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RESEARCH ARTICLE

Prime Fuzzy Ideals of a Gamma Semiring -1

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Abstract

Objectives: The main objective of this article is to introduce some fundamental results of fuzzy ideals and Prime fuzzy ideals of a $\Gamma-$ semiring R . **Methods:** We use some fundamental results of fuzzy ideals and conditions like semi subtractive, centreless etc. for developments of the main results. **Findings:** In this connection, we establish some important results of fuzzy ideals and prime fuzzy ideals of semirings to $\Gamma-$ semirings. **Novelty:** We find a special class of fuzzy ideals of a $\Gamma-$ semiring R and with this, construct a lattice ordered $\Gamma-$ semiring. Further, we find that a fuzzy ideal of $\Gamma-$ semiring R can be a decreasing map from [0, 1] into the set of R together with an empty set. As an application of this result, the prime fuzzy ideals of a $\Gamma-$ semiring R are described in terms of prime ideals of a $\Gamma-$ semiring R.

Keywords: Fuzzy Ideals; Prime Ideals; Prime Fuzzy Ideals; Lattice Ordered Γ-Semiring; Strong Ideal

1 Introduction

Semiring is one of the universal algebras which is a generalization not only of ring but also of distributive lattice. The notion of semiring was introduced by an author in [(1)] and then many researchers developed the theory of semirings. The notion of Γ in algebra was introduced by [(2)] as a generalization of the ring and the concept of $\Gamma-$ semiring was introduced in [(3)] as a generalization of $\Gamma-$ ring. The theory of fuzzy sets is the most appropriate theory for dealing with uncertainty and was first introduced by an author in [(4)] in 1965. After that many mathematicians have applied the concept of fuzzy subsets to the theory of groups, rings and semirings in algebra and many results on fuzzy theory appeared, showing the importance of the concept and its applications to logic, set theory, group theory, ring theory, real analysis, topology etc. The main aim of this study is to generalize some fundamental results of fuzzy ideals of semirings in Refs [(5,6)] and investigate that a fuzzy ideal of a $\Gamma-$ semiring R can be a decreasing map from [0, 1] into the set of R together with the empty set. As an application of this result, the prime fuzzy ideals of R are described in terms of the prime ideals of a $\Gamma-$ semiring R.

2 Preliminaries and examples

The purpose of this section is to provide a concise but reasonably complete exposition of the background material for the subsequent sections of this paper.

Definition 2.1.[7] Let S and Γ be non-empty sets. Then S is called a $\Gamma-$ semigroup if there exists a mapping $S\times\Gamma\times S\to S$ denoted by $(x,\alpha,y)\to x\alpha y$ satisfying the condition $x\alpha(y\beta z)=(x\alpha y)\beta z$ for all $x,y,z\in S$ and for all $\alpha,\beta\in\Gamma$.

Definition 2.2.[7] Let $(R,+)$ and $(\Gamma,+)$ be two commutative semigroups. Then R is called a $\Gamma-$ semiring if there exists a mapping $R\times\Gamma\times R\to R$ denoted by $x\alpha y$ for all $x,y\in R$ and $\alpha\in\Gamma$ satisfying the following condition:

(i) $x\alpha(y+z)=x\alpha y+x\alpha z$.

(ii) $(y+z)\alpha x=y\alpha x+z\alpha x$.

(iii) $x(\alpha+\beta)z=x\alpha z+x\beta z$

(iv) $x\alpha(y\beta z)=(x\alpha y)\beta z$ for all $x,y,z\in R$ and $\alpha,\beta\in\Gamma$.

Example 2.3. Let A and B be semirings and let R = Hom(A, B) and Γ = Hom(B, A) denote the sets of homomorphisms from A to B and B to A respectively. Then R is a $\Gamma-$ semiring with operations of pointwise addition and composition of mappings.

Example 2.4. Let M be a $\Gamma-$ ring and let R be the set of ideals of M. Define addition in the natural way and if A, B $\in$ R , $\gamma\in\Gamma$, let AγB denote the ideal generated by {xγy | x, y $\in$ M }. Then R is a $\Gamma-$semiring.

Definition 2.5.[7] A $\Gamma-$ semiring R is said to have zero element if $0\alpha x=0=x\alpha 0$ and $x+0=x=0+x$ for all $x\in R$ and $\alpha\in\Gamma$.

Definition 2.6.[7] A $\Gamma-$ semiring R is said to have identity element if for all $x\in R$ there exist $\alpha\in\Gamma$ such that $1\alpha x=x=x\alpha 1$.

Definition 2.7.[8] A $\Gamma-$ semiring R is said to have a strong identity if for all $x\in R$, $1\alpha x=x=x\alpha 1$ for all $\alpha\in\Gamma$.

Definition 2.8.[8] A $\Gamma-$ semiring R is said to be semi-subtractive if for every x and y in R there exists $z\in R$ such that either $z+x=y$ or $z+y=x$.

Definition 2.9.[8] A $\Gamma-$ semiring R with zero element is centreless if non zero elements do not sum to zero. Formally for all $x,y\in R$, $x+y=0$ implies that $x=y=0$.

Definition 2.10. Let X be a non-empty set. A mapping $\mu:X\to[0,1]$ is called a fuzzy subset of X.

Definition 2.11. Let R be a $\Gamma-$ semiring. A fuzzy subset μ of R is a function $\mu:R\to[0,1]$. For a fixed $0\le t\le 1$, the set $\mu_t=\{x\in R\mid \mu(x)\ge t\}$ is called the level subset of μ.

Definition 2.12. Let μ be a fuzzy subset of a $\Gamma-$semiring R. Then μ is called a fuzzy left (right) ideal of R if $\mu(x+y)\ge min\{\mu(x),\mu(y)\}$ and $\mu(x\alpha y)\ge\mu(y)(\mu(x\alpha y)\ge\mu(x))$, for all $x,y\in R$ and $\alpha\in\Gamma$.

A fuzzy ideal of a $\Gamma-$ semiring R is a nonempty fuzzy subset of R which is both fuzzy left and fuzzy right ideal of R.

Definition 2.14. Let R be a $\Gamma-$ semiring. A non constant fuzzy ideal μ of R is called prime fuzzy ideal if for any two fuzzy ideals λ_1 and λ_2 of R, $\lambda_1\Gamma\lambda_2\subseteq\mu$ implies that either $\lambda_1\subseteq\mu$ or $\lambda_2\subseteq\mu$.

Remark 2.15. Throughout this paper, R will denote a $\Gamma-$ semiring with zero element 'O' and identity element '1' unless otherwise stated.

3 Special class of fuzzy ideals of a $\Gamma-$semiring

In this section, we investigate some fundamental results of fuzzy ideals of semirings to $\Gamma-$semirings and as an application of these results, a lattice ordered $\Gamma-$semiring of a special class of its fuzzy ideals is constructed.

We start this section with definitions required to construct a $\Gamma-$semiring of fuzzy ideals of R.

Definition 3.1. A fuzzy ideal μ of a $\Gamma-$semiring R is said to be $k-$fuzzy ideal if $\mu(x)\ge min\{\mu(x+y),\mu(y)\}$ for all $x,y\in R$.

OR

A fuzzy ideal μ of a $\Gamma-$semiring R is said to be $k-$fuzzy ideal of R if $\mu(x+y)\ge\lambda,\mu(y)\ge\lambda$ implies that $\mu(x)\ge\lambda$, for all $x,y\in R,\lambda\in[0,1]$.

Example 3.2. Let μ and λ be two fuzzy subsets of a $\Gamma-$semiring N , set of all non-negative integers defined by

$$\mu(x)=\begin{cases}1/3 & \text{if } x \text{ is odd}\\ 1/2 & \text{if } x \text{ is even}\\ 0 & \text{if } x=0\end{cases}$$

and

$$\lambda(x)=\begin{cases}1 \text{ if } x>7\\ 1/2 \text{ if } 5<x<7\\ 0 \text{ if } 0\le x<5\end{cases}$$

Then μ is a $k-$fuzzy ideal of N. Also λ is a fuzzy ideal of N but not a $k-$ fuzzy ideal of N.

Definition 3.3. Let μ be a fuzzy ideal of R. Then we say that μ is strong, if for $x,y \in R$, $min\{\mu(x),\mu(y)\} \geq \mu(x+y)$.

Note: Since for a fuzzy ideal μ of R we have $\mu(x+y) \geq min\{\mu(x),\mu(y)\}$. Therefore μ is strong if and only if $\mu(x+y) = min(\mu(x),\mu(y)\},\ for\ x,y \in R$.

Theorem 3.4. Let λ and μ be fuzzy ideals of a $\Gamma-$ semiring R then $\lambda \cap \mu$ is a fuzzy ideal of R.

Theorem 3.5. Let λ and μ be $k-$fuzzy ideals of a $\Gamma-$ semiring R then $\lambda \cap \mu$ is a $k-$fuzzy ideal of R.

Proof. Let λ and μ be $k-$ fuzzy ideals of R. then by theorem 3.4, $\lambda \cap \mu$ is a fuzzy ideal of R. Let $x,y \in R$. Then

$(\lambda \cap \mu)(x) = min(\lambda(x),\mu(x)\}$

$$\geq min(min(\lambda(x+y),\lambda(y)), min(\mu(x+y),\mu(y))\}$$

$$= min(min(\lambda(x+y),\mu(x+y)),\ min(\lambda(y),\mu(y))\}$$

$= min\{(\lambda \cap \mu)(x+y),(\lambda \cap \mu)(y)\}$.

Therefore, $\lambda \cap \mu$ is a $k-$ fuzzy ideal of R

Theorem 3.6. Let λ be a fuzzy ideal of a $\Gamma-$ semiring R. Then λ is a strong fuzzy ideal of R if and only if each $\lambda_t, t \in$ range λ, is a strong ideal of R.

The following theorems are proved in [7].

Theorem 3.7. Let λ and μ be two fuzzy subset of R with strong identity and $\alpha \in \Gamma$. Then

(i) $\lambda(1) \leq \lambda(x) \leq \lambda(0)$, for all $0 \neq x \in R$, if λ is fuzzy ideal of R.

(ii) $\lambda \subseteq \mu$ if and only if $\lambda_t \subseteq \mu_t$ for all $t \in [0,1]$.

(iii) $(\lambda \cap \mu)_t = \lambda_t \cap \mu_t$ for all $t \in [0,1]$.

Theorem 3.8. Let λ and μ be two fuzzy subset of R and $\alpha \in \Gamma$. Then

(i) Any fuzzy subset λ of R is a fuzzy ideal if and only if each $\lambda_t, t \in$ range λ, is an ideal of R.

(ii) If $(\lambda_i \mid i \in \Lambda\}$ is a fuzzy ideal of R. Then arbitrary intersection is a fuzzy ideal of R.

(iii) $\lambda\Gamma\mu \subseteq \lambda \cap \mu$, if λ is a right and μ is a left fuzzy ideal of R. Hence $(\lambda\Gamma\mu)_t \subseteq \lambda_t \cap \mu_t$ for all $t \in (0,1]$.

Theorem 3.9. Let λ,μ and τ be fuzzy ideals of R. Then

(i) $\lambda + \mu \supseteq \lambda$, if $\mu(0) = 1$.

(ii) $\lambda + \lambda = \lambda$.

(iii) $\lambda + \mu$ is a fuzzy ideal of R.

(iv) If $\mu(0) = \tau(0) = 1$, then $(\lambda \cap \mu) + (\lambda \cap \tau) \subseteq \lambda \cap (\mu + \tau)$. Equality holds if λ,μ and τ are strong fuzzy ideals of R.

(v) $\lambda + \chi_0$ (the characteristic function of zero) $= \lambda$ if $\lambda(0) = 1$.

Proof. The proof of (i), (ii) and (v) are simple and straightforward.

(iii) Since $(\lambda+\mu)+(\lambda+\mu) = (\lambda+\mu)$ for all $x,y \in R$. So we have $(\lambda+\mu)(x+y) = ((\lambda+\mu)+(\lambda+\mu))(x+y) = sup_{x+y=a+b}\{min((\lambda+\mu)(a),(\lambda+\mu)(b))\} \geq min((\lambda+\mu)(x),(\lambda+\mu)(y))$. Further, If $x = a+b,\ y = c+d$ and $\alpha \in \Gamma$, then $x\alpha y = (a+b)\alpha(c+d) = a\alpha(c+d)+b\alpha(c+d) = (a+b)\alpha c+(a+b)\alpha d$.

Thus, $(\lambda+\mu)(x\alpha y) = sup_{x\alpha y=u+v}\{min(\lambda(u),\mu(v)\} \geq min\{\lambda(a\alpha(c+d)),\mu(b\alpha(c+d))\} \geq min(\lambda(a),\mu(b))$. Therefore, $(\lambda+\mu)(x\alpha y) \geq sup_{x=a+b}\{min(\lambda(a),\mu(b))\} = (\lambda+\mu)(x)$. Similarly, by using $x\alpha y = (a+b)\alpha c+(a+b)\alpha d$, we get $(\lambda+\mu)(x\alpha y) \geq (\lambda+\mu)(y)$. Hence, $(\lambda+\mu)$ is a fuzzy ideal of R.

(iv) Using (i), we have $\mu \subseteq \mu+\tau$ and $\tau \subseteq \mu+\tau$. This implies that $(\lambda \cap \mu) \subseteq \lambda \cap (\mu+\tau)$ and $(\lambda \cap \tau) \subseteq \lambda \cap (\mu+\tau)$. Thus, for $z = x+y$, we have $(\lambda \cap \mu)(x) \leq \lambda \cap (\mu+\tau)(x)$ and $(\lambda \cap \tau)(y) \leq \lambda \cap (\mu+\tau)(y)$. Hence, $min_{z=x+y}((\lambda \cap \mu)(x),(\lambda \cap \tau)(y) \leq min_{z=x+y}(\lambda \cap (\mu+\tau)(x),\lambda \cap (\mu+\tau)(y)) \leq \lambda \cap (\mu+\tau)(z)$. Thus, using (iii), $\lambda \cap (\mu+\tau)$ is a fuzzy ideal of R. This implies that $((\lambda \cap \mu)+(\lambda+\tau))(z) = sup_{z=x+y}\{min((\lambda \cap \mu)(x),(\lambda \cap \tau)(y)\} \leq \lambda \cap (\mu+\tau)(z)$, proving that $((\lambda \cap \mu)+(\lambda \cap \tau)) \subseteq \lambda \cap (\mu+\tau)$. Now suppose that $\lambda,\ \mu$ and τ are strong and $z = x+y \in R$. Then

$$((\lambda \cap \mu)+(\lambda \cap \tau))(z) = \sup_{z=x+y}\{\min((\lambda \cap \mu)(x),(\lambda \cap \tau)(y)\} = \sup_{z=x+y}\{\min(\min(\lambda(x),\mu(x)),\min(\lambda(y),\tau(y)))\} =$$

$$\sup_{z=x+y}\{\min(\min(\lambda(x),\lambda(y)),\min((\mu(x),\tau(y))\} \geq$$

$$\sup_{z=x+y}\{\min(\lambda(x+y)),\min((\mu(x),\tau(y))\} =$$

$$\min\left(\lambda(z),\ \sup_{z=x+y}\{\min((\mu(x),\tau(y))\}\right) = \lambda \cap (\mu+\tau)(z).$$

Let R be a $\Gamma-$ semiring. Then FLI(R), FRI(R) and FI(R) denote respectively the set of all fuzzy left ideals, set of all fuzzy right ideals and set of all fuzzy ideals of R.

Now the following results are proved in [9].

Proposition 3.10. Let λ and μ be two fuzzy left ideals (fuzzy right ideals, fuzzy ideal) of a $\Gamma-$ semiring R. Then $\lambda + \mu$ is the unique minimal number of the family of all fuzzy left ideals (fuzzy right ideals, fuzzy ideals) of R containing λ and μ. $\lambda \cap \mu$ is the unique maximal member of family of all fuzzy left ideals (fuzzy right ideals, fuzzy ideals)of R contained in λ and μ.

Proposition 3.11. (FLI(R), +, $\cap$) [(FRI(R), +, $\cap$),(FI(R), +, $\cap$)] is a complete lattice.

Proposition 3.12. If R is a $\Gamma-$ semiring then the lattice (FLI(R), +, $\cap$) [(FRI(R), +, $\cap$),(FI(R), +, $\cap$)] is modular if each of its member is a fuzzy left $k-$ ideal(fuzzy right k - ideal, fuzzy k - ideal).

Let R_f be the set of all fuzzy ideals of a $\Gamma-$ semiring R such that $\lambda(0) = 1$. Then in view of Theorem 3.9 ((iii) and (v)) and triviality of associativity and commutativity, we have

Theorem 3.13. The set $\left(R_f, +\right)$ is a commutative monoid with identity χ_0, the characteristic function of 0.

Let I be an ideal of R and $\chi_1^t, t \in [0,1)$, denote the fuzzy ideal of a $\Gamma-$emiring R defined by

$$\chi_1^t = \begin{cases} 1 & \text{if } x \in I \\ t & \text{if } x \notin I \end{cases}$$

Theorem 3.14. Let R be a Γ-semiring.

(i) If A and B are two strong ideals of R, then

(a) $A \cap B$ is strong ideal of R, and

(b) $A \cup B$ is strong ideal of R, if R is semi-subtractive.

(ii) For any ideal I of R, I is strong if and only if each $\chi_1^t, t \in [0,1)$ is strong.

(iii) R is centreless if and only if χ_0 is strong

Proof. (i) (a) and **(b)** are obvious.

(ii) This follows from definition 3.3, since for $t_1 \in [0,1)$ and $t_2 \in [0,1)$ we have

$$\left(\chi_A^{t_1}\right)_{t_2} == \begin{cases} A & \text{if } t_2 > t_1 \\ R & \text{if } t_2 \leq t_1 \end{cases}$$

(iii) It is easy to see that R is centreless if and only if the ideal (0) is strong. Now the desired result follows from (ii).

Let $R_\chi = (\chi_A^t, t \in [0,1), A$ is a strong ideal of $(R\}$.

Theorem 3.15. If R is semi-subtractive and centreless $\Gamma-$ semiring, then $\left(R_\chi, +\right)$ is a commutative monoid with identity χ_0.

Proof. The result follows directly by theorem 3.14((i)(b),(iii)) and theorem 3.13.

Now, fix $t \in [0,1)$ and let $R\chi^t = (\chi_A^t \mid A$ is a strong ideal of $(R\}$.

Finally, in this section we have

Theorem 3.16. Let R be semi subtractive and centreless $\Gamma-$ semiring. Then

(i)$(R\chi^t, +, \cap)$ is a commutative $\Gamma-$ semiring with χ_0^t the additive identity and χ_R the identity with respect to $\cap$.

(ii)$R\chi^t$ is centreless.

(iii)$(R\chi^t, +, \cap)$ is a lattice ordered $\Gamma-$ semiring .

Proof. The (i) and (ii) part follows from Theorem 3.15 and theorem 3.9$((ii),(iv)$ and (v)). Further, using $\chi_A^t + \chi_B^t = \chi_{A\cup B}^t$ it follows that $R\chi^t$ is lattice ordered $\Gamma-$ semiring with the corresponding algebraic system $(R\chi^t, \vee, \wedge)$ where $\chi_A^t \vee \chi_B^t = \chi_A^t \cup \chi_B^t$ and $\chi_A^t \wedge \chi_B^t = \chi_A^t \cap \chi_B^t$.

4 Prime Fuzzy ideals of a $\Gamma-$ Semiring

In this section, we observe that a fuzzy ideal of a $\Gamma-$ semiring R can be viewed as a decreasing map from $[0,1]$ into the set of ideals of R together with an empty set. As an application of this result, the prime fuzzy ideals of $\Gamma-$ semiring R are described in terms of prime ideals of $\Gamma-$ semiring.

Definition 4.1. Let R be a $\Gamma-$semiring. A fuzzy ideal μ of R is called prime fuzzy ideal if either $\mu = \chi_R$ or μ is a non constant function and for any two fuzzy ideals λ_1 and λ_2 of R, $\lambda_1\Gamma\lambda_2 \subseteq \mu$ implies that either $\lambda_1 \subseteq \mu$ or $\lambda_2 \subseteq \mu$.

We now state the following lemma and theorem, proof of which are analogous to the corresponding theorems in semirings [7], so we omit the proofs.

Lemma 4.2. If T, A and B are bounded subset of real numbers and sup $< min(supA, \sup B)$, then there exist $i_0 \in A$ and $j_0 \in B$ such that $j = min(i_0, j_0) \notin T$

Theorem 4.3. Let $I(R)$ denote the set of two sided ideals of a $\Gamma-$ semiring R together with an empty set ϕ. Let $\psi:[0,1]\rightarrow I(R)$ be a decreasing function. Then the function ψ give rise to a fuzzy ideal λ of R defined by $\lambda(x)=sup\{i\in[0,1]\mid x\in\psi(i)\}$. Further all fuzzy ideals of R arise in this manner.

As an application of the above result, we have

Theorem 4.4. Let P be a non-constant fuzzy ideal of a $\Gamma-$semiring R. Then P is prime if and only if there exist a prime ideal π of $R\,(\pi\neq R)$ such that $P_r\in(\pi,R\}$ for all $r\in(0,1]$.

Proof. Let P be a non-constant prime fuzzy ideal of a $\Gamma-$ semiring R. We first show that $P(0)=1$, so that each P_r, for all $r\in[0,1]$ is an ideal of R. On the contrary, suppose that $0<P(0)<1,(P(0)\neq 0$ as P is non-constant. Thus by theorem 3.7(i), we have $u=P(1)<v=P(0)<1$, Since the function $r\rightarrow P_r$ is a decreasing function, we have $\varphi\neq P_v\subsetneq P_u=R$. Let λ and μ be two fuzzy ideal of R arising from decreasing function $r\rightarrow\lambda_r, r\rightarrow\mu_r$ from $[0,1]$ into $I(R)$, where

$$\lambda_r=\begin{pmatrix} R & \text{if } r=0 \\ P_v & \text{if } r\in(0,1] \end{pmatrix} \text{ and } \mu_r=\begin{pmatrix} P_r & \text{if } r>v \\ R & \text{if } r\leq v \end{pmatrix}.$$

Using theorem 3.7(ii), we find that $\mu\nsubseteq P$ and $\lambda\nsubseteq P$ since $\lambda_1=P_v\nsubseteq P_1=\phi$ and $\mu_v=R\nsubseteq P_v$. Finally, we show that $(\lambda\Gamma\mu)_r\subseteq P$.For if $r>v$, then by the theorem 3.7(iii) and 3.8(iii), $(\lambda\Gamma\mu)_r\subseteq(\lambda\cap\mu)_r=\lambda_r\cap\mu_r=P_v\cap P_r\subseteq P_r$. Similarly, if $0<r\leq v$, then $(\lambda\Gamma\mu)_r\subseteq(\lambda\cap\mu)_r=\lambda_r\cap\mu_r=P_v\cap R=P_v\subseteq P_r$, because $r\rightarrow P_r$ is a decreasing function. Moreover, since $P_0=R,(\lambda\Gamma\mu)_0\subseteq P_0$. This contradicts the fact that P is prime. Hence $P(0)=1$. Therefore $P_r\neq\phi$ for all $r\in[0,1]$ and hence P_r is an ideal of R.

Now we show that P_r takes on at most two values and if it takes on two values, then one one must be R. If this is not the case , then there exist $u,v\in(0,1]$ such that $u<v$ and $\phi\neq P_v\subsetneq P_u\subsetneq R$. This contradicts the primness of P as shown above. It remains to prove that π is a prime ideal of R. Let σ and τ be two ideals of R with $(\sigma\Gamma\tau)_r\subseteq\pi$. Then the characteristics function χ_σ and χ_τ satisfy $\chi_\sigma\Gamma\chi_\tau\subseteq\chi_{\sigma\Gamma\tau}\subseteq\chi_\pi\subseteq P$, because $(\chi_\pi)_r=\pi$ if $r>0$ and $(\chi_\pi)_r=R$ if $r=0$. Now the primeness of P yields that either $\chi_\sigma\subseteq P$ and $\chi_\tau\subseteq P$ that is either $\sigma\subseteq\pi$ or $\tau\subseteq\pi$. Thus, π is the prime ideal of R. Conversely, assume that $P_r\in\{\pi,R\}$, for all $r\in[0,1]$, where π is prime. Suppose by the way of contradiction that P is not prime. Then there exist fuzzy ideal λ and μ of a $\Gamma-$ semiring R with $\lambda\Gamma\mu\subseteq P$, but neither is contained in P. The later implies that $\lambda_s\nsubseteq P_s$ and $\mu_t\nsubseteq P_t$ for some s, t $\in[0,1]$. Since every ideal is contained in R, $P_s=P_t=\pi$. Let $m=min(s,t)$. Then $P_m=\pi$ and $\lambda_m\nsubseteq\pi$ and $\mu_m\nsubseteq\pi$, where λ_m and μ_m are ideals of R. For any $a,b\in R,\lambda\Gamma\mu\subseteq P$ implies that $(a\alpha b)\geq min(\lambda(a),\mu(b))$. Thus, for any $z=\sum_{i=1}^{n}x_i\alpha_i y_i\in\lambda_m\Gamma\mu_m, x_i\in\lambda_m, y_i\in\mu_m$ and $\alpha_i\in\Gamma$, we have $P(z)\geq m$, since $\lambda(x_i)\geq m$ and $\mu(y_i)\geq m$. Thus, $z\in P_m=\pi$. This shows that $\lambda_m\Gamma\mu_m\subseteq\pi$, contradicting that π is prime ideal of R. Hence, P is prime.

Corollary 4.5. Let P be a prime ideal of a $\Gamma-$semiring R. Then range $P=(1,a\},\quad a\in[0,1)$

5 Conclusion

In this paper, we establish a special class of fuzzy ideals of a $\Gamma-$semiring and by using the concept of semi subtractive and centreless, an application in terms of a lattice ordered $\Gamma-$semiring is constructed. In the continuation of this paper, we characterize the prime fuzzy ideals of R in terms of prime ideals of a $\Gamma-$ semiring R, which is again an application such that a fuzzy ideal of $\Gamma-$ semiring R can be a decreasing map from [0, 1] into the set of R together with an empty set. Therefore, this article is very useful as it invites the researchers to explore more for other type of fuzzy ideals in $\Gamma-$semirings.

6 Declaration

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