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RESEARCH ARTICLE

Plane Wave Problem in a Generalized Thermo-elastic Solid in the Presence of Voids

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Abstract

Objective: To investigate the effect of voids on the propagation of plane waves in a generalized thermo-elastic solid. **Method:** The method of plane harmonic solution is employed to solve the basic equations of generalized thermo-elastic void solids. **Findings:** Under the effect of voids and non-voids, three sets of longitudinal waves are derived, and they are not appeared in any classical theory of elasticity. But one transverse wave is derived and these results are coinciding with the theory of classical elasticity. **Novelty:** Under the MATLAB programme the speed of longitudinal waves are shown in the frequency relation. Longitudinal waves are propagating with high speed in non-thermal voids solids.

Keywords: Thermoelasticity; Voids; Plane Harmonic Solution; Plane Longitudinal Waves; Plane Transverse Waves

1 Introduction

An important generalization on classical theory of elasticity is that "theory of linear elasticity materials having voids". This theory is very essential to investigate the different types of biological and geological materials for whose classical theory of elasticity is not adequate. The "linear elasticity theory with voids" leads that the materials with a small voids or pores, where the volume of the voids is included among the variables of Kinematic. This theory reduces to classical theory when voids tending to zero.

Adam G et al.[(1)] discussed the elastic solutions to 2D plane strain problems. The linear theory of elastic materials with voids was studied by De Cicco and De Angelis[(2)] in their paper entitled a plane strain problem in the theory of elastic materials with voids. The effect of voids on plane waves in micro stretch elastic solids was studied by Dilabag Singh[(3)]. Lesan[(4)] developed the "linear theory of thermo-elastic materials with voids". Marin Marin[(5)] presented a detailed study on the effect of voids in a heat-flux dependent theory for thermo elastic bodies. Stoneley and Rayleigh waves in thermo elastic materials with voids discussed by SS Singh[(6)]. Some results on thermo-elastic materials with voids were studied by a Ciarletta and Scarpetta[(7)]. Plane waves in

Thermo-viscoelastic material with voids under different theories of Thermo elasticity studied by Tomar et.al[8]. Kumar and Tomar[9] discussed Coupled dilational waves at a plane interface between two dissimilar magneto-elastic half-spaces containing voids. Mitthias et al. [10] studied a rate dependent non-linear mechanical behavior of thermo elastic composites.

S.K Tomar and Suraj Kumar[11] discussed the effects of voids on wave propagation in elastic-plastic materials, while the micropolar thermo elastic void solids linear theory was developed by Ciarletta et.al. [12]. Effects of voids on Rayleigh waves was studied by Chandrashekaraiah[13]. Tomar and Ashishkumar [14] studied about propagating waves in elastic materials with voids subjected electro-magnetic interaction. The equation of the linear theory of thermo elastic diffusion in porous media based on the concept of volume fraction derived by Aoudai[15].

In recent, Somaiah and Ravi Kumar[16] studied the plane waves in a micro-isotropic, micro-elastic solid. In this paper, we studied the effect of voids on plane longitudinal waves in generalized thermo-elastic solids with voids. It is observed that one set of transverse wave and three sets of longitudinal waves are derived. The longitudinal waves are travelled with high speed in the non-void solids.

2 Methodology

The governing equations in terms of displacement $\vec{u}$, volume fraction ϕ and temperature Φ for generalized thermo elastic void solids with the absence of body forces, equilibrated body forces and heat are given Iesan[4] , Lord and Shulman[17] as

$$\rho\frac{\partial^2 u}{\partial t^2}=(\lambda+\mu)\nabla(\nabla\cdot\vec{u})+\mu\nabla^2\vec{u}+b\nabla\phi-\beta\nabla\Phi \tag{1}$$

$$K\nabla^2\Phi=\beta T_0\left(\nabla\cdot\dot{\vec{u}}+\tau_0\nabla\cdot\ddot{\vec{u}}\right)+\rho c_e\left(\dot{\Phi}+\tau_0\ddot{\Phi}\right)+mT_0\left(\dot{\phi}+\tau_0\ddot{\phi}\right) \tag{2}$$

$$\rho\chi\frac{\partial^2\phi}{\partial t^2}=a\nabla^2\phi-c\phi-b\nabla\cdot\vec{u}+m\Phi \tag{3}$$

where λ,μ are Lame's constants, ρ is the density of the medium, τ_0 is the thermal relaxation time, $\Phi = T - T_0$, with T is the temperature, T_0 being uniform temperature of the medium and it is assumed to be such that $\left|\frac{\Phi}{T_0}\right|\leq 1, K$ is the coefficient of thermal conductivity, superpose dot is the partial derivative with respect to time t, c_e is the specific heat at a constant strain. The quantities a,b and c are the material constants due to the presence of voids, m is the thermo void coefficient, χ is the equilibrated inertia, $\beta=(3\lambda+2\mu)\alpha_t, \alpha_t$ being the coefficient of linear thermal expansion.

Equation (1) to (3) written as

$$c_1^2\nabla^2\vec{u}+c_2^2\nabla(\nabla\cdot\vec{u})-\beta_1\nabla\Phi+\mathrm{b}_1\nabla\phi=\frac{\partial^2\vec{u}}{\partial t^2} \tag{4}$$

$$K^*\nabla^2\Phi-\frac{\partial\Phi}{\partial t}-\beta^*\left(\nabla\cdot\frac{\partial\vec{u}}{\partial t}+\tau_0\nabla\cdot\frac{\partial^2\vec{u}}{\partial t^2}\right)-m^*\left(\frac{\partial\phi}{\partial t}+\tau_0\frac{\partial^2\phi}{\partial t^2}\right)=\tau_0\frac{\partial^2\Phi}{\partial t^2} \tag{5}$$

$$c_3^2\nabla^2\phi-c_4^2\phi-c_5^2\nabla\cdot\vec{u}+c_6^2\Phi=\frac{\partial^2\phi}{\partial t^2} \tag{6}$$

where

$$\begin{gathered}\nabla=\hat{i}_1\frac{\partial}{\partial x_1}+\hat{i}_2\frac{\partial}{\partial x_2}+\hat{i}_3\frac{\partial}{\partial x_3};\nabla^2=\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}+\frac{\partial^2}{\partial x_3^2};\\ c_1^2=\frac{\mu}{\rho};c_2^2=\frac{\lambda+\mu}{\rho};\beta_1=\frac{\beta}{\rho};b_1=\frac{b}{\rho};K^*=\frac{K}{\rho c_e};\beta^*=\frac{\beta T_0}{\rho c_e};\\ m^*=\frac{mT_0}{\rho c_e};c_3^2=\frac{a}{\rho\chi};c_4^2=\frac{c}{\rho\chi};c_5^2=\frac{b}{\rho\chi};c_6^2=\frac{m}{\rho\chi}\end{gathered} \tag{7}$$

For the plane wave propagation in the positive direction of a unit vector $\hat{n}$,

we may take

$$[\vec{u},\Phi,\phi]=[\vec{A},B,C]\exp[ik(\hat{n}\cdot\vec{r}-vt)] \tag{8}$$

where $\vec{A}$ is vector constant, B and C . are the scalar constants representing the amplitudes, $\vec{r}$ is the position vector, v is the phase velocity and k

is the wave number related with angular frequency ω defined by $\omega = vk$.
Equations (4) to (6) becomes with the help of equation (8) as

$$\left(\omega^2 - k^2c_1^2\right)\vec{A} - \left[c_2^2k^2(\hat{n}\cdot\vec{A}) + i\beta_1 kB - ib_1kC\right]\hat{n} = 0 \tag{9}$$

$$\beta^*\left(K^*\omega - \tau_0 k\omega^2\right)(\hat{n}\cdot\vec{A}) + \left(K^*k^2 + i\omega - \tau_0\omega^2\right)B + \left(im^*\omega - m^*\tau_0\omega^2\right)C = 0 \tag{10}$$

$$ikc_5^2(\hat{n}\cdot\vec{A}) - \left(\omega^2 - c_6^2\right)B + \left(k^2c_3^2 + c_4^2\right)C = 0 \tag{11}$$

We obtain B and C by solving equations (10) and (11) as

$$\begin{aligned} B &= (\hat{n}\cdot\vec{A})J \\ C &= (\hat{n}\cdot\vec{A})H \end{aligned} \tag{12}$$

where

$$\begin{gathered} J = \left[ikc_5^2m^*(i-\tau_0\omega) - \left(k^2c_3^2 + c_4^2\right)k\left(\beta^* - \tau_0\omega\right)\right] \\ \left\{\left(K^*\frac{k^2}{\omega} + i - \tau_0\omega\right)\left(k^2c_3^2 + c_4^2\right) - m^*\left(c_6^2 - \omega^2\right)(i-\tau_0\omega)\right\}^{-1} \end{gathered} \tag{13}$$

and

$$\begin{gathered} H = \left[\left(c_6^2 - \omega^2\right)\left(\beta^*k - k\tau_0\omega\right) - ikc_5^2\left(iK^*k^2 - \tau_0\omega\right)\right] \\ \left\{\left(K^*\frac{k^2}{\omega} + i - \tau_0\omega\right)\left(k^2c_3^2 + c_4^2\right) - m^*\left(c_6^2 - \omega^2\right)(i-\tau_0\omega)\right\}^{-1} \end{gathered} \tag{14}$$

After substituting the values of B and C . from Eq. (12) in equation (9) we obtain

$$\left(\omega^2 - k^2c_1^2\right)\vec{A} - \left[c_2^2k^2 + i\beta_1kJ - ib_1kH\right](\hat{n}\cdot\vec{A})\hat{n} = 0 \tag{15}$$

Taking scalar product of equation (15) with $\vec{A}$ we get,

$$\left(\omega^2 - k^2c_1^2\right)\vec{A} - \left[c_2^2k^2 + i\beta_1kJ - ib_1kH\right](\hat{n}\cdot\vec{A})^2 = 0 \tag{16}$$

where

$$\vec{A}\cdot\vec{A} = |A|^2 = A^2$$

2.1 Transverse waves

For plane transverse waves, we have $\hat{n}\cdot\vec{A} = 0$ and equation (16) becomes $\omega^2 - k^2c_1^2 = 0$.
Therefore,
$v^2 = c_1^2$
The speed of the transverse wave v_T is given by

$$v_T = c_1 = \sqrt{\frac{\mu}{\rho}} \tag{17}$$

The speeds of the above transverse waves are coinciding with the results of classical elasticity[18].

2.2 Longitudinal waves

For plane longitudinal waves, we have $\hat{n}\cdot\vec{A}=A$ and equation (16) yields to

$$k\left(v^2-c_1^2-c_2^2\right)+\left(ib_1H-i\beta_1J\right)=0 \tag{18}$$

Inserting the values of J and H from equation (13) and (14) in equation (18) we obtain the sixth degree equation in v

$$\alpha_0v^6+\alpha_1v^4+\alpha_2v^2+\alpha_3=0 \tag{19}$$

where

$$\begin{gathered}
\alpha_0=\tau_0\left[m^*\left(\omega^2+c_6^2\right)-c_4^2\right]\omega^3+i\left[c_4^2+m^*\left(\omega^2-c_6^2\right)\right]\omega^2\\
\alpha_1=-\tau_0\left[c_3^2+m^*\left(c_1^2+c_2^2\right)\right]\omega^5+\left[c_4^2K^*+\tau_0\left\{c_4^2\left(c_1^2+c_2^2\right)-m^*c_4^2\left(c_1^2+c_2^2\right)-b_1c_5^2+\beta_1m^*c_5^2\right\}\right]\omega^3\\
+i\left[\begin{array}{l} m^*\left(c_1^2+c_2^2\right)c_6^2-\left\{c_3^2\omega^2+c_4^2\left(c_1^2+c_2^2\right)+m^*\left(c_1^2+c_2^2\right)\omega^2\right\}+\\ b_1\left(\beta^*c_6^2-\tau_0c_6^2\omega-\beta^*\omega^2+\tau_0\omega^3\right)+\beta_1\left(c_4^2\tau_0\omega-\beta^*c_4^2+m^*c_5^2\right)\end{array}\right]\omega^2\\
\alpha_2=c_3^2\left(K^*-i\beta_1\tau_0\right)\omega^5+i\left[c_3^2\left\{\beta^*\beta_1-\left(c_1^2+c_2^2\right)\right\}+K^*b_1c_5^2\right]\omega^4\\
\alpha_3=c_3^2\left(c_1^2+c_2^2\right)\left(\tau_0-K^*\right)\omega^5
\end{gathered} \tag{20}$$

For solving equation (19) by Cardan's method, transform V
as

$$V=\alpha_0v^2+\frac{\alpha_1}{3} \tag{21}$$

we obtain

$$V^3+A_0V+A_1=0 \tag{22}$$

where

$$A_0=\alpha_0\alpha_1-\frac{1}{3}\alpha_1^2;A_1=\frac{2}{27}\alpha_1^3+\alpha_0^2\alpha_3-\frac{1}{3}\alpha_1\alpha_2\alpha_3 \tag{23}$$

Now roots of the equation (22) are

$$V_1=\left[\frac{-A_1}{2}+\left(\frac{A_1^2}{4}+\frac{A_0^3}{27}\right)^{\frac{1}{2}}\right]^{\frac{1}{3}}+\left[\frac{-A_1}{2}-\left(\frac{A_1^2}{4}+\frac{A_0^3}{27}\right)^{\frac{1}{2}}\right]^{\frac{1}{3}}$$

$$V_2=\left(\frac{-1+i\sqrt{3}}{2}\right)\left[\frac{-A_1}{2}+\left(\frac{A_1^2}{4}+\frac{A_0^3}{27}\right)^{\frac{1}{2}}\right]^{\frac{1}{3}}-\left(\frac{1+i\sqrt{3}}{2}\right)\left[\frac{-A_1}{2}-\left(\frac{A_1^2}{4}+\frac{A_0^3}{27}\right)^{\frac{1}{2}}\right]^{\frac{1}{3}}$$

and

$$V_3=\left(\frac{-1+i\sqrt{3}}{2}\right)\left[\frac{-A_1}{2}-\left(\frac{A_1^2}{4}+\frac{A_0^3}{27}\right)^{\frac{1}{2}}\right]^{\frac{1}{3}}-\left(\frac{1+i\sqrt{3}}{2}\right)\left[\frac{-A_1}{2}+\left(\frac{A_1^2}{4}+\frac{A_0^3}{27}\right)^{\frac{1}{2}}\right]^{\frac{1}{3}} \tag{24}$$

Hence by using equations (22) and (24), the roots of equation (19) are

$$v_{L_1}^2=\frac{1}{\alpha_0}\left(V_1-\frac{1}{3}\alpha_1\right);v_{L_2}^2=\frac{1}{\alpha_0}\left(V_2-\frac{1}{3}\alpha_1\right)and v_{L_3}^2=\frac{1}{\alpha_0}\left(V_3-\frac{1}{3}\alpha_1\right) \tag{25}$$

Equation (25) represents that three sets of longitudinal waves are propagate with distinct phase speeds and they are influenced by the voids of the body and they are dispersive in nature, not appeared in any classical theory of elasticity.

2.3 Particular cases of plane waves

Case (i): If we neglect the voids i.e., $a=b=c=0$, then the speed of the longitudinal waves $v_{L_1}^{*2}, v_{L_2}^{*2}$ and $v_{L_3}^{*2}$ are given by

$$v_{L_1}^{*2}=\frac{1}{\alpha_0^*}\left(V_1^*-\frac{1}{3}\alpha_1^*\right); v_{L_2}^{*2}=\frac{1}{\alpha_0^*}\left(V_2^*-\frac{1}{3}\alpha_1^*\right) \text{ and } v_{L_3}^{*2}=\frac{1}{\alpha_0^*}\left(V_3^*-\frac{1}{3}\alpha_1^*\right) \tag{26}$$

where

$$\begin{aligned}
\alpha_0^* &= m^*\left[c_6^2\left(\tau_0\omega-i\right)+\omega^2\left(i+\omega\tau_0\right)\right]\omega^2\\
\alpha_1^* &= -\tau_0 m^*\left(c_1^2+c_2^2\right)\omega^5+i\left[m^* c_6^2\left(c_1^2+c_2^2\right)-m^*\left(c_1^2+c_2^2\right)\omega^2+b_1\left(\beta^* c_6^2-c_6^2\tau_0\omega-\beta^*\omega^2+\tau_0\omega^3\right)\right]\omega^2\\
V_1^* &= \frac{\alpha_1^*}{3}\left[\left(-1+\sqrt{\frac{35}{216}}\right)^{\frac{1}{3}}-\left(1+\sqrt{\frac{35}{216}}\right)^{\frac{1}{3}}\right];\\
V_2^* &= \frac{\alpha_1^*}{3}\left[\left(\frac{-1+i\sqrt{3}}{2}\right)\left(-1+\sqrt{\frac{35}{216}}\right)^{\frac{1}{3}}+\left(\frac{1+i\sqrt{3}}{2}\right)\left(1+\sqrt{\frac{35}{216}}\right)^{\frac{1}{3}}\right];\\
V_3^* &= \frac{-\alpha_1^*}{3}\left[\left(\frac{-1+i\sqrt{3}}{2}\right)\left(1+\sqrt{\frac{35}{216}}\right)^{\frac{1}{3}}+\left(\frac{1+i\sqrt{3}}{2}\right)\left(-1+\sqrt{\frac{35}{216}}\right)^{\frac{1}{3}}\right]
\end{aligned} \tag{27}$$

Case (2): If we neglect the voids and thermo voids (i.e., $a=b=c=m=0$) in the body, then $\alpha_0=0$ and hence, $v_{L_1}^2, v_{L_2}^2, v_{L_3}^2 \rightarrow \infty$, i.e., we get high speed longitudinal waves in the body.

3 Results and Discussion

To study the effects of voids on the speed of plane longitudinal waves in a generalized thermo-elastic solid, we consider the magnesium solid as model of our problem and adopt the relevant parameters of magnesium solid from [19] as

Table 1. Parameters of Magnesium solid

Symbol	Value	Unit
λ	$1\cdot 027\times 10^{11}$	N/m^2
μ	1.510×10^{11}	N/m^2
K	0.690×10^{2}	$Wm^{-1}\text{deg}^{-1}$
ρ	8.836×10^{3}	kgm^{-3}
β	7.07×10^{6}	$\text{Nm}^{-2}\text{deg}^{-1}$
c_e	4.27×10^{2}	$J\,\text{kg}^{-1}\text{deg}^{-1}$
T_0	0.0298×10^{4}	K^0
τ_0	$0\cdot 18\times 10^{-12}$	sec

Table 2. The void parameters

Symbol	Value	Unit
a	$3\cdot 688\times 10^{-5}$	N
b	$1\cdot 13849\times 10^{10}$	Nm^{-2}
c	1.473×10^{10}	Nm^{-2}
m	2×10^{6}	$\text{Nm}^{-2}\text{deg}^{-1}$
χ	1.753×10^{-15}	m^2

Consider the non-dimensional angular frequency ω ratio with $0\cdot 1\leq\omega\leq 1$. The variation of angular frequency versus speed of longitudinal wave-1, wave-2 and wave-3 (i.e., $v_{L_1}^2, v_{L_2}^2, v_{L_3}^2$ and $v_{L_1}^{*2}, v_{L_2}^{*2}, v_{L_3}^{*2}$) (i.e.,and with voids and without voids are shown in Figures 1, 2 and 3.

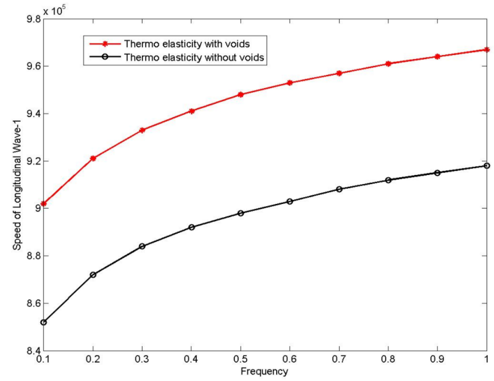

Fig 1. Frequency versus speed of longitudinal wave-1

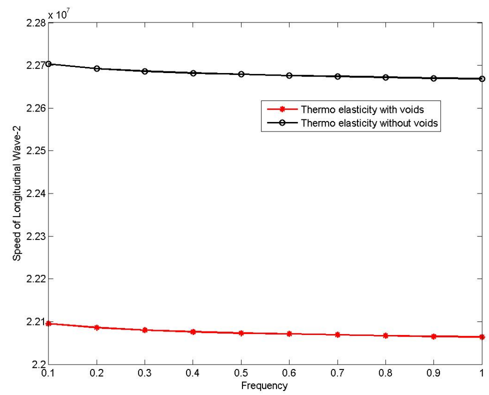

Fig 2. Frequency versus speed of longitudinal wave-2

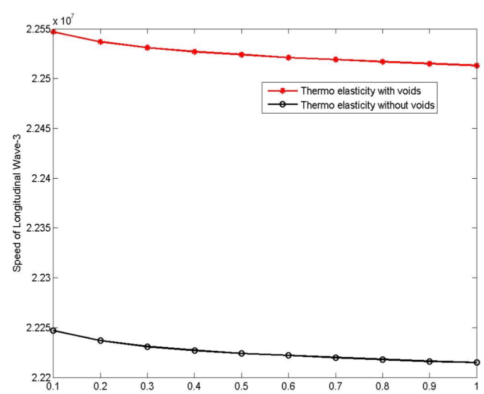

Fig 3. Frequency versus speed of longitudinal wave-3

From Figure 1 and Figure 3 we observed that the phase speeds $v_{L_1}^2$ and $v_{L_3}^2$ of wave-1 and wave-3 in thermo elastic solid with voids are faster than the phase speeds $v_{L_1}^{*2}, v_{L_3}^{*2}$ in thermo-elastic non-void solids. From Figure 2 we say that the phase speed $v_{L_2}^2$ of wave-2 in thermo-elastic solid with voids is slower than the phase speed $v_{L_2}^{*2}$ in thermo-elastic non-void solids.

The phase speed profiles $v_{L_1}^2, v_{L_2}^2$ and $v_{L_3}^2$ of longitudinal waves in void solids for different thermal relaxation times $0\,\mathrm{sec}, 5\times 10^{-4}\,\mathrm{sec}$, and $6\times 10^{-4} sec$ are shown in Figures 4, 5 and 6.

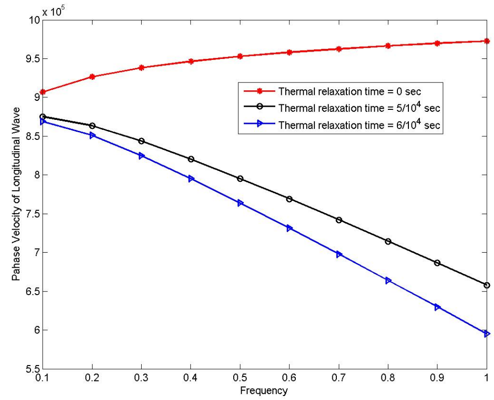

Fig 4. Frequency versus phase velocity of longitudinal wave

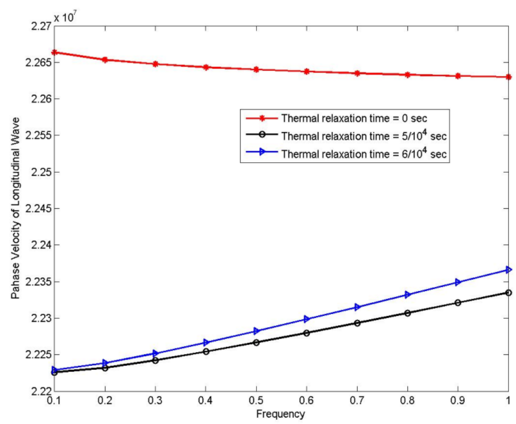

Fig 5. Frequency versus phase velocity of longitudinal wave

From these figures we observed that, the phase speed $v_{L_1}^2$ of wave-1 is inverse proportional to the thermal relaxation time, while $v_{L_3}^2$ of wave-3 is proportional to the thermal relaxation time. But the longitudinal wave-2 propagates with high speed as relaxation time $\tau_0 \rightarrow 0$ and it is proportional to the non-vanishing relaxation time. The effects of density on transverse waves are studied in two different experiments on purely solids and solids mixed with liquids are shown in Figures 7 and 8.

From these figures we observed that transverse waves are slower in high density materials.

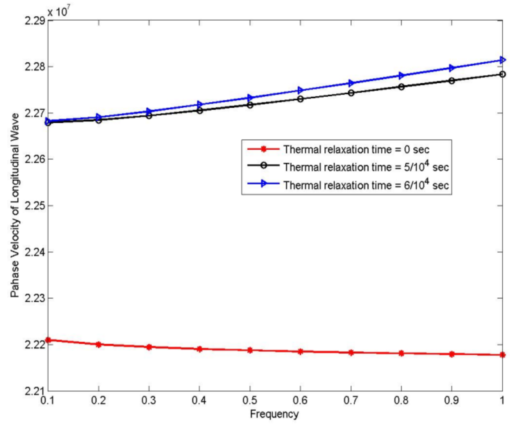

Fig 6. Frequency versus phase velocity of longitudinal wave

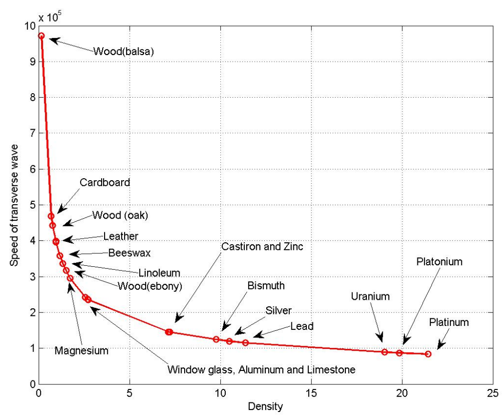

Fig 7. Density versus Speed of transverse wave

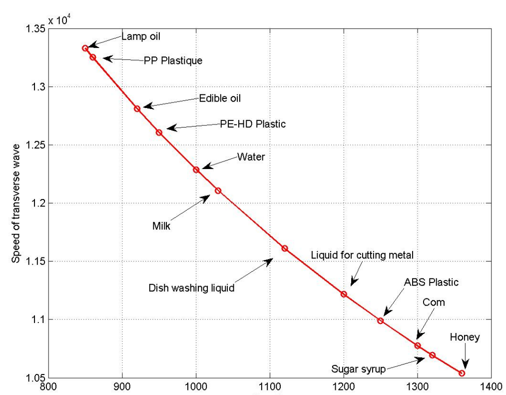

Fig 8. Density versus Speed of Transverse wave

4 Conclusion

The governing equations of generalized thermo elastic void solids are solved for the study of effects of voids on plane waves. From the illustrations we conclude that

i) One set of transverse wave is propagating and it is coinciding with the theory of classical elasticity.

ii) Three sets of coupled longitudinal waves are derived with the effects of voids and non-voids. These waves are not appeared in any theory of classical elasticity.

iii) Longitudinal waves are propagating with high speed in non-thermal void solids.

iv) Longitudinal waves-1 and 3 in thermo-elastic void solids are faster than in non-void solids.

v) Longitudinal wave-2 in thermo elastic void solids is slower than in non-void solids.

vi) Plane transverse waves are faster in low density materials.

vii) Longitudinal wave-1 is inverse proportional but wave-3 is proportional to thermal relaxation time.

viii) Longitudinal waves are propagating with high speed as thermal relaxation time tending to zero

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