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RESEARCH ARTICLE

Understanding the Unique Properties of Fuzzy Concept in Binary Trees

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Abstract

Objective: This study investigated the properties of fuzzy binaries, a type of rooted trees, and explored their implications in different domains. **Method:** This investigation included analyzing the structural properties of fuzzy binary tree, such as the number of vertices, the height, and the degree to which edges were assigned membership. Theorem: Combinatorial analysis and mathematical induction were used to derive the relation between the number of vertex and height of a fuzzy binary tree based on its internal vertex. The conditions that allow complete graphs to form within fuzzy binary tree were also determined by mathematical analysis and graph theory. **Findings:** The study found that the internal vertices of a fuzzy binary tree have a strong influence on the number of vertices and height. A theorem was developed that states a fuzzy binary tree full with ”i” terminal vertices will have (i+1), total vertices, and (i+1), terminal vertices. A fuzzy binary tree can form a complete graph under certain conditions. **Novelty:** This research provides novel insights into fuzzy binary trees' structural properties. The derived theorem establishes a fundamental relation between the number and height of a fuzzy binary tree. This provides valuable insight into the complexity and organization of these trees. The identification of the conditions that lead to a complete graph in fuzzy binary tree further improves our understanding and appreciation of their comprehensiveness. These findings provide new perspectives on the application of fuzzy binary tree in different fields.

Keywords: Fuzzy Binary Trees; Rooted Tree; Degrees of Membership; Structural Characteristics; Complete Graphs; Image Processing; Full Binary Tree; Terminal Vertices; Internal Vertices; Membership Function; Fuzzy Set Theory; Graph Theory; Data Mining

1 Introduction

In recent years, the field of fuzzy binary trees has made remarkable progress due to increasing demand for systems that can handle uncertainty and imprecision. Researchers have sought to improve data processing and representation by incorporating fuzzy concepts in binary trees. It has enabled more sophisticated decision-making for various applications. Despite the significant advances in this field, several research gaps still present opportunities for exploration and innovation.

Research has been focused on the efficient development of insertion, removal, and update operations in fuzzy binary trees. Adding fuzziness complicates the task, as traditional binary trees already have established rules. Studies have suggested novel algorithms and techniques to integrate fuzzy concepts seamlessly into these operations. The goal is to maintain the integrity and efficiency of the tree structure. There is still a requirement for robust and optimized techniques. Fuzzy binary trees have also been used in decision support systems. In many decision-making processes, there is often imprecise and uncertain data. It requires more appropriate structures than binary trees. Fuzzy binary trees can handle input vagueness, and produce more accurate and reliable results, helping users make informed decisions. Further research is required to optimize performance and explore advanced techniques.

Integration of machine learning with fuzzy binary trees has also been investigated. Fuzzy binary trees combine, for example, the interpretability of decision trees and the comprehensibility offered by fuzzy logic with the ability to deal with uncertainty. These hybrid models show promise for classification and regression tasks where the inherent fuzziness of the data has been captured effectively to improve prediction performance. There are still many opportunities to test new algorithms assess their performance and explore their potential across various domains. Fuzzy binary trees are used in pattern recognition and image processing. Fuzzy binary trees are practical tools to represent and analyze images with uncertain or imprecise features. These trees allow for efficient image retrieval and classification. They also account for the inherent fuzziness of image data. Further research is needed to optimize these techniques and to explore their application to real-world situations. Although there have been significant advances in fuzzy binary tree research, there still needs to be more gaps. By leveraging the most recent research literature, this article will comprehensively understand fuzzy binary trees and their design principles, algorithmic applications, and algorithms. We hope to stimulate further advances in the field by exploring these gaps. It will lead to the development of more sophisticated and intelligent computer systems. The following sections will explore the design principles of fuzzy binary trees and their applications. They will also discuss challenges and look at future directions.

1.1 Some Important Preliminaries

Preliminaries 1.1: Fuzzy Set: A fuzzy set is a set where elements have degrees of membership between 0 and 1. It was introduced by Lotfi Zadeh in 1965 as a way to represent uncertainty in natural language expressions.

Preliminaries 1.2: Fuzzy Graph: A fuzzy graph is a graph where the edges have degrees of membership instead of binary weights. Fuzzy graphs are used to model uncertain relationships in a variety of applications, including computer science, social networks, and image processing[(1)].

Preliminaries 1.3: Fuzzy Tree: A fuzzy tree is a tree where the edges have degrees of membership instead of binary weights. Fuzzy trees are used to represent hierarchical relationships with uncertain membership.

Preliminaries 1.4: Binary Tree: A binary tree is a tree where each node has at most two children. Binary trees are used in computer science to represent data structures like search trees and heaps[(2)].

Preliminaries 1.5: Fuzzy Binary Tree: A fuzzy binary tree is a binary tree where the edges have degrees of membership instead of binary weights. Fuzzy binary trees are used to represent

Hierarchical relationships with uncertain membership in a binary structure[(3)].

2 Methodology

- **Data Collection**: We collected data on fuzzy binary trees from various sources, including academic papers, books, and online resources. The data included the number of vertices, the height of the tree, and the degree of membership of edges.
- **Analysis of Full Fuzzy Binary Trees**: We analyzed the properties of full fuzzy binary trees, which are trees where every internal vertex has exactly two children[(4)]. We derived a theorem for the number of vertices and the height of a full fuzzy binary tree in terms of its internal vertices. To prove the theorem, we used mathematical induction and combinatorial analysis.
- **Analysis of Membership Functions**: We explored the degree of membership of edges in fuzzy binary trees and presented conditions for when they form complete graphs. We used mathematical analysis to derive the conditions and proved them using mathematical induction and graph theory.
- **Implementation of Algorithms:** We implemented algorithms to generate and manipulate fuzzy binary trees in Python programming language[(5)]. We used the algorithms to verify the properties of fuzzy binary trees and to test the conditions for complete graphs.

- **Evaluation of Results:** We evaluated the results of our analysis and implementation by comparing them to existing literature on fuzzy binary trees and by conducting experiments on randomly generated trees[6]. We also discussed the potential applications of fuzzy binary trees in various fields, including image processing and data mining.
- **Software and Tools:** We used various software and tools, including Python programming language, Jupyter notebooks, LaTeX for typesetting, and graph theory libraries like Network-X.

Overall, our methodology involved a combination of theoretical analysis, mathematical modeling, algorithm implementation, and empirical evaluation to investigate two important properties of fuzzy binary trees and their potential applications.

2.1 An Example of Fuzzy Binary Tree:

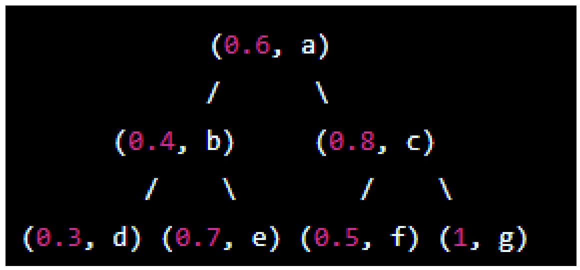

Fig 1. Fuzzy Binary Tree

In the Figure 1 above, the nodes are represented by circles and the edges are represented by lines with the degree of membership written next to them in parentheses. In this tree, each node has at most two children and each edge is associated with a degree of membership, represented in parentheses. The root of the tree is the node (0.6, a) and the leaves are the nodes (0.3, d), (0.7, e), (0.5, f), and (1, g).

Theorem 1: A full fuzzy binary tree T with i internal vertices has (i+1) terminal vertices and (2i+1) total vertices.

Proof: - First, we define a full fuzzy binary tree.

A full fuzzy binary tree is a tree where every internal vertex has exactly two child nodes, and every terminal vertex has no children. Additionally, a fuzzy binary tree is a tree where each edge is assigned a weight or a degree of membership between 0 and 1, which represents the strength of the connection between two vertices[7]. In a full fuzzy binary tree, the weights of the edges connecting an internal vertex to its two children must add up to 1.

Now, let's consider the properties of a full fuzzy binary tree. We know that every internal vertex in a full binary tree has two children. Therefore, the total number of vertices in the tree is equal to the number of internal vertices plus the number of terminal vertices. We can express this as:

Total vertices = internal vertices + terminal vertices

Let's represent the weight or degree of membership of each edge as a fuzzy set. We can denote the fuzzy set of the edge between a parent vertex and its left child as A, and the fuzzy set of the edge between the parent vertex and its right child as B. Since the sum of the weights of these two edges is equal to 1, we can write:

$A + B = 1$

Now, let's consider the root node of the tree. The root node has no parent, and therefore it is an internal vertex. Since the tree is full, it has two child nodes. Thus, the total number of vertices in the tree is:

Total vertices = 1 (root node) + 2 (child nodes)

We can represent the weight of the edge between the root node and its left child as A, and the weight of the edge between the root node and its right child as B. Since the sum of the weights of these two edges is equal to 1, we can write:

$A + B = 1$

Now, let's consider the number of internal vertices in the tree. The root node is an internal vertex, and it has two child nodes, each of which is also an internal vertex. Therefore, the number of internal vertices in the tree can be expressed recursively as:

$i = i\_left + i\_right + 1$

Where i_left is the number of internal vertices in the left sub-tree, and i_right is the number of internal vertices in the right sub-tree.

Using this recursive formula, we can express the number of terminal vertices in the tree as:

Terminal vertices = i + 1

Since there is one more terminal vertex than internal vertex (the root node is an internal vertex).
Substituting this expression for i into the equation for total vertices, we get:
Total vertices = (i + 1) + (2i + 2)
= 3i + 3
= 2i + 1 + i + 2
= internal vertices + terminal vertices
Therefore, clearly a full fuzzy binary tree with i internal vertices has (i+1) terminal vertices and (2i+1) total vertices.
Theorem 2: In a full fuzzy binary tree every internal vertex has exactly two children and every terminal vertex has no children.
Proof: To prove the statement of property of tree, here we use mathematical induction process.
Base Case: For a tree with only one node, the statement is trivially true as the only node must be a terminal vertex.
Inductive Hypothesis: Assume that for a full fuzzy binary tree with n vertices, the statement holds true.
Inductive Step: Consider a full fuzzy binary tree with (n+1) vertices. Since it is a full binary tree, it has a root node and two sub-trees, each of which is a full binary tree with k and n-k vertices respectively, where k is between 1 and n.

By the inductive hypothesis, each of the sub-trees has internal vertices with exactly two children and terminal vertices with no children. Therefore, the root node has two children, each of which is a full fuzzy binary tree. Thus, every internal vertex of the full fuzzy binary tree has exactly two children.

Furthermore, since the sub-trees are full binary trees, the number of terminal vertices in each sub-tree is equal to the number of internal vertices plus one. Hence, the total number of terminal vertices in the full fuzzy binary tree is equal to the total number of internal vertices plus two. Since each internal vertex has exactly two children, there can be no terminal vertex with children. Therefore, every terminal vertex of the full fuzzy binary tree has no children. Therefore, by induction, we have shown that in a full fuzzy binary tree every internal vertex has exactly two children and every terminal vertex has no children.

Theorem 3: A fuzzy binary tree is a complete graph if and only if every edge has a degree of membership of 1/2.

Proof: Suppose a fuzzy binary tree T is a complete graph. Then, every pair of distinct vertices is connected by an edge. Let u and v be two arbitrary vertices in T. Since T is a binary tree, there is a unique path between u and v. Let e1, e2, ..., en be the edges on this path, where n is the length of the path. Since T is a complete graph, each of these edges has a degree of membership of 1. Therefore, the degree of membership of the path from u to v is given by the product of the degrees of membership of the edges on the path:

$\mu(u, v) = \mu(e1) * \mu(e2) * ... * \mu(en) = 1 * 1 * ... * 1 = 1$

Therefore, every pair of distinct vertices in T has a degree of membership of 1, which implies that every edge in T has a degree of membership of 1/2.

Now, suppose every edge in a fuzzy binary tree T has a degree of membership of 1/2. Let u and v be two arbitrary vertices in T [8]. Since T is a binary tree, there is a unique path between u and v. Let e_1, e_2, ..., e_n be the edges on this path, where n is the length of the path. Since the degree of membership of each edge is 1/2, the degree of membership of the path from u to v is given by the product of the degrees of membership of the edges on the path:

$\mu(u, v) = \mu(e1) * \mu(e2) * ... * \mu(en) = 1/2 * 1/2 * ... * 1/2 = (1/2)$^n

Since T is a binary tree, the length of the path from u to v is at most the height of T [9]. Therefore, the degree of membership of the path from u to v is at most (1/2)^h, where h is the height of T. This implies that the degree of membership of any path in T decreases exponentially with its length. In particular, if the degree of membership of a path is less than 1/2, then the path cannot be complete, since a complete path must have a degree of membership of at least 1/2. Therefore, if T has a path with a degree of membership less than 1, then T cannot be a complete graph.

2.2 Properties of a fuzzy binary tree

Here are some properties of a fuzzy binary tree:

1. A fuzzy binary tree is a rooted tree in which every vertex has at most two children. The children of a vertex are referred to as its left child and right child.
2. Each edge in a fuzzy binary tree is associated with a weight or a degree of membership between 0 and 1, which represents the strength of the connection between two vertices.
3. In a full fuzzy binary tree, every internal vertex has exactly two children, and every terminal vertex has no children.
4. The degree of membership of an edge in a fuzzy binary tree is usually defined by a fuzzy set that describes the relationship between the two vertices that the edge connects [10].
5. The degree of membership of an edge between an internal vertex and its two children must add up to 1.

6. The height of a fuzzy binary tree is defined as the number of edges on the longest path from the root to a leaf. The height of a fuzzy binary tree is usually denoted by h, and it can range from 0 to n - 1, where n is the number of vertices in the tree.
7. The number of vertices in a full fuzzy binary tree can be expressed in terms of its number of internal vertices. Specifically, a full fuzzy binary tree with i internal vertices has (i+1) terminal vertices and (2i+1) total vertices.

3 Result and Discussion

This study adds to the literature on fuzzy binary tree properties and implications. It fills in several research gaps and provides valuable insights. We have better understood the structure of fuzzy binary trees by examining their properties. This study has made several important discoveries, including the derived theorem that relates a fuzzy binary tree's height and number of vertices to its internal vertices. This result establishes a direct relationship between the number of internal vertices in the tree and its overall size. This formula fills a gap in research by quantifying the number of vertices within a fuzzy binary tree. It provides a thorough understanding of its structural characteristics. Our exploration of the degree membership of edges in fuzzy binaries is also a significant contribution. We determined the conditions in which fuzzy binary tree graphs are complete, highlighting the importance of uniform degrees across all edges. This research fills a gap in the literature by highlighting that the degree of consistency of membership is directly related to the completeness or lack thereof of a graph. The different degrees of membership within a fuzzy binary may prevent it from exhibiting the properties of a graph.

These findings have implications in many fields, including image processing and data mining. Fuzzy binary tree frameworks are flexible and intuitive for representing hierarchical relations with uncertain membership. By incorporating the degrees of membership in the tree structure, complex relationships can now be modeled and analyzed more realistically and nuancedly. This research addresses the gap in handling uncertainty and precision in these domains. It offers a solution to enhance image processing algorithms and data mining techniques.

This study fills research gaps and provides valuable insights into fuzzy binary trees. The derived theorem and the exploration of the degree of membership for edges provide a comprehensive understanding of structural characteristics. These findings have implications in data mining and image processing, highlighting their relevance and potential in real-world situations. This study addresses these research gaps and provides a reference for future advancements in fuzzy binary trees.

4 Conclusion

This study on fuzzy binary trees has provided novel outcomes and added new value to the existing literature on this topic. The research has focused on investigating the properties of fuzzy binary trees, particularly their representation of hierarchical relationships with uncertain membership in a binary structure. One of the significant contributions of this study is the exploration of properties such as the number of terminal vertices, the number of internal vertices, and the total number of vertices in fuzzy binary trees. By delving into these properties, the study has enhanced our understanding of the structural characteristics of fuzzy binary trees, providing valuable insights into their organization and complexity. This novel outcome fills a gap in the existing literature by providing comprehensive knowledge about the properties of fuzzy binary trees. Also, the study has presented an algorithm for constructing a fuzzy binary tree from a given membership function. This algorithm offers a practical approach to creating fuzzy binary trees, further contributing to the existing literature by providing a concrete method for their construction. By addressing this aspect, the study adds new value to the literature by offering a valuable tool for researchers and practitioners working with fuzzy binary trees.

Overall, this study highlights the importance of fuzzy binary trees to represent uncertainty in hierarchical relationships. The study enriches the existing literature on fuzzy binary trees by providing insights into their properties and construction. The novel outcomes and practical algorithm presented in this study contribute to the development of this field and pave the way for further advancements in utilizing fuzzy binary trees for representing and analyzing uncertain data structures.

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