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RESEARCH ARTICLE

Setting up Waiting Time Targets for Out Patients using Fuzzy Linear Programming

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Abstract

Objective: To introduce fuzzy linear programming to find the patients' waiting time by satisfying some defined satisfaction targets of patients at out-patient department (OPD) of a healthcare unit. **Method:** The concept of tolerance limit has been used for obtaining fuzzy linear programming formulation. Fuzzy linear programming is transformed into its equivalent crisp linear programming. The software TORA 3.0 is then used to solve the obtained crisp linear programming. **Findings:** Waiting times for patients at their different satisfaction targets have been obtained by solving equivalent crisp linear programming. The solution of the present model suggests that At least 95.30 % patients should be checked-up within half an hour of their appointment and all of them should be checked up within 45 minutes for punctual patients and at least 26.51 % of them should be checked up within half an hour of their arrival and all of them should be checked up within 45 minutes for late patients. Due to assumption of tolerance limit, patients' satisfaction targets regarding waiting time are relaxed up to predefined limits. Consequently the efficiency of healthcare unit increases in the proportion of relaxed limits. **Novelty:** Patients' waiting time with satisfying targets have already been obtained using statistical analyses and classical linear programming. In this article we introduce the fuzzy linear programming to get waiting time for patients by fuzzifying the patients' satisfaction constraints which are indeed novel idea in setting patients' waiting time. Furthermore, some crisp constraints are also included to make the waiting time standards consistent for patients.

Keywords: Fuzzy Linear Programming; Tolerance Limit; Fuzzy Constraints; Crisp Constraints; Out Patients

1 Introduction

A healthcare unit need to improve the quality culture for satisfaction of the out - patient's waiting time at the day of appointment in OPDs; waiting time is often the most annoying factor. As far as the out-patient's services concerned, it is an important and critical area that a healthcare unit pursues to achieve. To keep attention on this area enables the healthcare unit to meet the quality management effectively. So, time standards for waiting time of an out-patient need to be observed differently. This may not only depends upon the services provided for health examination but also depend upon the punctuality of the patients to report at the appointment time.

In the connection of the area there are relevant studies which are devoting to establish patient's satisfaction regarding waiting times. Some of these studies were illustrated using statistical method. The work done in[1] represents that the average total waiting time was 137.02 $\pm$ 53.64 minutes and 83% patients waited for less than 180 minutes and 17% waited for more than 180 minutes before checked up by the doctors. The study was based on statistical analysis. Statistical package for the social sciences for windows version 23.0 was used to manipulate the data. The results in[2] show that the mean time spent in the hospital with standard deviation was 142.58$\pm$23.17 minutes and divided as in different stages was as 113.15$\pm$18.01 minutes – waiting time, 24.43$\pm$10.38 minutes – consultation time, nurse's bay – 23.79$\pm$6.47 minutes and in the queue – 22.94$\pm$8.98 minutes. Most of the respondents (66.6%) were highly satisfied with the service provided by facilities in the healthcare service facility and 45.2 minutes was the mean waiting time. IBM SPSS version 24.0 was used for analyzing the data[3]. In the study[4], patients' waiting time was studied with different diagnosis. Mean total waiting time was 116 minutes and concluded that efforts need to be adopting to reduce the long waiting time which is identified by the WHO as an important index for choosing the quality culture and satisfactory health services.

A comprehensive review of appointment scheduling in healthcare service providing units has been performed and literature was categorized based on several criteria the flow of patients patient preferences and random arrival time and service[5]. Individual unpunctuality among doctors, nurses and patients is one of the areas which have been suggested to focus in this review. A fuzzy programming model for improving outpatient appointment scheduling developed[6]. A mathematical-programming model which can be used to help determining the out-patient waiting time targets in a systematic way was introduced[7]. The results show that for punctual patients: at least 26.8 % of them should be checked-up within 15 minutes of their appointment and all of them should be seen within 30 minutes. For late patients: at least 59 % of them should be checked up within half an hour of their arrival and all of them should be checked up within 45 minutes.

Fuzzy set theory is well known for its ability to model decision making problems involving vagueness, imprecision etc. This ability has been successfully exploited for modeling different problems in various disciplines. Specifically, fuzzy linear programming has been developed and applied in various fields. Very recently, fuzzy linear programming with triangular fuzzy numbers is used to solve business problems[8]. Lexicographic approach was used to study the fuzzy linear programming in[9]. In the work[10], the mathematical model of fuzzy linear programming was studied, and then under the restriction of elastic constraints, the objective functions were optimized. Different methods to solve fuzzy linear programming were introduced in[11]. A fully intuitionistic fuzzy multi-objective linear fractional programming problem applied in e-education system[12]. An integrated fuzzy goal programming theory of constraints model introduced for production planning and optimization[13].

Due to applicability of fuzzy sets to deal uncertainty and complexity in efficient way than classical approaches. The present article is the study of setting up the waiting time keeping attention on different satisfaction targets for patients using fuzzy linear programming over classical linear programming discussed in[7]. The model considers the punctual patients and late patients separately as suggested in[5]. In Section 2, we recall the relevant concept of fuzzy linear programming. Section 4 is devoted to introduce the formulation part, membership functions for linguistic terms and fuzzy probabilities for these linguistic terms used in different constraints. In section 4, results and discussions are given in comprehensive way. In the last but not least conclusion of the whole work is presented.

1.1 Fuzzy linear programming problem (FLPP)

Linear programming problems are special kinds of decision models. The decision space is defined by the constraints and the goal is defined by the objective function. The linear programming problem is stated in matrix form as follows:

Max or Min $Z = C^T Y$

Subject to

$AY \;\; \leq = \geq B$

Where

$$B \in \mathbb{R}^m, C \in \mathbb{R}^n, Y \in \mathbb{R}^n and A = [a_{ij}]_{m\times n} and Y \geq 0 \tag{1}$$

Components of A, B and C are crisp values.

When linear programming problem is formulated in fuzzy environment, it would mean that the decision maker might really not want to actually optimize the objective function; rather he might want to reach to some aspiration levels. Another possibility may be that the constraints might be vague in nature and small violations in the constraint with strict inequalities might well be acceptable. The relevant fuzzy linear programming problem can be expressed as:

Min Z = $C^T Y$
Subject to

$$AY \gtrsim B$$

$$A'Y \geq B'$$

$$A''Y \leq B''$$

$C, Y \in \mathbb{R}^n, B \in \mathbb{R}^m, B' \in \mathbb{R}^{m'}, B'' \in \mathbb{R}^{m''} A = [a_{ij}]_{m\times n}, A' = [a'_{ij}]_{m'\times n}, A'' = [a''_{ij}]_{m''\times n}\, and Y \geq 0. C, Y, B, B' and B''$ are column vectors.

$$A = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ a_{21} & \cdots & a_{2n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix} = \begin{bmatrix} A_1 \\ \vdots \\ A_i \\ \vdots \\ A_m \end{bmatrix}$$

$$A_i = [a_{i1}, a_{i2}, \dots, a_{in}]$$

and

$$B = \begin{bmatrix} b_1 \\ \vdots \\ b_i \\ \vdots \\ b_m \end{bmatrix} \text{ where } 1 \leq i \leq m$$

where $1 \leq i \leq m$

Where $\gtrsim$ (essentially greater than) stands for the fuzzy version of the symbol $\geq$ having interpretation as below. The membership function for the fuzzy set representing i^{th} fuzzy constraint in (2a) is as follows:

$$\mu_i(Y) = \left(\begin{array}{cc} 0 & A_iY \leq b_i - p_i \\ (A_iY - (b_i - p_i))/p_i & b_i - p_i \leq A_iY \leq b_i \\ 1 & b_i \leq A_iY \end{array} \right.$$

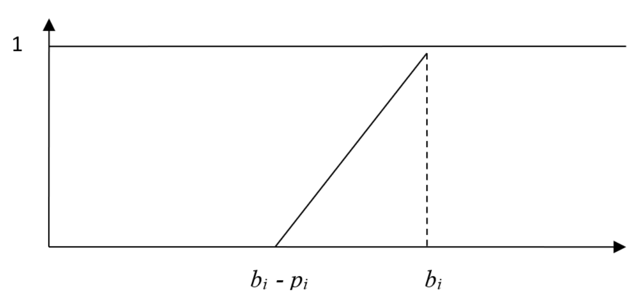

Fig 1. Represents i^{th} fuzzy constraint pictorially

Now the objective functions Z_u and Z_l can be obtained by solving following two crisp linear programming problems:

$$\text{Min}\, Z_u = C^T Y \tag{1}$$

Subject to

$$AY \geq B, A'Y \geq B \text{ and } A''Y \leq B'' \quad Y \geq 0 \tag{3}$$

$$\text{Min}\, Z_l = C^T Y$$

$$Subjectto AY \geq B - p, A'Y \geq B', A''Y \leq B'' \quad Y \geq 0 \tag{4}$$

$$p = (p_1, p_2, \ldots, p_m)^T$$

The membership function of resultant goal is given as:

$$\mu_{\tilde{G}} = \begin{cases} 0 \text{ if } Z_u \leq C^T Y \\ \dfrac{Z_u - C^T Y}{Z_u - Z_l} \text{ if } Z_l \leq C^T Y \leq Z_u \\ 1 \text{ if } C^T Y \leq Z_l \end{cases} \tag{5}$$

Pictorially, the membership function in (5) is represented as:

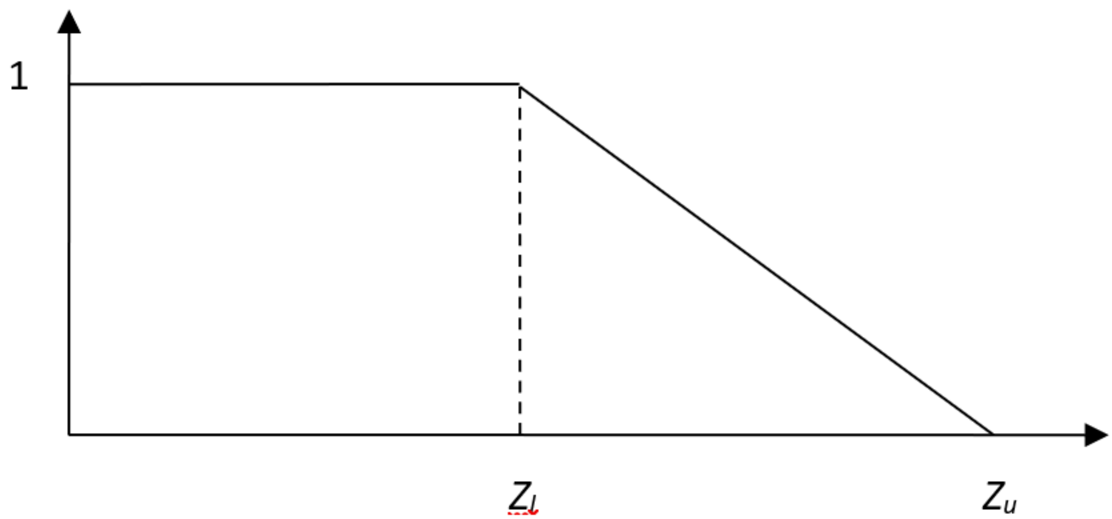

Fig 2. Membership function

Now conversion of FLP to LP is intuitively defined as
Max λ
Subject to
$\mu_{\tilde{G}} \geq \lambda$ and $\mu_i(Y) \geq \lambda \;\; for\ each\ i = 1, 2, \ldots, m.$
Thus, the crisp equivalent of the fuzzy linear program (2) is defined as

$$\text{Max}\, \lambda\, Subjectto \begin{array}{l} C^T Y + \lambda (Z_u - Z_l) \leq Z_u \\ AY - \lambda p \geq B - p \\ A'Y \geq B', \quad A''Y \leq B'' \\ 0 \leq \lambda \leq 1, Y \geq 0 \end{array} \tag{6}$$

2 Methodology

Following three assumptions are defined for patients' satisfaction levels regarding waiting time:

1.The probability that waiting times are "very satisfactory" to patients should be essentially greater than or equal to a specified level.

2. The probability that waiting times are "satisfactory or very satisfactory" to patients should be essentially greater than or equal to another specified level.

3. The probability that waiting times are "at least little satisfactory" essentially greater than or equal to another specified level.

Further there should be a common agreement that patients waiting times will be counted from a patient's appointment time instead of his arrival time, unless they arrive late. Hence, patients who arrive early would be treated as if they came on time. The treatment of late patients is not as simple as it seems. It may be argued that patients should have to wait as long as necessary if they miss their appointments. It is obviously not reasonable to keep a patient waiting time for very long time, should the appointment be missed by few minutes.

We use following notations to formulate the model:

t_j j^{th}waiting time limits where $j = \{1,2,\ldots n\}$;

t_m the maximum time that a patient can possibly wait;

$X_{1j}$the minimum probability that punctual patients do not have to wait longer than t_j $(j=1,2,\ldots,n-1)$;

$X_{1n}$the maximum probability that punctual patients have to wait longer than t_n;

X_{2j} the minimum probability that late patients do not have to wait longer than t_j; $(j=1,2,\ldots,n-1\}$

$X_{2n}$the maximum probability that punctual patients have to wait longer than t_n;

$\tilde{P}$ represents the fuzzy set for the concept that the waiting time is "very satisfactory";

$\tilde{Q}$represents the fuzzy set for the concept that the waiting time is "satisfactory or very satisfactory";

$\tilde{R}$ represents the fuzzy set for the concept that the waiting time is "at least little satisfactory (little satisfactory or satisfactory or very satisfactory";

$S_{\tilde{P}}$ specified minimum probability that waiting times are "very satisfactory" for patients;

$S_{\tilde{Q}}$ specified minimum probability that waiting times are "satisfactory or very satisfactory" for the patients;

$S_{\tilde{R}}$ specified minimum probability that waiting times are "at least little satisfactory" for the patients;

$p_{\tilde{P}_1}$ the lower tolerance of the probability that waiting times are "very satisfactory" for the punctual patients;

$p_{\tilde{P}_2}$ the lower tolerance of the probability that waiting times are "very satisfactory" for the late patients;

$p_{\tilde{Q}_1}$ the lower tolerance of the probability that waiting times are "satisfactory or very satisfactory" for the punctual patients;

$p_{\tilde{Q}_2}$ the lower tolerance of the probability that waiting times are "satisfactory or very satisfactory" for the late patients;

$p_{\tilde{R}_1}$ the lower tolerance of the probability that waiting times are "at least little satisfactory (little satisfactory or satisfactory or very satisfactory)" for the punctual patients;

$p_{\tilde{R}_2}$ the lower tolerance of the probability that waiting times are "at least little satisfactory (little satisfactory or satisfactory or very satisfactory)" for the late patients;

$X_l(\tilde{P})$ the probability that waiting times would be "very satisfactory" to punctual patients if the waiting time targets are achieved;

$X_2(\tilde{P})$the probability that waiting times would be "very satisfactory" to late patients if the waiting time targets are achieved;

$X_1(\tilde{Q})$ the probability that waiting times would be "satisfactory or very satisfactory" to punctual patients if the waiting time targets are achieved;

$X_2(\tilde{Q})$ the probability that waiting times would be "satisfactory or very satisfactory" to late patients if the waiting time targets are achieved;

$X_1(\tilde{R})$ the probability that waiting times would be "at least little satisfactory" to punctual patients if the waiting time targets are achieved;

$X_2(\tilde{R})$ the probability that waiting times would be "at least little satisfactory" to late patients if the waiting time targets are achieved;

Now in order to form a fuzzy mathematical model our aim is to minimize (relax) the patient's waiting time. The probabilities X_{ij} $(i=1,2; j=1,2,\ldots n)$ are the decision variables. The fuzzy model is described as

$$\text{Minimize} \sum_{i=1}^{2} w_{i1}X_{i1} + w_{i1}X_{i1} + w_{i1}X_{i1} - w_{i1}X_{i1}$$

where w_{ij} is a weight that indicates the relative importance of reducing a unit of X_{ij}

Fuzzy constraints:

The fuzzy constraints in the model to compute the probabilities X_{ij} on the basis of the patients' satisfactions are specified as follows:

$$X_I(\widetilde{P}) \tilde{>} S_{\widetilde{P}} \tag{7}$$

$$X_2(\tilde{P}) \tilde{>} S_{\tilde{P}} \tag{8}$$

$$X_l(\tilde{Q}) \tilde{>} S_{\tilde{Q}} \tag{9}$$

$$X_2(\tilde{Q}) > S_{\tilde{Q}} \tag{10}$$

$$X_l(\tilde{R}) \tilde{>} S_{\tilde{R}} \tag{11}$$

$$X_2(\tilde{R}) \tilde{>} S_{\tilde{R}} \tag{12}$$

Crisp constraints:

Except these constraints on patient's satisfactions, there are other constraints also which need to be considered.

$$X_{i(j-1)} - X_{ij} \leq 0(i = 1,2; j = 1,2,\ldots,n-1). \tag{13}$$

To make the waiting time standards consistent in the sense that waiting time targets should be set in such way that punctual patients will not be negatively discriminated in any circumstances, we have

$$X_{1j} - X_{2j} \geq 0(j = 1,2,\ldots,n-1) \tag{14}$$

$$X_{1n} - X_{2n} \leq 0 \tag{15}$$

$$X_{1(n-1)} + X_{in} \leq 1 \quad (i = 1,2) \tag{16}$$

and

$$X_{ij} \geq 0, \quad X_{ij} \leq 1$$

So, we have formulated the fuzzy model consisting the objective function, fuzzy constraints (7)-(12) and crisp constraints (13) – (16). Now the fuzzy sets for the linguistic terms "very satisfactory", "satisfactory or very satisfactory" and "at least little satisfactory" are defined as below.

2.1 Determination of linguistic terms

The membership functions of the linguistic terms "very satisfactory", "satisfactory or very satisfactory" and "little satisfactory or satisfactory or very satisfactory" for punctual and late patients can be approximated by the semi normal functions. These are given as follows:

The membership function of "very satisfactory" for punctual patient is defined by

$$\mu_{\tilde{P}}^{1}(x) = \begin{cases} 1 & \text{if } x \le 2 \\ e^{\left[-0.001204(x-2)^2\right]} & \text{if } x > 2 \end{cases} \tag{17}$$

The membership function of "satisfactory or very satisfactory" for punctual patient is defined by

$$\mu_{\tilde{Q}}^{1}(x) = \begin{cases} 1 & \text{if } x \le 5 \\ e^{\left[-0.000794(x-5)^2\right]} & \text{if } x > 5 \end{cases} \tag{18}$$

The membership function of "at least little satisfactorily" for punctual patient is defined by

$$\mu_{\tilde{R}}^{1}(x) = \begin{cases} 1 & \text{if } x \le 13 \\ e^{\left[-0.000398(x-13)^2\right]} & \text{if } x > 13 \end{cases} \tag{19}$$

The membership function of "very satisfactory" for late patient is defined by

$$\mu_{\tilde{P}}^{2}(x) = \begin{cases} 1 & \text{if } x \le 4 \\ e^{\left[-0.000395(x-4)^2\right]} & \text{if } x > 4 \end{cases} \tag{20}$$

The membership function of "satisfactory or very satisfactory" for late patient is defined by

$$\mu_{\tilde{Q}}^{2}(x) = \begin{cases} 1 & \text{if } x \le 13 \\ e^{\left[-0.000372(x-13)^2\right]} & \text{if } x > 13 \end{cases} \tag{21}$$

The membership function "at least little satisfactorily" for late patient is defined by

$$\mu_{\tilde{R}}^{2}(x) = \begin{cases} 1 & \text{if } x \le 20 \\ e^{\left[-0.000213(x-20)^2\right]} & \text{if } x > 20 \end{cases} \tag{22}$$

2.2 Computation of probabilities $X_i(\tilde{P}), X_i(\tilde{Q}), X_i(\tilde{R})$

We assume that patient waiting times are uniformly distributed within each interval over $[0, t_m]$ as divided by t_j *(j = 1,2,…,n),* i.e. $[0,t_1]$, $[t_1, t_2]$, $[t_2, t_3]$,…, $[t_n, t_m]$. The probabilities,$X_i(\tilde{P}), X_i(\tilde{Q}), X_i(\tilde{R})$ *used* to measure the different satisfaction levels are determined by using the following relations:

$$\begin{aligned} X_i(\tilde{P}) = & \frac{X_{i1}}{t_1}\int_0^{t_1} \mu_{\tilde{P}}^{i}(x)dx + \frac{X_{i2}-X_{i1}}{t_2-t_1}\int_{t_1}^{t_2} \mu_{\tilde{P}}^{i}(x)dx + \ldots \\ & \ldots + \frac{X_{i(n-1)}-X_{i(n-2)}}{t_{n-1}-t_{n-2}}\int_{t_{n-2}}^{t_{n-1}} \mu_{\tilde{P}}^{i}(x)dx + \frac{1-X_{i(n-1)}-X_{in}}{t_n-t_{n-1}}\int_{t_{n-1}}^{t_n} \mu_{\tilde{P}}^{i}(x)dx, \\ & + \frac{X_{in}}{t_m-t_n}\int_{t_n}^{t_m} \mu_{\tilde{P}}^{i}(x)dx \quad (i=1,2) \end{aligned} \tag{23}$$

$$\begin{aligned} X_i(\tilde{Q}) = & \frac{X_{i1}}{t_1}\int_0^{t_1} \mu_{\tilde{Q}}^{i}(x)dx + \frac{X_{i2}-X_{i1}}{t_2-t_1}\int_{t_1}^{t_2} \mu_{\tilde{Q}}^{i}(x)dx + \ldots \\ & \ldots + \frac{X_{i(n-1)}-X_{i(n-2)}}{t_{n-1}-t_{n-2}}\int_{t_{n-2}}^{t_{n-1}} \mu_{\tilde{Q}}^{i}(x)dx + \frac{1-X_{i(n-1)}-X_{in}}{t_n-t_{n-1}}\int_{t_{n-1}}^{t_n} \mu_{\tilde{Q}}^{i}(x)dx \\ & + \frac{X_{in}}{t_m-t_n}\int_{t_n}^{t_m} \mu_{\tilde{Q}}^{i}(x)dx \quad (i=1,2) \end{aligned} \tag{24}$$

$$\begin{aligned}X_i(\tilde{R}) =&\frac{X_{i1}}{t_1}\int_0^{t_1}\mu_{\tilde{R}}^i(x)dx+\frac{X_{i2}-X_{i1}}{t_2-t_1}\int_{t_1}^{t_2}\mu_{\tilde{R}}^i(x)dx+\ldots\\ &\ldots+\frac{X_{i(n-1)}-X_{i(n-2)}}{t_{n-1}-t_{n-2}}\int_{t_{n-2}}^{t_{n-1}}\mu_{\tilde{R}}^i(x)dx+\frac{1-X_{i(n-1)}-X_{in}}{t_n-t_{n-1}}\int_{t_{n-1}}^{t_n}\mu_{\tilde{R}}^i(x)dx\\ &+\frac{X_{in}}{t_m-t_n}\int_{t_n}^{t_m}\mu_{\tilde{R}}^i(x)dx \quad (i=1,2)\end{aligned} \tag{25}$$

The integrals are computed by replacing the original membership function with its Taylor series expansion approximation about the midpoint of the corresponding integral interval. The higher the order of the Taylor series, more precisely the membership function is approximated within interval. The membership function is approximated as follows:

Let t_0 be the middle point of the interval $[t_L, t_H]$ i.e. $t_0 = \frac{1}{2}[t_L, t_H]$

In our model, the membership functions defined in (17) – (22) are of type

$$f(x) = e^{-k(x-a)^2}$$

and the Taylor series expansion of any function about t_0 is

$$\begin{aligned} f(x) &= f(t_0) + (x-t_0) f'(t_0) + \frac{(x-t_0)^2}{2} f'(t_0) + \ldots \\ e^{-k(x-a)^2} &= e^{-k(x-a)^2} - 2k(t_0-a)e^{-k(x-a)^2}(x-t_0) \\ &\quad + k(t_0-a)e^{-k(x-a)^2}\left[2k(t_0-a)^2-1\right](x-t_0)^2+\ldots \end{aligned}$$

Using third order approximation and integrating the function we get

$$\begin{aligned}&\int_{t_L}^{t_H} e^{-k(x-a)^2}dx = e^{-k(x-a)^2}\\ &\times\left\{(t_H-t_L)+\frac{k}{3}\left[2k(t_0-a)^2-1\right]\left[(t_H-t_0)^3-(t_L-t_0)^3\right]\right\}\end{aligned} \tag{26}$$

Using (26), the values of the integrals defined for $X_i(\tilde{P}), X_i(\tilde{Q}), X_i(\tilde{R})$ in (23) – (25). For i = 1, 2i.e.are obtained for punctual and late patients, we have

$$X_1(\tilde{P}) = 0.3415X_{11} + 0.3735X_{12} + 0.1776X_{13} - 0.0511X_{14} + 0.05178 \tag{27}$$

$$X_2(\tilde{P}) = 0.1197X_{21} + 0.2274X_{22} + 0.2440X_{23} - 0.3126X_{24} + 0.3974 \tag{28}$$

$$X_1(\tilde{Q}) = 0.2046X_{11} + 0.3415X_{12} + 0.2635X_{13} - 0.1621X_{14} + 0.1731 \tag{29}$$

$$X_2(\tilde{Q}) = 0.03918X_{21} + 0.1639X_{22} + 0.23649X_{23} - 0.4082X_{24} + 0.5603 \tag{30}$$

$$X_l(\tilde{R}) = 0.0419X_{11} + 0.1736X_{12} + 0.2461X_{13} - 0.4019X_{14} + 0.5384 \tag{31}$$

$$X_2(\tilde{R}) = 0.0047X_{21} + 0.0616X_{22} + 0.1368X_{23} - 0.4201X_{24} + 0.7968 \tag{32}$$

2.3 Numerical example

In this section an example has been used to demonstrate the application of the fuzzy linear programming approach to measure waiting time for out – patients in a healthcare unit. It is the extend version of discussed in [7]. Let us first assume that waiting time limits:

t_1 = 15 minutes; t_2 = 30 minutes; t_3 = 45 minutes; t_4 = 60 minutes; and t_m = 120 minutes.

The weights are chosen as

w_{i1} = 4, w_{i2} = 3 , w_{i3}= 2 , w_{i4} =1 for both punctual and late patients (i =1,2).

Our aim is to get patients' waiting time satisfying satisfaction targets subject to the following:

1)The probability that waiting times are "very satisfactory" to patients should be essentially greater than or equal to 0.5.

2)The probability that waiting times are "satisfactory or very satisfactory" to patients should be essentially greater than or equal to 0.75.

3) The probability that waiting times are "at least little satisfactory" essentially greater than or equal to 0.97.

Now the fuzzy linear model for waiting time targets is given as

Minimize

$4 X_{11} + 3 X_{12} + 2 X_{13} - X_{14} + 4X_{21} + 3 X_{22} + 2 X_{23} - X_{24}$

Subject to

$$
\begin{gathered}
0.3415\, X_{11} + 0.3735\, X_{12} + 0.1776\, X_{13} - 0.0511\, X_{14} \mathrel{\tilde{>}} 0.4482, \\
0.1197\, X_{21} + 0.2274\, X_{22} + 0.2440\, X_{23} - 0.3126\, X_{24} \mathrel{\tilde{>}} 0.1026, \\
0.2046\, X_{11} + 0.3415\, X_{12} + 0.2635\, X_{13} - 0.1621\, X_{14} \mathrel{\tilde{>}} 0.5769, \\
0.03918\, X_{21} + 0.1639\, X_{22} + 0.23649\, X_{23} - 0.4082\, X_{24} \mathrel{\tilde{>}} 0.1897, \\
0.0419 X_{11} + 0.1736\, X_{12} + 0.2461\, X_{13} - 0.4019\, X_{14} \mathrel{\tilde{>}} 0.4316, \\
0.0047 X_{21} + 0.0616\, X_{22} + 0.1368\, X_{23} - 0.4201\, X_{24} \mathrel{\tilde{>}} 0.1732, \\
X_{i(j-1)} - X_{ij} \leq 0 \; (i = 1,2;\; j = 1,2,\ldots,n-1), \\
X_{1j} - X_{2j} \geq 0 \; (j = 1,2,\ldots,n-1), \\
X_{1n} - X_{2n} \leq 0, \\
X_{1(n-1)} + X_{in} \leq 1 \quad (i = 1,2),
\end{gathered}
$$

and $X_{ij} \geq 0, \mathrm{Xij} \leq 1$

Solving the above fuzzy linear program similar to (3) and (4). Using (3) we get the optimal solution, Z_u= 9.9088.

Choose the lower tolerance limits $p_{A1} = p_{A2} = p_{B1} = p_{B2} = p_{C1} = p_{C2} = 0.05$and similar to (4), the optimal solution, Z_l= 6.1428.

Now by using (5) and (6), the crisp equivalent of fuzzy program is

Max λ

Subject to

$$
\begin{gathered}
4X_{11} + 3X_{12} + 2X_{13} - X_{14} + 4X_{21} + 3X_{22} + 2X_{23} - X_{24} + 3.766\lambda \leq 9.9088 \\
0.3415X_{11} + 0.3735X_{12} + 0.1776X_{13} - 0.0511X_{14} - 0.05\lambda \geq 0.3982 \\
0.1197X_{21} + 0.2274X_{22} + 0.2440X_{23} - 0.3126X_{24} - 0.05\lambda \geq 0.0526 \\
0.2046X_{11} + 0.3415X_{12} + 0.2635X_{13} - 0.1621X_{14} - 0.05\lambda \geq 0.5269 \\
0.03918X_{21} + 0.1639X_{22} + 0.23649X_{23} - 0.4082X_{24} - 0.05\lambda \geq 0.1397 \\
0.0419X_{11} + 0.1736X_{12} + 0.2461X_{13} - 0.4019X_{14} - 0.05\lambda \geq 0.3816 \\
0.0047X_{21} + 0.0616X_{22} + 0.1368X_{23} - 0.4201X_{24} - 0.05\lambda \geq 0.1232 \\
X_{11} - X_{12} \leq 0, X_{12} - X_{13} \leq 0, X_{21} - X_{22} \leq 0, X_{22} - X_{23} \leq 0 \\
X_{11} - X_{21} \geq 0, X_{12} - X_{22} \geq 0, X_{13} - X_{23} \geq 0, X_{14} - X_{24} \leq 0 \\
X_{13} + X_{14} \leq 1, X_{23} + X_{24} \leq 1 \\
\text{and } X_{ij} \geq 0, X_{ij} \leq 1
\end{gathered}
$$

The linear programming problem was solved with software TORA 3.0, the optimal solution for waiting time targets is

$\lambda = 0.5987$
$X_{11} = 0, X_{12} = 0.9530, X_{13} = 1, X_{14} = 0$
$X_{11} = 0, X_{12} = 0.2651, X_{13} = 1, X_{14} = 0$

3 Results and Discussion

The results of the model based on linear programming[7]: at least 26.8 % of them should be checked-up within 15 minutes of their appointment and all of them should be seen within 30 minutes. For late patients: at least 59 % of them should be checked up within half an hour of their arrival and all of them should be checked up within 45 minutes.

The results of present model based on fuzzy linear programming:

For punctual patients: at least 95.30 % of them should be checked-up within half an hour of their appointment and all of them should be checked up within 45 minutes.

For late patients: at least 26.51 % of them should be checked up within half an hour of their arrival and all of them should be checked up within 45 minutes.

Three type of satisfactory levels 'very satisfactory', satisfactory or very satisfactory and at least little satisfactory, with some tolerance values have been defined. For these vague terms exponential type membership functions are chosen. To calculate the probabilities for these defined fuzzy sets, $X_i(\tilde{P}), X_i(\tilde{Q}), X_i(\tilde{R})$expressions are defined in (23)-(25). For computation of these expressions,expansion up to some specific degree term of exponential membership functions has been used.vary the results in [1, 2, 3, 47]and the present model suggests the fast service than the other models i.e. patients' waiting time is lesser than the others' work[1–4]. The variance of the result with[7] is due to take into account the fuzzy linear programming. The results in our approach do not only satisfy patients' satisfaction but also relaxation to the health care unit. It is possible by the assumption of tolerance limit. Thus the present approach suggests more credible policy because of optimizing the problem with patients' satisfaction and efficient performance of healthcare unit.

4 Conclusion

In this study, fuzzy linear programming model to set up waiting time for out-patients for a healthcare unit has been proposed. Constraints for the different level of patient's satisfaction of waiting time are considered in fuzzy sense using tolerance approach in fuzzy constraints. The assumption regarding satisfaction constraints is the advancement of the earlier study of[7]. To make the waiting time standards more consistent, some crisp constraints are also considered. All these constraints are taken for punctual patients and late patients simultaneously. A numerical example of[7] has been taken for manifestation of the present method. Results of the numerical example using present method exist in section-4 suggest the healthcare unit that almost all punctual patients and late patients should be checked up within 45 minutes. The model based on mathematical programming of[7] was based on strict constraints. It suggests that all the patients must be checked with in 45 minutes and reflects strict boundary lines for healthcare units. In our approach there is slight delay for check up of patients yet all the patients are checked up within 45 minutes and it eases off the healthcare unit to improve the services with fulfilling the satisfaction of patients. For future work, fuzzy goal programming can be introduced in the model by imposing some other goals of healthcare unit.

References

1) Usman SO, Olowoyeye E, Adegbamigbe OJ, Olubayo GP, Ibijola AA, Tijani AB, et al. Patient Waiting Time: Gaps And Determinants Of Patients Waiting Time In Hospitals In Our Communities To Receive Quality Services. *European Journal of Medical and Health Sciences*. 2020;2(1):1–4. Available from: https://ej-med.org/index.php/ejmed/article/view/136/59.
2) Anyanwu OU, Ezeonu T, Orji LL, Ezeanosike O, Ikegwuonu C, Omeje KK, et al. Waiting time analysis in a paediatric outpatient clinic in South East Nigeria. *Medical Science and Discovery*. 2021;8(2):50–59. Available from: https://doi.org/10.36472/msd.v8i2.459.
3) Yenuganti VV, D SR, J SKP, R N. Patient satisfaction and waiting times in the primary health centres of South Chennai. *International Journal Of Community Medicine And Public Health*. 2021;8(3):1386. Available from: https://doi.org/10.18203/2394-6040.ijcmph20210832.
4) Langstieh AJ, Sarkar C, Wahlang JB, Adjourno CC. A study on patient waiting time in neurology OPD of a tertiary healthcare centre and its association with patients' demographic profile. 2022. Available from: https://doi.org/10.53771/ijstra.2022.3.2.0133.
5) Ali A, Chen F. Appointment Scheduling Problem in Complexity Systems of the Healthcare Services: A Comprehensive Review. *Journal of Healthcare Engineering*. 2022;p. 1–16. Available from: https://doi.org/10.1155/2022/5819813.
6) Farahbakhsh B, Moosavirad SH, Asadi Y, Amirbeigi A. Developing a fuzzy programming model for improving outpatient appointment scheduling. *Iranian Journal of Fuzzy Systems*. 2021;18(4):169–184. Available from: https://doi.org/10.22111/IJFS.2021.6183.
7) Huang XMM, Thompson A. Setting up waiting-time targets to satisfy patients: An application of mathematical programming. *Mathematical Medicine and Biology*. 1995;12(3-4):235–247. Available from: https://doi.org/10.1093/imammb/12.3-4.235.
8) Michael G. Applications of fuzzy linear programming to business problems. *WSEAS Transactions on Mathematics*. 2020;19:343–348. Available from: https://www.wseas.org/multimedia/journals/mathematics/2020/a685106-050.pdf.

9) Canedo BP, Verdegay JL, Eduardo R, R A. Lexicographic methods for fuzzy linear programming. *Mathematics*. 2020;8:1540. Available from: https://doi.org/10.3390/math8091540.
10) Wang Y. Application of Fuzzy Linear Programming Model in Agricultural Economic Management. *Journal of Mathematics*. 2022;2022:1–13. Available from: https://doi.org/10.1155/2022/6089072.
11) Das S, Chakraborty D. Graphical method to solve fuzzy linear programming. *Sādhanā*. 2022;47(4):1–11. Available from: https://doi.org/10.1007/s12046-022-02025-8.
12) Malik M, Gupta SK. An Application of Fully Intuitionistic Fuzzy Multi-objective Linear Fractional Programming Problem in E-education System. *International Journal of Fuzzy Systems*. 2022;24(8):3544–3563. Available from: https://doi.org/10.1007/s40815-022-01348-2.
13) Zare H, Saraji MK, Tavana M, Streimikiene D, Cavallaro F. An Integrated Fuzzy Goal Programming—Theory of Constraints Model for Production Planning and Optimization. *Sustainability*. 2021;13(22):12728. Available from: https://doi.org/10.3390/su132212728.