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RESEARCH ARTICLE

The Best Proximity Point Problem for New Types of Ciric $\alpha-\beta-$ Contraction with Application

Satyaj Tiwari[1], **Monika Dewangan**[2]*

1 Associate Professor, Department of Mathematics, Shri Rawatpura Sarkar University, Raipur, 492015, Chhattisgarh, India
2 Research Scholar, Department of Mathematics, Shri Rawatpura Sarkar University, Raipur, 492015, Chhattisgarh, India

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* **Corresponding author**.

monidewangan1995@gmail.com

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Abstract

Objectives: To generate the best proximity point theorem for new types of Ciric $\alpha-\beta-$ contraction. **Methods:** By using generalized $\alpha-\beta-$ proximal quasi contraction and Ciric type G-contraction. **Findings:** We made new types of Ciric $\alpha-\beta-$ contraction. **Novelty/Improvement:** we give new types of Ciric $\alpha-\beta-$contraction which is a generalization of generalized $\alpha-\beta-$ proximal quasi contraction and Ciric type G-contraction.

Keywords: New types of Ciric α-β-contraction; Best proximity point; Ciric type G-contraction; generalized α-β- proximal quasi contraction; Cauchy Sequence

1 Introduction

The Banach-contraction principal on complete metric spaces was introduced in 1922. Every self mapping T on complete metric space (X,d) satisfying $d(Tx,Ty) \leq kd(x,y)$, for all $x,y \in X$, where $k \in (0,1)$, has a unique fixed point in X. This result has been extended and generalized by many authors in different ways [1–3]. Some other extensions of Banach contraction principle have been presented by considering the concept of best proximity point. A point x in A for which $d(x,Tx) = d(A,B)$ is called a best proximity point of T, whenever a non-self mapping T has no fixed point. A best proximity point represent optimal approximate solution to the equation $Tx = x$.

The first result on cyclic contraction and best proximity point was reported by Kirk et. al. [4]. Their results are the generalization of the usual contraction and fixed point. Eldered and Veeramani [5] proved the existence of a best proximity point for proximal pointwise contraction maps. Anuradha and Veeramani [6] proved the existence of a best proximity point for proximal pointwise contraction. Recently many authors have studied and generalize various concept related to the best proximity points [7–14].

The main objective of this paper is to generalized the result of Mohammad Ladh. Ayari [15] introduced new types of Ciric $\alpha-\beta-$contractive mapping on metric spaces involving β comparison function and Ciric type G-contraction.

In this paper to prove the existence of best proximity problem for new types of Ciric $\alpha-\beta-$ contraction on metric spaces endowed with binary relations.

Considering a pair (A,B) of non-empty subset of (X,d), we will use the following notations in this paper:
$(A,B) = inf\{d(x,y) : x \in A,\ y \in B\}$;

$$A_0 = \{x \in A : d(x,y) = d(A,B)\ \ for\ some\ y \in B\};$$

$$B_0 = \{y \in B : d(x,y) = d(A,B)\ \ for\ some\ x \in A\}.$$

Definition 1.1. [16] Let (A,B) be a pair of non-empty subsets of a metric space (X,d) such that A_0 is non-empty. Then the pair (A,B) is said to have the P-property iff $d(x_1,y_1) = d(x_2,y_2) = (A,B) \Rightarrow d(x_1,x_2) = d(y_1,y_2)$ for all $x_1,x_2 \in A_0$ and $y_1,y_2 \in B_0$.

Definition 1.2. [17] Let $\beta \in (0,\infty)$. A β -comparison function is a map $\phi : (0,+\infty) \to (0,+\infty)$ fulfilling the following properties:

(1) ϕ is non-decreasing;

(2) $\lim_{n\to\infty} \phi_\beta^n(t) = 0$ for all $t > 0$, where ϕ_β^n denotes the n^{th} iterate of ϕ_β and $\phi_\beta(t) = \phi(\beta.t)$;

(3) there exist $s \in (0,\infty)$ such that $\sum_{n=1}^{\infty} \phi_\beta^n(s) < \infty$.

The set of all β comparison functions ϕ satisfying (1)-(3) will be denoted by ϕ_β.

Definition 1.3. [18] Let $T : A \to B$ and $\alpha : A \times A \to [0,+\infty)$. We say that T is $\alpha-$ proximal admissible if $\alpha(x_1,x_2) \geq 1$ and $d(u_1,Tx_1) = d(u_2,Tx_2) = d(A,B)$, then $\alpha(u_1,u_2) \geq 1$, for all $x_1,x_2,u_1,u_2 \in A$.

Definition 1.4. [18] A non-self mapping $T : A \to B$ is said to be (α,d) regular, where $\alpha : A \times A \to [0,+\infty)$, if for all (x,y) such that $0 \leq \alpha(x,y) < 1$, there exists $u_0 \in A_0$ such that $\alpha(x,u_0) \geq 1$ and $\alpha(y,u_0) \geq 1$.

In an arbitrary graph G, a link is an edge of G with distinct ends and a loop is an edge of G with identical ends. Two or more links of G with the same pairs of ends are called parallel edges of G.

Let (X,d) be a metric space and G be a directed graph with vertex set $V(G) = X$ such that the edge set $E(G)$ contains all loops, that is $(x,x) \in E(G)$ for all $x \in X$. Assume further that G has no parallel edges. Under these hypothesis, the graph G can be easily denoted by the ordered pair $(V(G),E(G))$ and it is said that the metric space (X,d) is endowed with the graph G.

Definition 1.5. [19] A non self mapping $T : A \to B$ is G-proximal if T satisfies

$$\left\{\begin{array}{c}(y_1,y_2) \in E(G)\\ d(x_1,Ty_1) = d(A,B)\\ d(x_2,Ty_2) = d(A,B)\end{array}\right\} \Rightarrow (x_1,x_2) \in E(G),$$

for all $x_1,x_2,y_1,y_2 \in A$.

Definition 1.6. [20] A non-self mapping $T : A \to B$ is a Ciric type G-contraction, if there exists $\alpha \in [0,1)$ such that $d(Tx,Ty) \leq \alpha.Q_T(x,y)$, for all $x,y \in A$ with $(x,y) \in E(G)$, where

$Q_T(x,y) = max\left\{d(x,y), d(x,Tx) - d(A,B), d(y,Ty) - d(A,B), \frac{d(x,Ty)+d(y,Tx)}{2} - d(A,B)\right\}$.

Definition 1.7. [15] A non-self mapping $T : A \to B$ is said to be a generalized $\alpha - \beta-$ proximal quasi-contractive, where $\alpha : A \times A \to [0,+\infty)$ iff there exist $\varphi \in \varphi_\beta$ and positive numbers $\alpha_0,...,\alpha_4$ such that

$\alpha(x,y)d(Tx,Ty) \leq \varphi(M_T(x,y)),\ \forall x,y \in A$,

Where

$M_T(x,y) = max\{\alpha_0 d(x,y), \alpha_1[d(x,Tx) - d(A,B)], \alpha_2[d(y,Ty) - d(A,B)], \alpha_3[d(y,Tx) - d(A,B)],$
$\alpha_4[d(x,Ty) - d(A,B)]\}$.

To prove our main results, we need the following lemma:

Lemma 1.8. [15] Let $T : A \to B$ be a non-self-mapping and $\alpha : A \times A \to [0,+\infty)$, satisfying the following conditions:

(1) $T(A_0) \subset B_0$;

(2) T is $\alpha-$ proximal admissible;

(3) There exist elements $x_0,x_1 \in A_0$ such that $d(x_1,Tx_0) = d(A,B)$ and $\alpha(x_0,x_1) \geq 1$.

Then there exists a sequence $\{x_n\} \subset A_0$ such that $d(x_{n+1},Tx_n) = d(A,B)$ and $\alpha(x_n,x_{n+1}) \geq 1$, such a sequence $\{x_n\}$ is a Cauchy sequence.

2 Results and Discussion

Definition 2.1. We introduced a new type of Ciric $\alpha - \beta-$contraction. A non-self mapping $T : A \to B$ is a new type of Ciric $\alpha - \beta-$contraction where $\alpha : A \times A \to [0,+\infty)$ if there exist $\varphi \in \varphi_\beta$ and positive numbers $\alpha_0,\alpha_1,\alpha_2,\alpha_3$ such that,

$\alpha(x,y).d(Tx,Ty) \leq \varphi(M_T(x,y)),\ \forall x,y \in A,$
where
$M_T(x,y) = max\{\alpha_0 d(x,y), \alpha_1 d(x,Tx) - d(A,B), \alpha_2 d(y,Ty) - d(A,B),$
$\alpha_3[\frac{d(x,Ty)+d(y,Tx)}{2} - d(A,B)]\}.$

Example : Let $X = R^2$ with metric
$d((x_1,y_1),(x_2,y_2)) = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$
and put $A = \{(x,1) : x \in [0,1]\}$ and $B = \{(y,0) : y \in [0,1]\}$, and $T : A \to B$ is defined by
$T(x,1) = \left(\begin{array}{ll} (0,0), & if\ 0 \leq x < 1 \\ (\frac{1}{3},0), & if\ x = 1. \end{array} \right.$
Then it is easy to see that $d(A,B) = 1$ and $A_0 = A$ and $B_0 = B$. So that T is a new types of Ciric $\alpha - \beta -$ contraction with $\varphi(t) = t$, $\alpha \in [0,1]$ and $\alpha_i = \frac{1}{3^{i+1}}$ for $i = 0,1,2,3$.

Theorem 2.2. Let A and B be non-empty closed subsets of a complete metric space (X,d) such that A_0 is non-empty. Let $\alpha : A \times A \to [0,+\infty)$ and $\varphi \in \varphi_\beta$. Assume that non-self mapping $T : A \to B$ satisfying the following conditions:
(1) $T(A_0) \subset B_0$ and the pair (A,B) satisfies the P-property;
(2) T is α proximal admissible;
(3) There exist elements $x_0, x_1 \in A_0$ such that $d(x_1,Tx_0) = d(A,B)$ and $\alpha(x_0,x_1) \geq 1$;
(4) T is continuous on new types of Ciric $\alpha - \beta$ contraction;
(5) if $\{x_n\}$ be a sequence in A such that $\alpha(x_n,x_{n+1}) \geq 1$ and $\{x_{n(k)}\}$ of $\{x_n\}$ such that $\alpha(x_{n(k)},x^*) \geq 1$ for all k;
(6) there exist $\alpha - \beta$ contraction;
(7) φ is continuous, $\beta > max\{\alpha_2, \alpha_3\}$.
Then T has a best proximity point $x^* \in A$ such that $d(x^*,Tx^*) = d(A,B)$.

Proof. From given to condition (3), there exist $x_0,x_1 \in A$ such that, $d(x_1,Tx_0) = d(A,B)$ and $\alpha(x_0,x_1) \geq 1$. Since $T(A_0) \subset B_0$, there exist $x_2 \in A_0$ such that $d(x_2,Tx_1) = d(A,B)$. Since T is α proximal admissible and using $\alpha(x_0,x_1) \geq 1, d(x_1,Tx_0) = d(x_2,Tx_1) = d(A,B)$. This implies that $\alpha(x_1,x_2) \geq 1$.

In a similar condition, by induction sequence $\{x_n\} \subset A_0$ such that

$$d(x_{n+1},Tx_n) = d(A,B), \tag{1}$$

and $\alpha(x_n,x_{n+1}) \geq 1$ for all $n \in N \cup \{0\}$.

Using the P-property we deduce that

$$d(x_n,x_{n+1}) = d(Tx_{n-1},Tx_n), \forall n \in N \tag{2}$$

Since T is new types of Ciric $\alpha - \beta$ contractive, there exist a function $\varphi \in \varphi_\beta$ such that

$$\alpha(x_{n-1},x_n) \cdot d(Tx_{n-1},Tx_n) \leq \varphi(M_T(x_{n-1},x_n)), \quad \forall n \in N \tag{3}$$

Where

$$M_T(x_{n-1},x_n) = max\{\alpha_0 d(x_{n-1},x_n), \alpha_1[d(x_{n-1},Tx_{n-1}) - d(A,B)], \alpha_2[d(x_n,Tx_n) - d(A,B)]$$

$$\alpha_3[\frac{d(x_{n-1},Tx_n) + d(x_n,Tx_{n-1})}{2} - d(A,B)]\}$$

$$= max\{\alpha_0 d(x_{n-1},x_n), \alpha_1[d(x_{n-1},Tx_{n-1}) - d(A,B)], \alpha_2[d(x_n,Tx_n) - d(A,B)],$$

$$\alpha_3[\frac{d(x_{n-1},Tx_n) - d(A,B)}{2} + \alpha_3 \frac{d(x_n,Tx_{n-1}) - d(A,B)}{2}]\}$$

$$= max\{\alpha_0 d(x_{n-1},x_n), \alpha_1[d(x_{n-1},Tx_{n-1}) - d(A,B)], \alpha_2[d(x_n,Tx_n) - d(A,B)],$$

$$\alpha_3[\frac{d(x_{n-1},Tx_n) - d(A,B)}{2}]\}$$

$$\leq max\{\alpha_0 d(x_{n-1},x_n), \alpha_1 d(x_{n-1},x_n), \alpha_2 d(x_n,x_{n+1}), \alpha_3[\frac{d(x_{n-1},x_n)d(x_n,x_{n+1})}{2}]\}$$

$$M_T(x_{n-1},x_n) \le \beta max\{d(x_{n-1},x_n),d(x_n,x_{n+1})\}, \tag{4}$$

where φ is non-decreasing, then we get that

$$d(x_{n+1},x_n) \le \varphi(\beta max\{d(x_{n-1},x_n),d(x_n,x_{n+1})\})$$

$= \varphi_\beta(max\{d(x_{n-1},x_n),d(x_n,x_{n+1})\}),$

Suppose that, for some n, we have

$$d(x_{n-1},x_n) \le d(x_n,x_{n+1}),$$

then we get

$$d(x_n,x_{n+1}) \le \varphi_\beta d(x_n,x_{n+1})$$

$< d(x_n,x_{n+1}),$

which is contradiction.

Therefore, for all $n \ge 0$,

$$d(x_n,x_{n+1}) \le \varphi_\beta d(x_{n-1},x_n), \ \forall n \in N. \tag{5}$$

Now, by induction, we find

$$d(x_n,x_{n+1}) \le \varphi_\beta^n(d(x_0,x_1)), \ \forall n \in N \cup (0\}. \tag{6}$$

Now we will show that $\{x_n\}$ is a Cauchy sequence.

Consider $m,n \in N$ with $m > n$, using the triangle inequality and the above inequality (6), we get

$d(x_n,x_m) \le \sum_{k=n}^{m-1} d(x_k,x_{k+1})$

$\le \sum_{k=n}^{m-1} \varphi_\beta^k d(x_0,x_1) \to 0 \ as \ n,m \to +\infty.$

Hence $\{x_n\}$ is a Cauchy sequence in $A_0 \subset A$ and Since (X,d) is complete, there exist $x^* \in A$ (depending on x_0 and x_1) such that $x_n \to x^* \in A$.

We next to show that x^* is best proximity point for T.

Using hypothesis (5) of the theorem, $\exists$ a subsequence $\{x_{n(k)}\}$ to $\{x_n\}$ such that $\alpha(x_{n(k)},x^*) \ge 1 \ \forall k$.

Since T is new types of Ciric $\alpha - \beta$ contractive, then we have

$d(Tx_{n(k)},Tx^*) \le \alpha(x_{n(k)},x^*)d(Tx_{n(k)},Tx^*)$

$$\le \varphi(M_T(x_{n(k)},x^*)), \ \forall k \tag{7}$$

where

$$M_T(x_{n(k)},x^*) = max\{\alpha_0 d(x_{n(k)},x^*), \alpha_1 d(x_{n(k)},Tx_{n(k)}) - d(A,B), \alpha_2[d(x^*,Tx^*) - d(A,B)],$$

$$\alpha_3[\frac{d(x_{n(k)},Tx^*) + d(x^*,Tx_{n(k)})}{2}] - d(A,B)\}. \tag{8}$$

Using triangular inequality, We have

$$\begin{aligned} d\left(x^*,Tx^*\right) &\le d\left(x^*,x_{n(k)+1}\right) + d\left(x_{n(k)+1},Tx_{n(k)}\right) + d\left(Tx_{n(k)},Tx^*\right) \\ &= d\left(x^*,x_{n(k)+1}\right) + d(A,B) + d\left(Tx_{n(k)},Tx^*\right) \end{aligned} \tag{9}$$

$d(x^*,Tx^*) - d(x^*,x_{n(k)+1}) - d(A,B) \le d(Tx_{n(k)},Tx^*), \ \forall k.$

Using (7) and (9), we get

$$d\left(x^*, Tx^*\right) - d\left(x^*, x_{n(k)+1}\right) - d(A,B) \leq \varphi\left(M_T\left(x_{n(k)}, x^*\right)\right) \tag{10}$$

$$\leq \varphi(max\{\alpha_0 d(x_{n(k)}, x^*), \alpha_1[d(x_{n(k)}, Tx_{n(k)}) - d(A,B)],$$

$$\alpha_2[d(x^*, Tx^*) - d(A,B)], \alpha_3[\frac{d(x_{n(k)}, Tx^*) + d(x^*, Tx_{n(k)})}{2}] - d(A,B)\}).$$

Assume $\rho = d(x^*, Tx^*) - d(A,B) > 0$. Let us we consider two separate cases as follows. If φ is continuous, as $k \to \infty$, we get

$\rho \leq \varphi(max\{\alpha_0 d(x_{n(k)}, x^*), \alpha_1[d(x_{n(k)}, x_{n(k)+1}) + d(x_{n(k)+1}, Tx_{n(k)})] - d(A,B),$

$\alpha_2[d(x^*, Tx^*) - d(A,B)], \alpha_3[\frac{d(x_{n(k)}, Tx^*) - d(A,B)}{2} + \frac{d(x^*, Tx_{n(k)}) - d(A,B)}{2}]\})$

$\rho \leq \varphi(max\{\alpha_2, \alpha_3\}\rho) \leq \varphi(\beta\rho) < \rho,$

which is contradiction.

If $\beta > max\{\alpha_2, \alpha_3\}$ and we claim also that $\rho = 0$. Suppose by contradiction that $\rho > 0$, letting $k \to \infty$ in (8), we get

$M_T(x_{n(k)}, x^*) \to max(\alpha_2, \alpha_3\}\rho.$

If there exist$\varepsilon > 0$ and $N > 0$, such that for all $n > N$, we have

$$M_T(x_{n(k)}, x^*) < (max\{\alpha_2, \alpha_3\} + \varepsilon)\rho \tag{11}$$

and

$\beta > (max\{\alpha_2, \alpha_3\} + \varepsilon),$

therefore in equation (10) and (11)

$d(x^*, Tx^*) - d(x^*, x_{n(k)+1}) - d(A,B) \leq \varphi(M_T(x_{n(k)}, x^*))$

$< \varphi((max\{\alpha_2, \alpha_3\} + \varepsilon)\rho)$

$< \varphi_\beta(\frac{max\{\alpha_2, \alpha_3\} + \varepsilon}{\beta}\rho)$

$< \frac{max\{\alpha_2, \alpha_3\} + \varepsilon}{\beta}\rho$

$< \rho.$

Consequently, by letting $k \to \infty$, we get

$\rho < \frac{max\{\alpha_2, \alpha_3\} + \varepsilon}{\beta}\rho < \rho,$

which is contradiction as well.

Hence our claim holds.

Thus, we proved that x^* is a best proximity point of T, that is $d(x^*, Tx^*) = d(A,B)$.

3 Application

Best proximity point for metric spaces endowed with a binary relation.

Before apply our results, we need some preliminaries. Let (X,d) be a metric space and R be a binary relation over X.

The symmetric relation attached to R.

x, y in X, $xRy \Leftrightarrow xRy$ or yRx, Where $S = R \cup R^{-1}$.

Definition 3.1. [6] Let X be a non-empty set. A non-self mapping $T: A \to B$ is called β -quasi-contractive if there exist $\beta > 0$ and $\varphi \in \varphi_\beta$ such that

$x, y \in A : xRy \Rightarrow d(Tx, Ty) \leq \varphi(M_T(x,y))$

where $M_T(x,y) = max\left\{\alpha_0 d(x,y), \alpha_1 d(x,Tx) - d(A,B), \alpha_2 d(y,Ty) - d(A,B), \alpha_3[\frac{d(x,Ty)+d(y,Tx)}{2} - d(A,B)]\right\},$

with $\alpha_k \geq 0$ for $k = 0,1,2,3$.

Definition 3.2. [14] A non-self mapping $T: A \to B$ is a proximal comparative mapping if xRy and $d(u_1, Tx) = d(u_2, Ty) = d(A,B),\ \forall x, y, u_1, u_2 \in A$ then $u_1 R u_2$.

Theorem 3.3. Let A and B be non-empty closed subsets of a complete metric space (X,d) such that A_0 is non-empty. Let R be a binary relation over X. Assume that non-self mapping $T: A \to B$ satisfying the following conditions:

(1) $T(A_0) \subset B_0$ and the pair (A,B) satisfies the P-property;

(2) T is a proximal comparative mapping;

(3) There exist elements $x_0, x_1 \in A_0$ such that $d(x_1, Tx_0) = d(A,B)$ and $\alpha(x_0, x_1) \geq 1$;

(4) T is continuous on new types of Ciric $\alpha - \beta$ contraction;

(5) if $\{x_n\}$ be a sequence in A such that $\alpha(x_n, x_{n+1}) \geq 1$ and $\left\{x_{n(k)}\right\}$ of $\{x_n\}$ such that $\alpha(x_n, x_{n+1}) \geq 1$ for all $k(A,d,R)$ is regular;

(6) there exist $\alpha - \beta$ contractive;

(7) φ is continuous, $\beta > max\{\alpha_2, \alpha_3\}$,

then T has a best proximity point $x^* \in A$ such that $d(x^*, Tx^*) = d(A,B)$.

Proof. define the $\alpha : A \times A \to [0,\infty)$ by:

$$\alpha(x,y) = \begin{cases} 1 \; if \; xRy \\ 0 \; otherwise \end{cases}$$

then T is α -admissible.

Assume that $\alpha(x,y) \geq 1$, and $d(u_1, Tx) = d(u_2, Tx) = d(A,B)$ for some $x, y, u_1, u_2 \in A$. By the definition of α, we get xSy, $d(u_1, Tx) = d(u_2, Ty) = d(A,B)$.

Assertion (2) of the theorem implies that $u_1 R u_2$, which gives us $\alpha(u_1, u_2) \geq 1$.

Assertion (3) if T is α -admissible and $d(x_1, Tx_0) = d(A,B)$.

Assertion (4) T is β -quasi-contractive means that T is new types of Ciric $\alpha - \beta -$contractive.

Also the condition (5) hold.

Next in condition (6) A is R-directed implies that the non-self mapping $T : A \to B$ is (α, d) is regular.

Now all the condition of theorem 2.2 are satisfied, which implies the existence of best proximity point for the non-self mapping T.

4 Conclusion

This study adds an improvement to the best proximity point theorems[15,20], for Ciric type G-contraction and generalized $\alpha - \beta -$ proximal quasi contractive mappings. This improvement is obtained by introducing new types of Ciric $\alpha - \beta$ contraction involving Ciric type G-contraction by the generalization of the generalized $\alpha - \beta -$proximal-quasi-contractive mappings on metric space. We have established application of best proximity point result for the case of non-self mappings on metric spaces endowed with binary operation.

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