k}) \leq \mathrm{V}\{\varsigma(\mathrm{k}\cdot\mathrm{m}),\varsigma(\mathrm{m})\}, \forall \mathrm{k},\mathrm{m} \in \mathrm{M}$.

Thus, $(M,\varsigma)$is an N_{KM}-ideal of M.

3.2.2 Example

Consider the Cayley table of $(S,\ \bullet, n_{00})$ KM-algebra where $S = (n_{00}, n_{01}, n_{02}, n_{03}\}$, '•' is a binary operation, and n_{00} is the zero element.

Table 3. Binary Operation '•'

$\bullet$	n_{00}	n_{01}	n_{02}	n_{03}
n_{00}	n_{00}	n_{01}	n_{02}	n_{03}
n_{01}	n_{01}	n_{00}	n_{03}	n_{02}
n_{02}	n_{02}	n_{03}	n_{00}	n_{01}
n_{03}	n_{03}	n_{03}	n_{03}	n_{00}

Define an N_{KM}-function ς by

$$\begin{array}{c} S \\ \varsigma \end{array} \begin{pmatrix} n_{00} & n_{01} & n_{02} & n_{03} \\ -0.23 & -0.03 & -0.06 & -0.17 \end{pmatrix}$$

$$\varsigma(n_{00}) = -0.23 \leq -0.03 = \varsigma(n_{01})$$

$$\varsigma(n_{00}) = -0.23 \leq -0.06 = \varsigma(n_{02})$$

$$\varsigma(n_{00}) = -0.23 \leq -0.17 = \varsigma(n_{03})$$

Therefore Condition (i) in the definition of N_{KM}-ideal is satisfied.

Table 4. Condition $\varsigma(\mathrm{k}) \not\leq V\{\mathrm{s}(\mathrm{k}\cdot\mathrm{m}), \mathrm{s}(\mathrm{m})\}$

$\varsigma(k)$	$\varsigma(k \bullet m)$	$\varsigma(m)$	$(\varsigma(k \bullet m), \varsigma(m)\}$
$\varsigma(n_{00}) = -0.23$	$\varsigma(n_{00} \bullet n_{00}) = \varsigma(n_{00}) = -0.23$	-0.23	-0.23
$\varsigma(n_{00}) = -0.23$	$\varsigma(n_{00} \bullet n_{01}) = \varsigma(n_{01}) = -0.03$	-0.03	-0.03
$\varsigma(n_{00}) = -0.23$	$\varsigma(n_{00} \bullet n_{02}) = \varsigma(n_{02}) = -0.06$	-0.06	-0.06
$\varsigma(n_{00}) = -0.23$	$\varsigma(n_{00} \bullet n_{03}) = \varsigma(n_{03}) = -0.17$	-0.17	-0.17
$\varsigma(n_{01}) = -0.03$	$\varsigma(n_{01} \bullet n_{00}) = \varsigma(n_{01}) = -0.03$	-0.23	-0.03
$\varsigma(n_{01}) = -0.03$	$\varsigma(n_{01} \bullet n_{01}) = \varsigma(n_{00}) = -0.23$	-0.03	-0.03
$\varsigma(n_{01}) = -0.03$	$\varsigma(n_{01} \bullet n_{02}) = \varsigma(n_{03}) = -0.17$	-0.03	-0.03
$\varsigma(n_{01}) = -0.03$	$\varsigma(n_{01} \bullet n_{03}) = \varsigma(n_{02}) = -0.06$	-0.17	-0.06

Thus $\varsigma(n_{01}) \not\leq (\varsigma(n_{01} \bullet n_{03}), \varsigma(n_{03})\}$
Therefore (S, ς) is not an N_{KM}-ideal.
(i) If we define N_{KM}-function ς by

$$\begin{array}{c} S \\ \varsigma \end{array} \begin{pmatrix} n_{00} & n_{01} & n_{02} & n_{03} \\ \bar{s}_0 & \bar{s}_1 & \bar{s}_0 & \bar{s}_1 \end{pmatrix}, \ \bar{s}_0 < \bar{s}_1 \& \bar{s}_0, \bar{s}_1 \varepsilon (-1, 0).$$

$\varsigma(n_{00}) = \bar{s}_0 < \varsigma(n_{01}) \& \varsigma(n_{03})$
$\& \varsigma(n_{00}) \leq \varsigma(n_{02})$.

Table 5. Condition $\varsigma(\mathrm{k}) \leq \mathrm{V}\{\varsigma(\mathrm{k}\cdot\mathrm{m}), \varsigma(\mathrm{m})\}$

$\varsigma(k)$	$\varsigma(k \bullet m)$	$\varsigma(m)$	$(\varsigma(k \bullet m), \varsigma(m)\}$
$\varsigma(n_{00}) = \bar{s}_0$	$\varsigma(n_{00} \bullet n_{00}) = \varsigma(n_{00}) = \bar{s}_0$	$\bar{s}_0$	$\bar{s}_0$
$\varsigma(n_{00}) = \bar{s}_0$	$\varsigma(n_{00} \bullet n_{01}) = \varsigma(n_{01}) = \bar{s}_1$	$\bar{s}_1$	$\bar{s}_1$
$\varsigma(n_{00}) = \bar{s}_0$	$\varsigma(n_{00} \bullet n_{02}) = \varsigma(n_{02}) = \bar{s}_0$	$\bar{s}_0$	$\bar{s}_0$
$\varsigma(n_{00}) = \bar{s}_0$	$\varsigma(n_{00} \bullet n_{03}) = \varsigma(n_{03}) = \bar{s}_1$	$\bar{s}_1$	$\bar{s}_1$
$\varsigma(n_{01}) = \bar{s}_1$	$\varsigma(n_{01} \bullet n_{00}) = \varsigma(n_{01}) = \bar{s}_1$	$\bar{s}_0$	$\bar{s}_1$
$\varsigma(n_{01}) = \bar{s}_1$	$\varsigma(n_{01} \bullet n_{01}) = \varsigma(n_{00}) = \bar{s}_0$	$\bar{s}_1$	$\bar{s}_1$
$\varsigma(n_{01}) = \bar{s}_1$	$\varsigma(n_{01} \bullet n_{02}) = \varsigma(n_{03}) = \bar{s}_1$	$\bar{s}_0$	$\bar{s}_1$
$\varsigma(n_{01}) = \bar{s}_1$	$\varsigma(n_{01} \bullet n_{03}) = \varsigma(n_{02}) = \bar{s}_0$	$\bar{s}_1$	$\bar{s}_1$
$\varsigma(n_{02}) = \bar{s}_0$	$\varsigma(n_{02} \bullet n_{00}) = \varsigma(n_{02}) = \bar{s}_0$	$\bar{s}_0$	$\bar{s}_0$
$\varsigma(n_{02}) = \bar{s}_0$	$\varsigma(n_{02} \bullet n_{01}) = \varsigma(n_{03}) = \bar{s}_1$	$\bar{s}_1$	$\bar{s}_1$
$\varsigma(n_{02}) = \bar{s}_0$	$\varsigma(n_{02} \bullet n_{02}) = \varsigma(n_{00}) = \bar{s}_0$	$\bar{s}_0$	$\bar{s}_0$
$\varsigma(n_{02}) = \bar{s}_0$	$\varsigma(n_{02} \bullet n_{03}) = \varsigma(n_{01}) = \bar{s}_1$	$\bar{s}_1$	$\bar{s}_1$
$\varsigma(n_{03}) = \bar{s}_1$	$\varsigma(n_{03} \bullet n_{00}) = \varsigma(n_{03}) = \bar{s}_1$	$\bar{s}_0$	$\bar{s}_1$
$\varsigma(n_{03}) = \bar{s}_1$	$\varsigma(n_{03} \bullet n_{01}) = \varsigma(n_{03}) = \bar{s}_1$	$\bar{s}_1$	$\bar{s}_1$
$\varsigma(n_{03}) = \bar{s}_1$	$\varsigma(n_{03} \bullet n_{02}) = \varsigma(n_{03}) = \bar{s}_1$	$\bar{s}_0$	$\bar{s}_1$
$\varsigma(n_{03}) = \bar{s}_1$	$\varsigma(n_{03} \bullet n_{03}) = \varsigma(n_{00}) = \bar{s}_0$	$\bar{s}_1$	$\bar{s}_1$

Therefore, Condition (i) and (ii) for N_{KM}-ideal in the definition is satisfied.
Thus (S,ς) is N_{KM}-ideal.

3.2.3 Proposition

For all $k,m\, \varepsilon\, K,\;\; k \le m \;\Rightarrow\; \varsigma(k) \le \varsigma(m)$; where (K,ς) is an N_{KM}-ideal of K.

Proof:

Given $k \le m$, for all $k, m \varepsilon K$.

$\Rightarrow k \bullet m = 0$

Given (K,ς) is an N_{KM}-ideal of K

By definition, $\varsigma(\mathrm{k}) \le \mathrm{V}\{\varsigma(\mathrm{k}\cdot\mathrm{m}), \varsigma(\mathrm{m})\}$

$= \mathrm{V}\{\varsigma(0), \varsigma(\mathrm{m})\} = \varsigma(\mathrm{m})$ $\quad$ [Since $\varsigma(0) \le \varsigma(\mathrm{m})\}$

Therefore $\varsigma(k) \le \varsigma(m)$.

3.2.4 Proposition

Let (K,ς) be an N_{KM}-ideal of K. For all $k,m,n\, \varepsilon\, K$, the following assertion is satisfied:

$\mathrm{k}\cdot\mathrm{m} \le \mathrm{n} \Rightarrow \varsigma(\mathrm{k}) \le \mathrm{V}\{\varsigma(\mathrm{m}), \varsigma(\mathrm{n})\}$

Proof:

For all $k,m,n\, \varepsilon K$,

$$k \bullet m \le n \qquad \longrightarrow (1)$$

$\varsigma(k.m) \le V(\varsigma((k\bullet m)\bullet n), \varsigma(n)\}$ by definition

$\le V(\varsigma(n\bullet n), \varsigma(n)\}$ by (1)

$= V(\varsigma(0), \varsigma(n)\}$

$\varsigma(k.m) = \varsigma(n) \qquad \longrightarrow (2)$[since $\varsigma(0) \le \varsigma(n)\}$

Now,

$\varsigma(k) \le V(\varsigma(k\bullet m), \varsigma(m)\}$ and by (2)

$$\varsigma(k) \le V(\varsigma(n), \varsigma(m)\}$$

3.2.5 Theorem

Every N_{KM}-ideal of a KM-algebra K is an N_{KM}-subalgebra, and its converse is also true.

Proof:

Suppose (K,ς) be an N_{KM}-ideal of K and for all $k,m\, \varepsilon\, K$,

$$\varsigma(k\bullet m) \le V(\varsigma((k\bullet m)\bullet m), \varsigma(m)\}$$

$$= (\varsigma(k\bullet(m.m)), \varsigma(m)\}$$

$$= V(\varsigma(0), \varsigma(n)\}(\varsigma(k\bullet 0), \varsigma(m)\}$$

$$= V(\varsigma(k), \varsigma(m)\}$$

$$\varsigma(k\bullet m) \le V(\varsigma(k), \varsigma(m)\}.$$

Thus, (K,ς) is an N_{KM}-Subalgebra of K.

The converse of the above theorem is also true.

$$\varsigma(k\bullet m) \le V(\varsigma(k), \varsigma(m)\}$$

$$= V\left(\varsigma\left(k\bullet(m\bullet m)\right),\varsigma(m)\right\}$$

$$= V\left(\varsigma\left(m\bullet(k\bullet m),\varsigma(m)\right)\right\}$$

$$= V\left(\varsigma\left((k\bullet m)\bullet m\right),\varsigma(m)\right\}$$

$$\Sigma\left((k\bullet m)\bullet(k\bullet m)\right)\leq V\left(\varsigma(k\bullet m),\varsigma(k\bullet m)\right\}$$

$$\Rightarrow \varsigma(0)\leq\varsigma(k\bullet m)$$

Consider the set $K_v = (k\varepsilon K|\varsigma(k)\leq\varsigma(v)\}$ for any element $v\ in\ K$. Also, this set is a nonempty subset of K since $v\varepsilon K$.

3.2.6 Theorem

For any element $in\ K$, the set K_v is a KMM-ideal of K, and if (K,ς) is an N_{KM}-ideal of K.

Proof:

$$\varsigma(\theta)\leq\varsigma(k)\qquad\Rightarrow\theta\varepsilon K_v$$

For any elements $k,m\varepsilon K,\ \ let\ k\bullet m\varepsilon K_v\ and\ m\varepsilon K_v$.

$\Rightarrow\varsigma(k.m)\leq\varsigma(v)$ and $\varsigma(m)\leq\varsigma(v)$.

Also, given (K,ς) is an N_{KM}-ideal of K.$Therefore\ \varsigma(k)\leq V\left(\varsigma(k\bullet m),\varsigma(m)\right\}\leq V\left(\varsigma(v),\varsigma(v)\right\}$

$$=\varsigma(v)$$

$\Rightarrow k\varepsilon K_v$.

$\Rightarrow K_v\ is\ a\ KMM$-ideal of K.

3.2.7 Theorem

Let (K,ς) be anN_{KM}-structure of$K\ and\ \varsigma$. For any element $v\varepsilon K,\ K_v$ is a KMM-ideal of K if the following assertions are satisfied by (K,ς):

(i) $\varsigma(\theta)\leq\varsigma(k)$for all $k\ \varepsilon\ K$ and

$$\varsigma(k)\geq V\left(\varsigma(m\bullet n),\varsigma(n)\right\}\quad\Rightarrow\varsigma(k)\geq\varsigma(m);\ \ for\ all\ k,m,n\ \varepsilon\ K$$

Proof:

Given (K,ς) is an N_{KM}-structure of K and satisfies the assertions (i) and (ii) in the statement of the theorem.

Let $k,m\ \varepsilon\ K$ be such that $k,m\ \varepsilon\ K_v$ and $m\ \varepsilon\ K_v$ for all $v\ \varepsilon\ K$.

$\Rightarrow\varsigma(k.m)\leq\varsigma(v)$ and $\varsigma(m)\leq\varsigma(v)$.

$\Rightarrow\ (\varsigma(k\bullet m),\varsigma(m)\}\leq\varsigma(v)$.

$\Rightarrow\varsigma(v)\geq\varsigma(k)$ by (ii)

$\Rightarrow k\ \varepsilon\ K_v$.

Also, by (i) $\theta\ \varepsilon\ K_v$

Therefore K_v is an ideal of K.

3.2.8 Theorem

If K_v is an ideal of K, then the following assertion is satisfied by (K, ς) :

$\varsigma(k) \geq V(\varsigma(m \bullet n), \varsigma(n)\} \Rightarrow \varsigma(k) \geq \varsigma(m)$, for all $k, m, n\ \varepsilon\ K$.

Proof:

Let $k, m, n\ \varepsilon\ K$ be such that $\varsigma(k) \geq V(\varsigma(m \bullet n), \varsigma(n)\}$. Since given K_v is an ideal of K.

$\Rightarrow m \bullet n\ \varepsilon\ K_v$ and $n\ \varepsilon\ K_v$.

$\Rightarrow K_v$ is an ideal of K

$$\Rightarrow m \varepsilon K_k$$

$$\Rightarrow \varsigma(m) \leq \varsigma(k).$$

3.3 Strong and Weak p_{KM}-cut of $\mathcal{N}_{KM}$-structure $(s,\ \varsigma\ )$

A new idea of Strong and Weak p_{KM}-cut of N_{KM}-structure(K, ς) has been introduced with examples, and results with innovative ideas have been proved.

3.4 Definition

Let us define a set $W_{KM}(\varsigma; p) = (k\ \varepsilon\ K | \varsigma(k) \leq p\}$, for any N_{KM}-structure (K, ς)&$-1 \leq p < 0$. The above-defined set is called the Weak p_{KM}-cut of (K, ς). Similarly, if we define a set $S_{KM}(\varsigma; p) = (k\ \varepsilon\ K|\ \varsigma(k) < p\}$for any$N_{KM}$-structure$(K, \varsigma)$&$-1 \leq p < 0$, then $S_{KM}(\varsigma; p)$ is called Strongp_{KM}-cut of (K, ς).

3.5 Example

In example 3.1.2 (i),consider p= - 0.23.

Then $W_{KM}(\varsigma; p) = (n_{00}, n_{01}, n_{02}, n_{03}\}$, $S_{KM}(\varsigma; p) = (n_{01}, n_{02}, n_{03}\}$

Also if $p = -0.17$, then $W_{KM}(\varsigma; p) = (n_{01}, n_{02}, n_{03}\}$, $S_{KM}(\varsigma; p) = (n_{01}, n_{02}\}$

In example 2.1.2 (ii), Consider $p = \bar{s_0}$.

$$W_{KM}(\varsigma; p) = \{\ \}$$

$$S_{KM}(\varsigma; p) = \{\ \}$$

but if we consider $p = \bar{s_1}.W_{KM}(\varsigma; p) = (n_{00}, n_{01}, n_{02}, n_{03}\}$

$S_{KM}(\varsigma; p) = (n_{00}, n_{02}\}$.

3.6 Theorem

If the nonempty weak p_{KM}-cut of $(K, \varsigma)$is a KMM-ideal of K, for all $1 \leq p \leq 0$, then (K, ς) is an N_{KM}-ideal of K.

Proof:

Given $W_{KM}(\varsigma; p) \neq \phi$. Also given $W_{KM}(\varsigma; p)$ is a KMM-ideal of $K \longrightarrow (*)$

Suppose $\varsigma(0) > \varsigma(k)$, for some $k \varepsilon K$,

Then there exists $-1 \leq p < 0$, such that $\varsigma(0) > p \geq \varsigma(k)$.

$\Rightarrow k \varepsilon W_{KM}(\varsigma; p)$

but $0 \notin W_{KM}(\varsigma; p)$, which is a contradiction to $W_{KM}(\varsigma; p)$ is a KMM-ideal of K.

Therefore $\varsigma(0) \leq \varsigma(k) \longrightarrow (1)$

Now suppose that $\varsigma(k) > (\varsigma(k \bullet m), \varsigma(m)\}$, for some $k, m \varepsilon K$.

Then there exists $-1 \leq p < 0$ such that $\varsigma(k) > p \geq V(\varsigma(k \bullet m), \varsigma(m)\}$where

$$p = \frac{1}{2}(\varsigma(k) + V(\varsigma(k \bullet m), \varsigma(m)\}$$

$\Rightarrow k \bullet m \ \varepsilon \ W_{KM}(\varsigma;p)$ and $m \ \varepsilon \ W_{KM}(\varsigma;p)$, but $k \notin W_{KM}(\varsigma;p)$, which is a contradiction to $(*)$.

$$\therefore \ \varsigma(k) \leq V(\varsigma(k \bullet m), \varsigma(m)\}$$

Thus (K,ς) is an N_{KM}-ideal of K.

3.6.1 Theorem

The nonempty weak p_{KM}-cut of $(K,\varsigma)$is a KMM-ideal of K, for all $\varsigma(0) \leq p \leq 0$, if an N_{KM}-Structure(K,ς) is an N_{KM}-ideal of K.

Proof:

Given $\varsigma(0) \leq p$

Let $k \ \varepsilon \ W_{KM}(\varsigma;p) \implies \varsigma(k) \leq p$.

Immediately we can say that $\varsigma(0) \leq \varsigma(k)$

For any $k, m\varepsilon K$ & $\varsigma(0) \leq p \leq 0$, $k \bullet m\varepsilon W_{KM}(\varsigma;p)$and $m\varepsilon W_{KM}(\varsigma;p)$

Then it follows that $\varsigma(k \bullet m) \leq p$ and $\varsigma(m) \leq p$.

$$\therefore \ \varsigma(k) \leq V(\varsigma(k \bullet m), \varsigma(m)\} \leq p$$

$$\Rightarrow k\varepsilon W_{KM}(\varsigma;p).$$

Therefore $W_{KM}(\varsigma;p)$ is a KMM-ideal of K.

3.6.2 Theorem

For any KMM-ideal S_M of K, where S_M is the weak p_{KM}-cut of (K,ς), there exists an N_{KM}-ideal (K,ς) of K, for any fixed number p such that $-1 < p < 0$.

Proof:

Define ς of (K,ς) as follows $\varsigma(x) = p$, if x is in S_M &

$\varsigma(x) = 0$, if x is not in $S_M \longrightarrow (*)$

Let us consider elements $k, m \ \varepsilon \ K$.

Case (i):

Suppose m is not in S_M, then $\varsigma(m) = 0$ from $(*)$& we have $\varsigma(k) \leq 0 = (\varsigma(k \bullet m), \varsigma(m)\}$.

Case (ii)

Suppose m is in S_M. Also, if k is in S_M, then $k \bullet m$ may or may not be in S_M.

Thus $\varsigma(k) \leq V(\varsigma(k \bullet m), \varsigma(m)\}$

Case (iii)

Suppose k is not in S_M, then $k \bullet m$ is also not in S_M.

Thus $\varsigma(k) = 0 = V(\varsigma(k \bullet m), \varsigma(m)\}$

Also, $\varsigma(0) \leq \varsigma(k)$ in all the above cases.

Therefore (K,ς) is an N_{KM}-ideal of K.

3.7 Application of Negative Valued Fuzzy KM-Subalgebras

In Indian households, respect for one's parents runs deep. Children believe it is their moral obligation to care for their elderly parents and elders. In terms of family structure, our culture has changed dramatically over the previous several decades. As a result of societal change and changing lifestyles, more elderly parents are finding themselves in nursing homes. The proliferation of nuclear families, particularly in metropolitan India, has resulted in an increase in the number of old-age homes.

The movement of children from their hometowns to metropolitan centers in quest of better education, employment, and lifestyles is the most evident explanation for this trend. While the younger generation has no trouble moving out of their parents' houses and adjusting to new lives, the elderly choose to stay because of the place's sentimental value. Furthermore, many children are unable to care for their elderly parents who suffer from chronic illnesses. Due to differences in beliefs and perspectives, some elderly find it difficult to deal with their daughters-in-law and grandkids.

This heartbreaking tragedy does have a silver lining. Many working children are attempting to care for their aging parents at home. They are employing nurses to look after their elderly parents while they are at work in the city. Others have put in

Fig 1. Old Age Home

place security measures for the elderly, such as placing CCTV cameras to keep an eye on them while they are at work. Children are sometimes placed in old age homes when they reach an age where they are unable to care for their own elderly parents. It's heartwarming to witness some children and grandkids make every effort to care for their own parents at home. The elderly enjoy receiving the care and attention they deserve from their family members and in their own homes.

A typical old age home is a facility that provides nursing and care to senior citizens in a clinical environment. There are a number of advantages to living in a senior assisted living facility. To begin with, individuals are treated with love, compassion, and care. Second, these residences have a specialized medical staff that includes competent physicians, nurses, and carers who are accessible 24 hours a day, seven days a week. Second, our senior assisted living community, Aurum, has established relationships with several top hospitals, pathology labs, and ambulance services to provide fast medical attention in the event of an emergency. Third, in addition to assisted living services, elders have the opportunity to live in a senior community where they may make new friends, participate in group activities, and live a carefree lifestyle. Some of the facilities provided in the home are

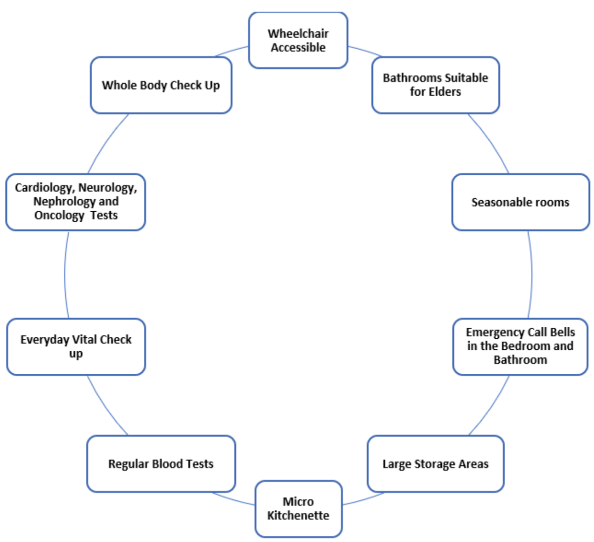

Fig 2. Ten Facilities of Old Age Home

Let us consider four old age homes in Trichy as OAH=$(oah_{00}, oah_{01}, oah_{02}, oah_{03}\}$. Let us consider the most comfortable old age home for an old man or woman may be the home which contains all the above ten facilities which are given above in the diagram. Let { OAH, ς } be a KM-algebra. The data which we have collected from the four old age homes have been tabulated below.

Table 6. Facilities in Old Age Homes

	Wheelchair Accessible	Bathroom Suitable for Elders	Seasonable rooms	Emergency Call Bells in the Bedroom and Bathroom	Large Storage Areas	Micro Kitchenette	Regular Blood Tests	Everyday Vital Checkup	Cardiology, Neurology, Nephrology, and Oncology Tests	Whole Body Check Up	Average
$\varsigma(oah_{00})$	-0.06	-0.07	-0.02	-0.06	-0.57	-0.47	-0.09	-0.79	-0.59	-0.77	-0.349
$\varsigma(oah_{01})$	-0.08	-0.45	-0.03	-0.09	-0.45	-0.50	-0.10	-0.99	-0.23	-0.81	-0.373
$\varsigma(oah_{02})$	-0.43	-0.67	-0.12	-0.23	-0.89	-0.56	-0.29	-0.86	-0.73	-0.96	-0.574
$\varsigma(oah_{03})$	-0.33	-0.34	-0.72	-0.25	-0.66	-0.69	-0.35	-0.82	-0.58	-0.79	-0.553

From these data, it is clear that oah_{00} has more facilities than other old age homes. Therefore, one of the greatest priorities is to choose this oah_{00} old age home. Since oah_{00} has an average of -0.349, which has the greatest facilities among all the other old age homes, the first preference to choose the old age home among the four may be oah_{00}. The Second one to choose is oah_{01}, since the average is -0.373, and the third one to prefer is oah_{03}, whose average is -0.553, and the last one to choose among the four is oah_{02}.

Let us consider p = -0.5, $W_{KM}(\varsigma;p) = S_{KM}(\varsigma;p) = \{-0.574,\ -0.553\}$. From this Strong and Weak p_{KM}-cut of N_{KM}-structure(K,ς) by considering p = -0.5, it is clear that preferring either oah_{00} or oah_{01} as the old age home. From the table, it is crystal clear that oah_{00} has good and more facilities rather than oah_{01}.

Also, we have uploaded the data in R-programming. The Pie-chart of R-programming that we have obtained also gives the same output as what we have obtained above. That is oah_{00} will be the one which is better in all the facilities than other old age homes.

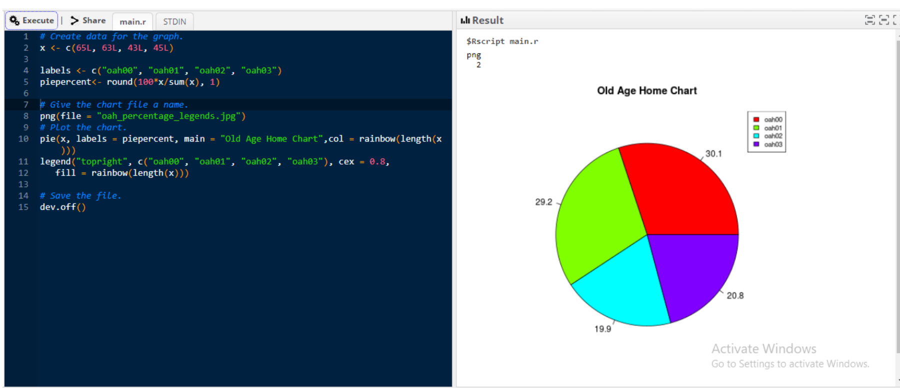

Fig 3. R-Programming Pie Chart

Even though old age homes are safer for elderly people, they dislike old age homes for a variety of reasons. The most obvious is the belief that there would be no one like their children in an old age home who will understand and care for their special requirements. They are afraid of being abandoned at the mercy of the home's management, which may or may not consider meeting their specific requirements. Each senior encounters their own set of issues as they age. While some individuals have geriatric issues, others are anxious about adjusting to a new environment and interacting with residents from various backgrounds.

However, we analyzed *that* oah_{00} is the best among the four; elderly people love to live with their children and grandchildren. So we should be supportive and affectionate to the elderly people, as they are the ladder for our growth and improvement.

4 Conclusion

We introduced a new notion of negative valued function in fuzzy KM-algebras,

N_{KM}-Structures and N_{KM}-Subalgebras in KM-algebras. Also, a new notion of Negative Valued ideals of KM-algebras has been introduced, and some interesting results have been proved. One such interesting result is every N_{KM}-ideal of a KM-algebra K is an N_{KM}-subalgebra. Even a new notion called Strong and Weak p_{KM}-cut of N_{KM}-structure(K, ς) has been introduced. One of the practical applications, considering the facilities of four old age homes and concluded which is the best old age home among the four having all the facilities expected by the senior citizen by using the Strong and Weak p_{KM}-cut of N_{KM}-structure(K, ς). And this has been checked using R-programming.

For future work,

• Negative valued function in fuzzy KM-algebra concepts and major conclusions can be used in a wide range of algebraic systems, particularly B-algebras like BV-algebra, BF-algebra, BM-algebra, BN-algebra, BO-algebra, and BT-algebra.

• To establish fuzzy KM-homomorphism in negative valued function of fuzzy KM-algebra.

• To develop the negative valued function of interval-valued, intuitionistic fuzzy KM-algebra.

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