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Contra Soft g^{**} - Continuous Functions in Soft Topological Spaces

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Abstract

Objectives: The goal of this paper is to introduce the ideas of Contra soft generalized ** (briefly Contra soft g^{**}). Continuous functions and almost Contra soft generalized ** (briefly almost Contra soft g^{**}) – Continuous functions in soft topological spaces. Additionally, we discussed some of its characteristics. **Methods:** To obtain the definition of contra soft g^{**} -continuous functions, we used the previously introduced definition⁽¹⁾ (i.e. the inverse image of every soft closed set in F_Q is soft g^{**} -closed set in F_P). Also, the inverse image of every soft g^{**} -closed set in F_Q is soft regular open in F_P is almost contra soft g^{**} -continuous functions. **Findings:** Using the existing contra soft continuous functions we find the new continuous functions namely contra soft g^{**} -continuous functions and almost contra soft g^{**} -continuous functions in soft topological spaces and also discuss their properties. **Novelty:** Some of the properties that are differentiated and discussed with the existing contra continuous functions and almost contra continuous functions in soft topological spaces. Also, the converse part of every property has been solved by the suitable example.

Keywords: Contra Soft g^{**} - Continuous Functions; Almost Contra Soft g^{**} - Continuous Function; Contra Soft g^{**} -Open Map; Contra Soft g^{**} -Closed Map; Contra Soft g^{**} -Irresolute Map

1 Introduction

In 1999, Molodtsov⁽²⁾ presented the theory of soft set as a generic mathematical tool for handling problems involving uncertainty. The concepts of interior and closure of the Soft sets, as well as soft separation axioms, were first developed by Muhammed shabir and Munazza Naz⁽³⁾. The term "contra continuity" was coined by J. Dontchev⁽⁴⁾ in 1996. The generalisation results of numerous soft closed sets have been expanded upon by numerous researchers. Soft contra g - continuous functions were first proposed in 2015 by I. Arockia rani and A.Selvi⁽⁵⁾. As well, the idea of soft contra – gb Continuous functions was developed by C. Janaki and D. Sreeja⁽⁶⁾. 2018 saw the debut of a brand-new class of contra - continuous functions, first proposed by C. Santhini and M. Santhiyaa⁽⁷⁾. Many researchers⁽⁸⁻¹⁰⁾ have developed on and explored various additional

varieties of contra Soft Continuous functions in soft topology spaces as an extension of these Contra soft Continuity. The purpose of this work is to present the concept of almost Contra soft g^{**} - Continuous functions. We also looked into the functions' characteristics.

2 Methodology

This present contra soft g^{**} - continuous functions and almost contra soft g^{**} - continuous functions is newly found and extended through contra soft continuous functions and almost contra soft continuous functions in soft topological spaces. We discuss the relationship between already existing contra soft continuous functions and almost contra soft continuous functions with this new type of continuous function by the method of applying definitions. And their converse parts is proved by suitable example.

Preliminaries

Definition : 2.1⁽¹¹⁾

(i) A soft set F_A on the universe U is defined by the set of ordered pairs, E be the set of parameters and $A \subseteq E$, then $F_A = \{(x, f_A(x)) : x \in E\}$ where $f_A : E \rightarrow P(U)$ such that $f_A(x) = \emptyset$ if $x \notin A$. Here the value of $f_A(x)$ may be arbitrary. Some of them may be empty some may have non-empty intersection. Note that the set of all soft sets with the parameter set E over U will be denoted by $S(U)$.

(ii) Let $F_A \in S(U)$. If $f_A(x) = \emptyset$ for all $x \in A$ then F_A is called an empty soft set, denoted by F_\emptyset .

(iii) Let $F_A \in S(U)$. If $F_A(x) = U$ for all $x \in A$ then F_A is called a A -universal soft set, denoted by F_A^- . If $A = E$, then the A -universal soft set is called universal soft set denoted by F_E^- .

(iv) Let $F_A, F_B \in S(U)$. Then soft union $F_A \cup F_B$, Soft intersection $F_A \cap F_B$, and soft difference $F_A \setminus F_B$ of F_A and F_B are defined by respectively.

$f_{A \cup B}(x) = f_A(x) \cup f_B(x)$, $f_{A \cap B}(x) = f_A(x) \cap f_B(x)$, $f_{A \setminus B}(x) = f_A(x) \setminus f_B(x)$, and the soft complement F_A^c of F_A is defined by $f_{F_A^c}(x) = f_A^c(x)$ where $f_A^c(x)$ is complement of the set $f_A(x)$, that is $f_A^c(x) = U \setminus f_A(x)$ for all $x \in E$.

(v) Let $F_A \in S(U)$. The relative complement of F_A is denoted by F'_A and is defined by $(F_A)' = (F'_A)$ where $F'_A : A \rightarrow P(U)$ is a mapping given by $F'_\alpha = U \setminus F_\alpha$ for all $\alpha \in A$.

(vi) Let $F_A \in S(U)$. A soft topology on F_A denoted by $\tilde{\tau}$, is a collection of soft subsets of F_A having following conditions.

(i) $F_A, F_\emptyset \in \tilde{\tau}$.

(ii) The union of any number of soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$

(iii) The intersection of any two soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$

Then the pair $(F_A, \tilde{\tau})$ is called a soft topological space.

(vii) Let $(F_A, \tilde{\tau})$ be a soft topological space, then every element of $\tilde{\tau}$ is called a soft open set in $\tilde{\tau}$.

(viii) Let $(F_A, \tilde{\tau})$ be a soft topological space. A soft set F_A is said to be a soft closed set, if its relative complement F'_A belongs to $\tilde{\tau}$.

(ix) Let $(F_A, \tilde{\tau})$ be a soft topological space, then soft interior of soft set F_A is defined as the union of all soft open sets contained in F_A . It is denoted by $int(F_A)$.

(x) Let $(F_A, \tilde{\tau})$ be a soft topological space, then soft closure of soft set F_A is defined as the intersection of all soft closed super sets containing in F_A . It is denoted by $cl(F_A)$.

Definition: 2.2^(1,11,12)

Let $(F_P, \tilde{\tau})$ and $(F_Q, \tilde{\sigma})$ be the two soft topological spaces. A function $f : (F_P, \tilde{\tau}) \rightarrow (F_Q, \tilde{\sigma})$ is known as,

- 1) Soft Continuous, if the inverse image of every soft closed (open) set in $(F_Q, \tilde{\sigma})$ is soft closed (open) in $(F_P, \tilde{\tau})$.
- 2) Soft-Semi-Continuous if the inverse image of every soft closed (Open) set in $(F_Q, \tilde{\sigma})$ is soft-semi- closed (open) in $(F_P, \tilde{\tau})$.
- 3) Soft-Pre-Continuous if the inverse image of every soft closed (open) set in $(F_Q, \tilde{\sigma})$ is soft-pre-closed (open) in $(F_P, \tilde{\tau})$.
- 4) Soft- α -Continuous if the inverse image of every soft closed (open) set in $(F_Q, \tilde{\sigma})$ is soft- α - closed (open) in $(F_P, \tilde{\tau})$.
- 5) Soft- β -Continuous if the inverse image of every soft closed (open) set in $(F_Q, \tilde{\sigma})$ is soft- β - closed (open) in $(F_P, \tilde{\tau})$.
- 6) Soft Regular Continuous if the inverse image of every soft closed (open) set in $(F_Q, \tilde{\sigma})$ is soft- α - closed (open) in $(F_P, \tilde{\tau})$.
- 7) Soft generalized (briefly soft g-continuous Continuous if the inverse image of every soft closed (open) set in $(F_Q, \tilde{\sigma})$ is soft- α - closed (open) in $(F_P, \tilde{\tau})$.
- 8) Soft semi generalized (briefly soft g-continuous Continuous if the inverse image of every soft closed (open) set in $(F_Q, \tilde{\sigma})$ is soft- α - closed (open) in $(F_P, \tilde{\tau})$.
- 9) Soft generalized Continuous (briefly for g-continuous) if their inverse image of every soft closed (open) set in $(F_Q, \tilde{\sigma})$ is soft g^* closed (open) in $(F_P, \tilde{\tau})$.
- 10) Soft generalized Continuous (briefly for g-continuous) if their inverse image of every soft closed (open) set in $(F_Q, \tilde{\sigma})$ is soft g^* closed (open) in $(F_P, \tilde{\tau})$.

11) Soft regular generalized Continuous (briefly for rg-continuous) if the inverse image of every soft closed (open) set in $(F_Q, \tilde{\sigma})$ is soft rg- closed (open) in $(F_P, \tilde{\tau})$.

12) Soft regular α - generalized Continuous (briefly for rg-continuous) if the inverse image of every soft closed (open) set in $(F_Q, \tilde{\sigma})$ is soft rg- closed (open) in $(F_P, \tilde{\tau})$.

13) Soft weakly Continuous (briefly soft ω g-continuous) if the inverse image of every soft closed (open) set in $(F_Q, \tilde{\sigma})$ is soft ω g- closed (open) in $(F_P, \tilde{\tau})$.

14) Soft generalized semi pre-Continuous (briefly soft gsp-cts) if the inverse image of every soft closed (open) set in $(F_Q, \tilde{\sigma})$ is soft gsp-closed (open) in $(F_P, \tilde{\tau})$.

15) Soft generalized semi pre regular continuous (briefly soft gpr-cts) if the inverse image of every soft closed (open) set in $(F_Q, \tilde{\sigma})$ is soft gpr-closed (open) in $(F_P, \tilde{\tau})$.

Definition: 2.3

Let $(F_P, \tilde{\tau})$ and $(F_Q, \tilde{\sigma})$ be two soft topological spaces. A function $f : (F_P, \tilde{\tau}) \rightarrow (F_Q, \tilde{\sigma})$ is said to be,

(i) Soft Contra Continuous, if $f^{-1}(F_Q)$ is soft closed in $(F_P, \tilde{\tau})$ for every soft open set F_Q of $(F_Q, \tilde{\sigma})$.

(ii) Soft Contra b - Continuous, if $f^{-1}(F_Q)$ is soft b - closed in $(F_P, \tilde{\tau})$ for every soft open set F_Q of $(F_Q, \tilde{\sigma})$.

(iii) Soft Contra g - Continuous, if $f^{-1}(F_Q)$ is soft g - closed in $(F_P, \tilde{\tau})$ for every soft open set F_Q of $(F_Q, \tilde{\sigma})$.

(iv) Soft Contra πg - Continuous, if $f^{-1}(F_Q)$ is soft πg - closed in $(F_P, \tilde{\tau})$ for every soft open set F_Q of $(F_Q, \tilde{\sigma})$.

(v) Soft Contra πgb - Continuous, if $f^{-1}(F_Q)$ is soft πgb - closed in $(F_P, \tilde{\tau})$ for every soft open set F_Q of $(F_Q, \tilde{\sigma})$.

(vi) Soft Contra $\#g$ - Continuous, if $f^{-1}(F_Q)$ is soft $\#g$ - closed in $(F_P, \tilde{\tau})$ for every soft open set F_Q of $(F_Q, \tilde{\sigma})$.

(vii) Soft Contra β - Continuous, if $f^{-1}(F_Q)$ is soft β - closed in $(F_P, \tilde{\tau})$ for every soft open set F_Q of $(F_Q, \tilde{\sigma})$.

(viii) Soft Contra JA - Continuous, if $f^{-1}(F_Q)$ is soft JA - closed in $(F_P, \tilde{\tau})$ for every soft open set F_Q of $(F_Q, \tilde{\sigma})$.

3 Results and Discussion

Contra Soft g^{**} - Continuous Functions

We introduce a new class of continuous functions in this section that is Contra soft g^{**} -Continuous functions.

Definition: 3.1

A function $f : (F_P, \tilde{\tau}) \rightarrow (F_Q, \tilde{\sigma})$ is said to be Contra soft g^{**} - Continuous, if $f^{-1}(F_Q)$ is soft g^{**} - closed in $(F_P, \tilde{\tau})$ for every soft open set F_Q of $(F_Q, \tilde{\sigma})$.

Example: 3.2

Let $X = Y = \{1, 2, 3\}$, $E = \{e_1, e_2, e_3\}$, $P, Q \subseteq E$ and $P = Q = \{e_1, e_2\} \subseteq E$, $F_P = \{(e_1, \{1, 2\}), (e_2, \{3\})\}$, $F_P = \{(e_1, \{1\})\}$, $F_{P_2} = \{(e_1, \{2\})\}$, $F_P = \{(e_1, \{1, 2\})\}$, $F_P = \{(e_1, \{1\}), (e_2, \{3\})\}$, $F_{P_5} = \{(e_1, \{2\}), (e_2, \{3\})\}$, $F_{P_6} = \{(e_2, \{3\})\}$, $F_{P_7} = F_P$, $F_{P_8} = F_\phi$, $\tilde{\tau}(sos) = F_P, F_\phi, F_{P_2}, F_P\}$, $\tilde{\tau}(scs) = \{F_\phi, F_P, F_{P_1}, F_{P_4}\}$. Soft g^{**} - Closed set of $(F_P, \tilde{\tau}) = \{F_P, F_\phi, F_{P_1}, F_{P_3}, F_{P_4}\}$. $F_Q = \{(e_1, \{1, 3\}), (e_2, \{2\})\}$, $F_{Q_1} = \{(e_1, \{1\})\}$, $F_{Q_2} = \{(e_1, \{3\})\}$, $F_{Q_3} = \{(e_1, \{1, 3\})\}$, $F_{Q_4} = \{(e_1, \{1\}), (e_2, \{2\})\}$, $F_{Q_5} = \{(e_1, \{3\}), (e_2, \{2\})\}$, $F_{Q_6} = \{(e_2, \{2\})\}$, $F_{Q_7} = F_Q$, $F_{Q_8} = F_\phi$, $\tilde{\sigma}(sos) = \{F_Q, F_\phi, F_{Q_2}, F_{Q_5}\}$, $\tilde{\sigma}(scs) = \{F_\phi, F_Q, F_{Q_4}, F_{Q_6}\}$. Let $f : (F_P, \tilde{\tau}) \rightarrow (F_Q, \tilde{\sigma})$ be the mapping defined by $f(1) = 3$, $f(2) = 1$, $f(3) = 2$ and $f^{-1}(1) = 2$, $f^{-1}(2) = 3$, $f^{-1}(3) = 1$. Let (i) $f(F_{Q_2}) = \{(e_1, \{3\})\}$ is soft open set in $(F_Q, \tilde{\sigma})$. Then $f^{-1}(F_{Q_2}) = \{(e_1, \{1\})\} = F_{P_1}$, which is soft g^{**} - closed set in $(F_P, \tilde{\tau})$. (ii) Let $f(F_{Q_5}) = \{(e_1, \{3\}), (e_2, \{2\})\}$ is soft open set in $(F_Q, \tilde{\sigma})$. Then $f^{-1}(F_{Q_5}) = \{(e_1, \{1\}), (e_2, \{3\})\} = F_{P_4}$ is a soft g^{**} - closed set in $(F_P, \tilde{\tau})$. Therefore, the inverse image of every closed set in $(F_Q, \tilde{\sigma})$ is soft g^{**} - Closed in $(F_P, \tilde{\tau})$. Therefore, $f : (F_P, \tilde{\tau}) \rightarrow (F_Q, \tilde{\sigma})$ is Contra Soft g^{**} - Continuous functions.

Theorem 3.3

Every Contra Soft Continuous Function is Contra soft g^{**} - Continuous function.

Proof:

Let $f : (F_P, \tilde{\tau}) \rightarrow (F_Q, \tilde{\sigma})$ be a Contra soft continuous function. Let F_Q be a soft open set in $(F_Q, \tilde{\sigma})$. Since f is contra soft continuous, $f^{-1}(F_Q)$ is soft - closed set in $(F_P, \tilde{\tau})$. We are aware that, each soft closed set is soft g^{**} - Closed set. So $f^{-1}(F_Q)$ is soft g^{**} Closed set in $(F_P, \tilde{\tau})$. Hence, f is Contra Soft g^{**} - Continuous function." The following example demonstrates why the inverse of the aforementioned theorem need not be true

Example: 3.4

Let $X = \{1, 2, 3\}$, $E = \{e_1, e_2, e_3\}$, and $P=Q=\{e_1, e_2\} \subseteq E$, $F_P = ((e_1, \{1, 2\}), (e_2, \{3\}))$, $F_{P_1} = ((e_1, \{1\}))$, $F_{P_2} = ((e_1, \{2\}))$, $F_{P_3} = ((e_1, \{1, 2\}))$, $F_{P_4} = ((e_1, \{1\}), (e_2, \{3\}))$, $F_{P_5} = ((e_1, \{2\}), (e_2, \{3\}))$, $F_{P_6} = ((e_2, \{3\}))$, $F_{P_7} = F_P$, $F_{P_8} = F_\phi$, $\tilde{\tau}(sos) = (F_P, F_\phi, F_{P_2}, F_{P_5})$, $\tilde{\tau}(scs) = (F_\phi, F_P, F_{P_1}, F_{P_4})$. Soft g^{**} - Closed sets are $= (F_P, F_\phi, F_{P_1}, F_{P_3}, F_{P_4})$ and Soft g^{**} -Open sets are $= (F_P, F_\phi, F_{P_5}, F_{P_6}, F_{P_2})$. $F_Q = ((e_1, \{1, 3\}), (e_2, \{2\}))$, $F_{Q_1} = ((e_1, \{1\}))$, $F_{Q_2} = ((e_1, \{3\}))$, $F_{Q_3} =$

$\{(e_1, (1, 3))\}$, $F_{Q_4} = \{(e_1, (1)), (e_2, \{2\})\}$, $F_{Q_5} = \{(e_1, (3)), (e_2, \{2\})\}$, $F_{Q_6} = \{(e_2, (2))\}$, $F_{Q_7} = F_Q$, $F_{Q_8} = F_\phi$, $\tilde{\sigma}(sos) = \{F_Q, F_\phi, F_{Q_3}, F_{Q_1}\}$, $\tilde{\sigma}(scs) = \{F_\phi, F_Q, F_{Q_5}, F_{Q_6}\}$. Let $f : (F_P, \tilde{\tau}) \rightarrow (F_Q, \tilde{\sigma})$ be the mapping defined by $f(1) = 1$, $f(2) = 3$, $f(3) = 2$ and $f^{-1}(1) = 1$, $f^{-1}(2) = 3$, $f^{-1}(3) = 2$. Here f is Contra Soft g^{**} -Continuous function. But not a contra soft Continuous function. Since $F_{Q_3} = \{(e_1, (1, 3))\}$ be a soft open set in $(F_Q, \tilde{\sigma})$ and $f^{-1}(F_{Q_3}) = \{(e_1, (1, 2))\} = F_P$ be a soft g^{**} -Closed set in $(F_P, \tilde{\tau})$ but it is not soft closed set in $(F_P, \tilde{\tau})$.

Theorem: 3.5

Every Contra Soft g^{**} -Continuous function is Contra Soft rg -Continuous function.

Proof:

Let $f : (F_P, \tilde{\tau}) \rightarrow (F_Q, \tilde{\sigma})$ be a Contra Soft g^{**} -Continuous function. Let F_B be a soft Open set over $(F_Q, \tilde{\sigma})$. Since f is Contra Soft g^{**} -continuous, $f^{-1}(F_B)$ is soft g^{**} -closed set over $(F_P, \tilde{\tau})$. We are aware that, Each soft g^{**} -closed set is soft rg -Closed set. So $f^{-1}(F_B)$ is soft rg -Closed set over $(F_P, \tilde{\tau})$. Hence f is Contra soft rg -Continuous function. The following example demonstrates why the inverse of the aforementioned theorem need not be true.

Example: 3.6

Let $X = (1, 2, 3)$, $E = \{e_1, e_2, e_3\}$, and $P = Q = \{e_1, e_2\} \subseteq E$. $F_P = \{(e_1, (1)), (e_2, \{2, 3\})\}$, $F_{P_1} = \{(e_1, (1))\}$, $F_{P_2} = \{(e_1, (1)), (e_2, \{2\})\}$, $F_{P_3} = \{(e_1, (2)), (e_2, \{3\})\}$, $F_{P_4} = F_P$, $F_{P_5} = \{(e_1, (2))\}$, $F_{P_6} = \{(e_2, (3))\}$, $F_{P_7} = \{(e_2, (2, 3))\}$, $F_{P_8} = F_\phi$. $\tilde{\tau}(sos) = \{F_P, F_\phi, F_{P_1}, F_{P_2}\}$, $\tilde{\tau}(scs) = \{F_\phi, F_P, F_{P_3}, F_{P_1}\}$. Soft g^{**} -Closed sets are $\{F_P, F_\phi, F_{P_1}, F_P, F_{P_6}\}$ and Soft rg -Closed sets are $\{F_P, F_\phi, F_{P_1}, F_{P_2}, F_{P_3}, F_{P_5}, F_{P_6}, F_{P_7}\}$. $F_Q = \{(e_1, (1, 3)), (e_2, \{2\})\}$, $F_{Q_1} = \{(e_1, (1))\}$, $F_{Q_2} = \{(e_1, (3))\}$, $F_{Q_3} = \{(e_1, (1, 3))\}$, $F_{Q_4} = \{(e_1, (1)), (e_2, \{2\})\}$, $F_{Q_5} = \{(e_1, (3)), (e_2, \{2\})\}$, $F_{Q_6} = \{(e_2, (2))\}$, $F_{Q_7} = F_Q$, $F_{Q_8} = F_\phi$, $\tilde{\sigma}(sos) = \{F_Q, F_\phi, F_{Q_1}, F_{Q_4}\}$, $\tilde{\sigma}(scs) = \{F_\phi, F_Q, F_{Q_2}, F_{Q_5}\}$. Let $f : (F_P, \tilde{\tau}) \rightarrow (F_Q, \tilde{\sigma})$ be the identity function. Here f is Contra Soft rg -Continuous function but not Contra Soft g^{**} -Continuous function. Since $F_{Q_4} = \{(e_1, (1)), (e_2, \{2\})\}$ be soft open set in $(F_Q, \tilde{\sigma})$ and $(F_{Q_4}) = \{(e_1, (1)), (e_2, (2))\} = F_{Q_2}$ is soft rg -closed in $(F_Q, \tilde{\tau})$ but it is not soft g^{**} -closed in $(F_Q, \tilde{\tau})$.

Theorem: 3.7

Every Contra soft g^{**} -Continuous function is weakly contra soft g -Continuous function.

Proof:

Let $f : (F_P, \tilde{\tau}) \rightarrow (F_Q, \tilde{\sigma})$ be a contra soft g^{**} -continuous function. Let F_Q be a soft open set over $(F_Q, \tilde{\sigma})$. Since f is contra soft g^{**} -Continuous function, $f^{-1}(F_Q)$ in soft g^{**} -Closed set in $(F_P, \tilde{\tau})$. We are aware that each soft g^{**} -Closed set is weakly soft g -closed set. So $f^{-1}(F_Q)$ is weakly soft g -closed set over $(F_P, \tilde{\tau})$. Hence f is weakly contra soft g -continuous function. The following example demonstrates why the inverse of the aforementioned theorem need not be true.

Example: 3.8

Let us take example (3.4), Let $X = (1, 2, 3)$, $E = \{e_1, e_2, e_3\}$, $P = Q = \{e_1, e_2\} \subseteq E$, $F_P = \{(e_1, (1)), (e_2, \{2, 3\})\}$, $F_{P_1} = \{(e_1, (1))\}$, $F_{P_2} = \{(e_1, (1)), (e_2, \{2\})\}$, $F_{P_3} = \{(e_1, (2)), (e_2, \{3\})\}$, $F_{P_4} = F_P$, $F_{P_5} = \{(e_1, (2))\}$, $F_{P_6} = \{(e_2, (3))\}$, $F_{P_7} = \{(e_2, (2, 3))\}$, $F_{P_8} = F_\phi$, $\tilde{\tau}(sos) = \{F_P, F_\phi, F_{P_1}, F_{P_2}\}$, $\tilde{\tau}(scs) = \{F_\phi, F_P, F_{P_3}, F_{P_1}\}$, Soft g^{**} -Closed sets $= \{F_P, F_\phi, F_{P_1}, F_{P_3}, F_{P_6}\}$ and Weakly Soft- g Closed sets $= \{F_P, F_\phi, F_{P_1}, F_{P_2}, F_{P_3}, F_{P_5}, F_{P_6}\}$. $F_Q = \{(e_1, (1, 3)), (e_2, \{2\})\}$, $F_{Q_1} = \{(e_1, (1))\}$, $F_{Q_2} = \{(e_1, (3))\}$, $F_{Q_3} = \{(e_1, (1, 3))\}$, $F_{Q_4} = \{(e_1, (1)), (e_2, \{2\})\}$, $F_{Q_5} = \{(e_1, (3)), (e_2, \{2\})\}$, $F_{Q_6} = \{(e_2, (2))\}$, $F_{Q_7} = F_Q$, $F_{Q_8} = F_\phi$, $\tilde{\sigma}(sos) = \{F_Q, F_\phi, F_{Q_1}, F_{Q_4}\}$, $\tilde{\sigma}(scs) = \{F_\phi, F_Q, F_{Q_2}, F_{Q_5}\}$. Let $f : (F_P, \tilde{\tau}) \rightarrow (F_Q, \tilde{\sigma})$ be the identity function. Here f is weakly Contra soft g -continuous but not Contra soft g^{**} -Continuous function. Since $F_{Q_6} = \{(e_2, (2))\}$ is soft open in $(F_P, \tilde{\tau})$ and $f^{-1}(F_{Q_6}) = \{(e_2, (2))\} = F_{P_5}$ is weakly Soft g -Closed in $(F_P, \tilde{\tau})$, but it is not soft g^{**} -Closed in $(F_P, \tilde{\tau})$.

Theorem: 3.9

Every Contra soft g^{**} -continuous function is contra soft gpr -continuous function.

Proof:

Let $f : (F_P, \tilde{\tau}) \rightarrow (F_Q, \tilde{\sigma})$ be a Contra soft g^{**} -continuous function. Let F_Q be a soft open set in $(F_Q, \tilde{\sigma})$. Since f is contra soft g^{**} -Continuous function, $f^{-1}(F_Q)$ in soft g^{**} -Closed set over $(F_P, \tilde{\tau})$. We are aware that, Each soft g^{**} -Closed set is soft gpr -closed set. Therefore $f^{-1}(F_Q)$ is soft gpr -closed set over $(F_P, \tilde{\tau})$. Hence f is contra soft gpr -continuous function. The following example demonstrates why the inverse of the aforementioned theorem need not be true

Example: 3.10

In the above example (3.6) we have, $F_P = \{(e_1, (1)), (e_2, \{2, 3\})\}$, $\tilde{\tau}(sos) = \{F_P, F_\phi, F_{P_1}, F_{P_2}\}$, $\tilde{\tau}(scs) = \{F_\phi, F_P, F_{P_3}, F_{P_1}\}$. Soft g^{**} -Closed sets are $\{F_P, F_\phi, F_{P_1}, F_{P_3}, F_{P_6}\}$ and Soft gpr -Closed sets are $\{F_P, F_\phi, F_{P_1}, F_{P_2}, F_{P_3}, F_{P_5}, F_{P_6}, F_{P_7}\}$. $F_B = \{(e_1, (1, 3)), (e_2, \{2\})\}$, $\tilde{\tau}(sos) = \{F_Q, F_\phi, F_{Q_1}, F_{Q_4}\}$, $\tilde{\tau}(scs) = \{F_\phi, F_Q, F_{Q_2}, F_{Q_5}\}$. Let $f : (F_P, \tilde{\tau}) \rightarrow (F_Q, \tilde{\sigma})$ be the identity function. Here f is Contra Soft gpr -Continuous function but not a Contra Soft g^{**} -Continuous. Since $F_{Q_4} = \{(e_1, (1)), (e_2, \{2\})\}$ be soft open set in $(F_Q, \tilde{\sigma})$ and $f^{-1}(F_{Q_4}) = \{(e_1, (1)), (e_2, (2))\} = F_{P_2}$ is soft gpr -closed in $(F_P, \tilde{\tau})$ but it is not soft g^{**} -closed in $(F_Q, \tilde{\tau})$.

Theorem: 3.11

A mapping from a soft space is defined as $f : (F_P, \tilde{\tau}) \rightarrow (F_Q, \tilde{\sigma})$ using a soft space F_P to soft space F_Q . Consequently, the following conditions are equivalent. (i) f is Contra Soft g^{**} -Continuous. (ii) The inverse image of each soft closed set in $(F_Q, \tilde{\sigma})$ is contra soft g^{**} -open in $(F_P, \tilde{\tau})$.

Proof: (i) \Rightarrow (ii)

Let $f : (F_P, \tilde{\tau}) \rightarrow (F_Q, \tilde{\sigma})$ be a soft mapping and let F_{Q_i} be a soft closed set in $(F_Q, \tilde{\sigma})$. Then F_{Q_i}' is soft open set in $(F_Q, \tilde{\sigma})$. Thus $f^{-1}(F_{Q_i}')$ is soft g^{**} -open set in $(F_P, \tilde{\tau})$. (ie) $F_P - f^{-1}(F_{Q_i})$ is soft g^{**} -Closed set in $(F_P, \tilde{\tau}) \Rightarrow f^{-1}(F_{Q_i})$ is soft g^{**} -Open set in $(F_P, \tilde{\tau})$.

(ii) \Rightarrow (i)

Let F_{Q_i} is Soft closed set in $(F_Q, \tilde{\sigma})$. Then F_{Q_i}' is soft open set in $(F_Q, \tilde{\sigma})$. Now by (ii) we have $f^{-1}(F_{Q_i})$ is soft g^{**} -Open set in $(F_P, \tilde{\tau})$. (ie) $F_P - f^{-1}(F_{Q_i})$ is soft g^{**} -Closed set in $(F_P, \tilde{\tau})$. So $f^{-1}(F_{Q_i})$ is soft g^{**} -Open set in $(F_P, \tilde{\tau})$. Therefore f is Contra soft g^{**} -Continuous function.

Theorem: 3.12

Suppose $(F_P, \tilde{\tau})$ in soft g^{**} -open set and is soft closed under arbitrary intersections. If a function $f : (F_P, \tilde{\tau}) \rightarrow (F_Q, \tilde{\sigma})$ is Contra Soft g^{**} -Continuous and U_A is soft open in $(F_A, \tilde{\tau})$, then $f/U_P : (U_P, \tilde{\tau}) \rightarrow (F_Q, \tilde{\sigma})$ is Contra soft g^{**} -Continuous.

Proof:

Let F_{Q_i} be soft closed in $(F_Q, \tilde{\sigma})$. Since $f : (F_P, \tilde{\tau}) \rightarrow (F_Q, \tilde{\sigma})$ is Contra soft g^{**} -Continuous, then $f^{-1}(F_{Q_i})$ is soft g^{**} -open in $(F_P, \tilde{\tau})$. Since U_P is soft open in $(F_P, \tilde{\tau}) \Rightarrow (f/U_P)^{-1}F_{Q_i} = f^{-1}(F_{Q_i}) \cap U_P$ is soft g^{**} -open in $(F_P, \tilde{\tau})$. Hence $(f/U_P)^{-1}F_{Q_i}$ is soft g^{**} -Open in $U_P \Rightarrow f/U_P : (U_P, \tilde{\tau}) \rightarrow (F_Q, \tilde{\sigma})$ is Contra Soft g^{**} -Continuous function.

Definition 3.13

Let $(F_P, \tilde{\tau})$ and $(F_Q, \tilde{\sigma})$ be soft topological spaces and $f : (F_P, \tilde{\tau}) \rightarrow (F_Q, \tilde{\sigma})$ be a soft function. Then the function f is (i) Contra Soft g^{**} -Open map, if $f(F_{P_i})$ is soft g^{**} -Open in $(F_Q, \tilde{\sigma})$ for every soft closed set F_{P_i} of $(F_P, \tilde{\tau})$. (ii) Contra Soft g^{**} -Closed map, if $f(F_{P_i})$ is soft g^{**} -Closed in $(F_Q, \tilde{\sigma})$ for every soft open set F_{P_i} of $(F_P, \tilde{\tau})$.

Example: 3.14

(i) Let $X = \{1, 2, 3\}$, $E = \{e_1, e_2, e_3\}$, $P = Q = \{e_1, e_2\} \subseteq E$, $F_P = \{(e_1, \{1, 2\}), (e_2, \{3\})\}$, $F_{P_1} = \{(e_1, \{1\})\}$, $F_{P_2} = \{(e_1, \{2\})\}$, $F_{P_3} = \{(e_1, \{1, 2\})\}$, $F_{P_4} = \{(e_1, \{1\}), (e_2, \{3\})\}$, $F_{P_5} = \{(e_1, \{2\}), (e_2, \{3\})\}$, $F_{P_6} = \{(e_2, \{3\})\}$, $F_{P_7} = F_P$, $F_{P_8} = F_\phi$. $\tilde{\tau}(sos) = \{F_P, F_\phi, F_{P_2}, F_{P_5}\}$, $\tilde{\tau}(scs) = \{F_\phi, F_P, F_{P_1}, F_{P_4}\}$. $F_Q = \{(e_1, \{1, 3\}), (e_2, \{2\})\}$, $F_{Q_1} = \{(e_1, \{1\})\}$, $F_{Q_2} = \{(e_1, \{3\})\}$, $F_{Q_3} = \{(e_1, \{1, 3\})\}$, $F_{Q_4} = \{(e_1, \{1\}), (e_2, \{2\})\}$, $F_{Q_5} = \{(e_1, \{3\}), (e_2, \{2\})\}$, $F_{Q_6} = \{(e_2, \{2\})\}$, $F_{Q_7} = F_Q$, $F_{Q_8} = F_\phi$. $\tilde{\tau}(sos) = \{F_Q, F_\phi, F_{Q_1}, F_{Q_4}\}$, $\tilde{\tau}(scs) = \{F_\phi, F_Q, F_{Q_2}, F_{Q_5}\}$. Soft g^{**} -Closed sets = $\{F_Q, F_\phi, F_{Q_2}, F_{Q_3}, F_{Q_5}\}$ and Soft g^{**} -Open sets = $\{F_Q, F_\phi, F_{Q_1}, F_{Q_4}, F_{Q_6}\}$. Let $f : (F_P, \tilde{\tau}) \rightarrow (F_Q, \tilde{\sigma})$ be the mapping defined by $f(1) = 1$, $f(2) = 3$, $f(3) = 2$. Let $F_{P_1} = \{(e_1, \{1\})\}$ be a soft closed in $(F_P, \tilde{\tau})$ and $f(F_{P_1}) = \{(e_1, \{1\})\} = F_{Q_1}$ is also soft g^{**} -open in $(F_Q, \tilde{\sigma})$. Let $F_{P_4} = \{(e_1, \{1\}), (e_2, \{3\})\}$ be a soft closed in $(F_P, \tilde{\tau})$ and $f(F_{P_4}) = \{(e_1, \{1\}), (e_2, \{3\})\} = F_{Q_4}$ is also soft g^{**} -open in $(F_Q, \tilde{\sigma})$. Therefore $f : (F_P, \tilde{\tau}) \rightarrow (F_Q, \tilde{\sigma})$ is Contra Soft g^{**} -open map. (i) Let $F_{P_2} = \{(e_1, \{2\})\}$ be a soft open set in $(F_P, \tilde{\tau})$ and $f(F_{P_2}) = \{(e_2, \{3\})\} = F_{P_2}$ is also a soft g^{**} -closed set in $(F_B, \tilde{\sigma})$. (ii) Let $F_{A_5} = \{(e_1, \{2\}), (e_2, \{3\})\}$ be a soft open set in $(F_P, \tilde{\tau})$ and $f(F_{P_5}) = \{(e_1, \{3\}), (e_2, \{2\})\} = F_{Q_5}$ is also soft g^{**} -closed set in $(F_Q, \tilde{\sigma})$. Therefore $f : (F_P, \tilde{\tau}) \rightarrow (F_Q, \tilde{\sigma})$ is Contra Soft g^{**} -closed map.

Definition 3.15

A Soft map $f : (F_P, \tilde{\tau}) \rightarrow (F_Q, \tilde{\sigma})$ is known as Contra soft g^{**} -irresolute if $f^{-1}(F_{Q_i})$ is soft g^{**} -closed in $(F_P, \tilde{\tau})$ for every soft g^{**} -open a set F_{Q_i} of $(F_Q, \tilde{\sigma})$.

Example: 3.16

From the previous example 3.6, we have $F_P = \{(e_1, \{1\}), (e_2, \{2, 3\})\}$, $\tilde{\tau}(sos) = \{F_P, F_\phi, F_{P_7}, F_{P_2}\}$, $\tilde{\tau}(scs) = \{F_\phi, F_P, F_{P_3}, F_{P_1}\}$. Soft g^{**} -Closed sets are $\{F_P, F_\phi, F_{P_1}, F_{P_3}, F_{P_4}\}$ and Soft g^{**} -Open sets are $\{F_P, F_\phi, F_{P_5}, F_{P_6}, F_{P_2}\}$. $F_Q = \{(e_1, \{1, 3\}), (e_2, \{2\})\}$, $\tilde{\sigma}(sos) = \{F_Q, F_\phi, F_{Q_1}, F_{Q_4}\}$, $\tilde{\sigma}(scs) = \{F_\phi, F_Q, F_{Q_2}, F_{Q_5}\}$. Soft g^{**} -Closed sets are $\{F_Q, F_\phi, F_{Q_2}, F_{Q_3}, F_{Q_5}\}$ and Soft g^{**} -Open sets are $\{F_Q, F_\phi, F_{Q_1}, F_{Q_4}, F_{Q_6}\}$. Let $f : (F_P, \tilde{\tau}) \rightarrow (F_Q, \tilde{\sigma})$ be the mapping defined by $f(1) = 1$, $f(2) = 3$, $f(3) = 2$ and $f^{-1}(1) = 1$, $f^{-1}(2) = 3$, $f^{-1}(3) = 3$.

(i) Let $F_{Q_1} = \{(e_1, \{1\})\}$ be Soft g^{**} -open set in $(F_Q, \tilde{\sigma})$ and $f^{-1}(F_{Q_1}) = \{(e_1, \{1\})\} = F_{P_1}$ is soft g^{**} -closed set in $(F_P, \tilde{\tau})$. (ii) $F_{Q_4} = \{(e_1, \{1\}), (e_2, \{2\})\}$ be Soft g^{**} -open set in $(F_Q, \tilde{\sigma})$ and $f^{-1}(F_{Q_4}) = \{(e_1, \{1\}), (e_2, \{3\})\} = F_{P_3}$ is soft g^{**} -closed set in $(F_P, \tilde{\tau})$. (iii) $F_{Q_6} = \{(e_2, \{2\})\}$ is Soft g^{**} -open set in $(F_Q, \tilde{\sigma})$ and $f^{-1}(F_{Q_6}) = \{(e_2, \{3\})\} = F_{P_6}$ is soft g^{**} -closed set in $(F_P, \tilde{\tau})$. Therefore $f : (F_P, \tilde{\tau}) \rightarrow (F_Q, \tilde{\sigma})$ is Contra Soft g^{**} -irresolute map.

Theorem 3.17

Let $f : (F_P, \tilde{\tau}) \rightarrow (G_P, \tilde{\sigma})$ and $g : (G_P, \tilde{\sigma}) \rightarrow (H_P, \tilde{\xi})$ be two maps in soft topological spaces such that $g \circ f : (F_P, \tilde{\tau}) \rightarrow (H_P, \tilde{\xi})$. (i) If g is soft g^{**} -continuous, if f is Contra soft g^{**} -irresolute, then $g \circ f$ is Contra soft g^{**} -Continuous function. (ii) If g is soft g^{**} -irresolute and f is Contra soft g^{**} -irresolute then $g \circ f$ is Contra soft g^{**} -irresolute function.

Proof:

(i) Let H_{P_i} be soft closed set in $(H_P, \tilde{\xi})$. Then $g^{-1}(H_{P_i})$ is soft g^{**} -closed in $(G_P, \tilde{\sigma})$. Since f is Contra soft g^{**} -irresolute, $f^{-1}(g^{-1}(H_{P_i}))$ is soft g^{**} -Open in $(F_P, \tilde{\tau})$. Hence $g \circ f$ is Contra Soft g^{**} -Continuous function.

(ii) Let H_{P_i} be soft g^{**} -closed set in $(H_P, \tilde{\xi})$. Then $g^{-1}(H_{P_i})$ is soft g^{**} -closed in $(G_P, \tilde{\sigma})$. Since f is Contra soft g^{**} -irresolute, $f^{-1}(g^{-1}(H_{P_i}))$ is soft g^{**} -Open in $(F_P, \tilde{\tau})$. Hence $g \circ f$ is Contra Soft g^{**} -irresolute function.

Theorem: 3.18

If $f : (F_P, \tilde{\tau}) \rightarrow (G_P, \tilde{\sigma})$ is soft closed function and $g : (G_P, \tilde{\sigma}) \rightarrow (H_P, \tilde{\xi})$ is Contra Soft g^{**} -Closed function, then $g \circ f$ is Contra Soft-closed.

Proof:

Let $f : (F_P, \tilde{\tau}) \rightarrow (G_P, \tilde{\sigma})$ be a Soft Closed function. Let F_{P_i} be a Soft Closed set in $(F_P, \tilde{\tau})$, then $f(F_{P_i})$ is soft closed set in $(G_P, \tilde{\sigma})$. Since g is contra soft g^{**} -closed function, $g(f(F_{P_i}))$ is Contra Soft g^{**} -Closed set in $(H_P, \tilde{\xi})$. $g(f(F_{P_i})) = (g \circ f)(F_{P_i})$ is Soft g^{**} -Open set in $(H_P, \tilde{\xi})$. Therefore, $(g \circ f)$ is soft g^{**} -Open function. $g \circ f$ is Contra Soft g^{**} -Closed.

Theorem: 3.19

Let $f : (F_P, \tilde{\tau}) \rightarrow (G_P, \tilde{\sigma})$ and $g : (G_P, \tilde{\sigma}) \rightarrow (H_P, \tilde{\xi})$ be two maps in soft topological spaces such that $g \circ f : (F_P, \tilde{\tau}) \rightarrow (H_P, \tilde{\xi})$ is Contra Soft g^{**} -open map, and f is Soft Continuous and Surjective, then g is Contra Soft g^{**} -closed map.

Proof:

Let G_{P_i} be a soft closed set in $(G_P, \tilde{\sigma})$. Now, $f^{-1}(G_{P_i})$ is soft closed set in $(F_P, \tilde{\tau})$ as f is soft continuous. Since $g \circ f$ is Contra soft g^{**} -open map,

$(g \circ f)(f^{-1}(G_{P_i})) = g(f(f^{-1}(G_{P_i}))) = g(G_{P_i})$ is contra soft g^{**} -Open set in $(H_P, \tilde{\xi})$. Hence $g : (G_P, \tilde{\sigma}) \rightarrow (H_P, \tilde{\xi})$ is Contra Soft g^{**} -Closed map.

Almost Contra Soft g^{} -Continuous Functions**

We introduce a new class of functions in this section that is Almost Contra soft g^{**} -Continuous functions.

Definition: 4.1

Let $f : (F_P, \tilde{\tau}) \rightarrow (F_Q, \tilde{\sigma})$ be a function and is known as almost Contra soft g^{**} -Continuous if $f^{-1}(F_{Q_i})$ is soft g^{**} -closed (open) in $(F_Q, \tilde{\tau})$ for every soft regular open (closed) F_{Q_i} of $(F_Q, \tilde{\sigma})$.

Example: 4.2

Let $X = \{1, 2, 3\}$, $E = \{e_1, e_2, e_3\}$, $P = Q = \{e_1, e_2\} \subseteq E$. $F_P = \{(e_1, \{1, 2\}), (e_2, \{3\})\}$, $F_{P_1} = \{(e_1, \{1\})\}$, $F_{P_2} = \{(e_1, \{2\})\}$, $F_{P_3} = \{(e_1, \{1, 2\})\}$, $F_{P_4} = \{(e_1, \{1\}), (e_2, \{3\})\}$, $F_{P_5} = \{(e_1, \{2\}), (e_2, \{3\})\}$, $F_{P_6} = \{(e_2, \{3\})\}$, $F_{P_7} = F_P$, $F_{P_8} = F_\emptyset$. $\tilde{\tau}(sos) = \{F_P, F_\emptyset, F_{P_2}, F_{P_3}\}$, $\tilde{\tau}(scs) = \{F_\emptyset, F_P, F_{P_1}, F_{P_4}\}$. Soft g^{**} -closed sets = $\{F_\emptyset, F_P, F_{P_1}, F_{P_3}, F_{P_4}\}$ and Soft g^{**} -open sets = $\{F_\emptyset, F_P, F_{P_2}, F_{P_6}, F_{P_7}\}$. $F_Q = \{(e_1, \{1, 3\}), (e_2, \{2\})\}$, $F_{Q_1} = \{(e_1, \{1\})\}$, $F_{Q_2} = \{(e_1, \{3\})\}$, $F_{Q_3} = \{(e_1, \{1, 3\})\}$, $F_{Q_4} = \{(e_1, \{1\}), (e_2, \{2\})\}$, $F_{Q_5} = \{(e_1, \{3\}), (e_2, \{2\})\}$, $F_{Q_6} = \{(e_2, \{2\})\}$, $F_{Q_7} = F_Q$, $F_{Q_8} = F_\emptyset$. Soft regular open sets = $\{F_Q, F_\emptyset\}$. Let $f : (F_P, \tilde{\tau}) \rightarrow (F_Q, \tilde{\sigma})$ be the mapping defined by $f(1) = 1$, $f(2) = 3$, $f(3) = 2$ and $f^{-1}(1) = 2$, $f^{-1}(2) = 3$, $f^{-1}(3) = 2$. Let $F_Q = \{(e_1, \{1, 3\}), (e_2, \{2\})\}$ be Soft regular open set in $(F_Q, \tilde{\sigma})$ and $f^{-1}(F_Q) = \{(e_1, \{1, 3\}), (e_2, \{2\})\} = F_P$ is soft g^{**} -closed set in $(F_P, \tilde{\tau})$. $\Rightarrow f$ is almost Contra Soft g^{**} -Continuous function.

Theorem: 4.3

Every Contra soft g^{**} -continuous function is almost Contra soft g^{**} -Continuous function.

Proof:

Let $f : (F_P, \tilde{\tau}) \rightarrow (F_Q, \tilde{\sigma})$ be Contra Soft g^{**} -Continuous function and F_{Q_i} be a soft regular open in $(F_Q, \tilde{\sigma})$. We aware that each Soft regular open set is Soft open set in $(F_Q, \tilde{\sigma})$. By hypothesis $f^{-1}(F_{Q_i})$ is soft g^{**} -closed in $(F_Q, \tilde{\tau})$. $\Rightarrow f$ is almost Contra Soft g^{**} -Continuous function.

Theorem: 4.4

Let $f : (F_P, \tilde{\tau}) \rightarrow (F_Q, \tilde{\sigma})$ be a soft function. Suppose Sg^{**} -open set in $(F_P, \tilde{\tau})$ is soft closed under arbitrary unions. Consequently, the following statements are equivalent. (i) f is almost Contra soft g^{**} -continuous. (ii) $f^{-1}(F_{Q_i}) \in Sg^{**}O(F_P)$ for every $F_{Q_i} \in SRC(F_Q)$. (iii) For each $F_{P_i} \in F_P$ and each soft regular closed set F_{Q_i} in F_Q containing $f(F_{P_i})$, there exists a soft g^{**} -open set F_{P_j} in F_P containing F_{P_i} such that $f(F_{P_j}) \subseteq F_{Q_i}$. (iv) For each $F_{P_i} \in F_P$ and each soft regular open set F_{Q_i} in F_Q not containing $f(F_{P_i})$, there exists a soft g^{**} -closed set F_{P_k} in F_P not containing F_{P_i} such that $f^{-1}(F_{Q_j}) \subseteq P_k$.

Proof:(i) \Rightarrow (ii)

Assume a soft function $f : (F_P, \tilde{\tau}) \rightarrow (F_Q, \tilde{\sigma})$. Let F_{Q_i} be soft regular closed set in $(F_Q, \tilde{\sigma})$. $\Rightarrow F_{Q_i} \in SRC(F_Q)$. Then $F_Q - F_{Q_i} \in SRO(F_Q)$. Now, $f^{-1}(F_Q - F_{Q_i}) = f^{-1}(F_Q) - f^{-1}(F_{Q_i}) = F_P - f^{-1}(F_{Q_i}) \in Sg^{**}C(F_P) \Rightarrow f^{-1}(F_{Q_i}) \in Sg^{**}O(F_P)$

(ii) \Rightarrow (i) Let F_{Q_i} be Soft regular open set in $(F_Q, \tilde{\sigma}) \Rightarrow F_{Q_i} \in SRO(F_Q)$, Then $F_Q - F_{Q_i} \in SRC(F_Q)$. Now, $f^{-1}(F_Q - F_{Q_i}) = f^{-1}(F_Q) - f^{-1}(F_{Q_i}) = F_P - f^{-1}(F_{Q_i}) \in Sg * * C(F_P) \Rightarrow f^{-1}(F_{Q_i}) \in Sg * * C(F_P)$. Therefore f is almost Contra Soft $g * *$ -Continuous.

(ii) \Rightarrow (iii) Let F_{Q_i} be Soft regular Closed set in $(F_Q, \tilde{\sigma})$ containing $f(F_{P_i})$. $f^{-1}(F_{Q_i})$ be a Soft $g * *$ -Open set in $(F_P, \tilde{\tau})$. $\Rightarrow f^{-1}(F_{Q_i}) \in Sg * * O(F_P)$ and $F_{P_i} \in f^{-1}(F_{Q_i})$. By (ii) Take $F_{P_j} = f^{-1}(F_{Q_i})$ then $f(F_{P_j}) \subseteq F_{Q_i}$

(iii) \Rightarrow (ii) Let F_{Q_i} be Soft regular Closed set in $(F_Q, \tilde{\sigma}) \Rightarrow F_{Q_i} \in SRC(F_Q)$ and $F_{P_i} \in f^{-1}(F_{Q_i})$ from (iii) there exist a soft $g * *$ -open set F_{P_j} in F_P containing F_{P_i} such that $F_{P_j} \subseteq f^{-1}(F_{Q_i})$. We have $f^{-1}(F_{Q_i}) = \cup \left\{ (F_{P_j})_{F_{P_i}} : F_{P_i} \in f^{-1}(F_{Q_i}) \right\}$. Then $f^{-1}(F_{Q_i})$ is Soft $g * *$ -Open.

(iii) \Rightarrow (iv) Let F_{Q_j} be any Soft regular open set in F_Q containing $f(F_{P_i})$. Then $F_Q - F_{Q_j}$ is a soft regular closed set containing $f(F_{P_i})$. By (iii), there exists a soft $g * *$ -Open set F_{P_j} in F_P containing F_{P_i} such that $f(F_{P_j}) \subseteq F_Q - f^{-1}(F_{Q_j})$. Hence, $F_{P_j} \subseteq f^{-1}(F_Q - F_{Q_j}) \subseteq f^{-1}(F_Q) - f^{-1}(F_{Q_j}) \subseteq F_P - f^{-1}(F_{Q_j})$. Then $f^{-1}(F_{Q_j}) \subseteq F_P - F_{P_j}$. Take $F_{P_k} = F_P - F_{P_j}$. So we obtain a Soft $g * *$ -Closed set in F_P not containing F_{P_i} such that $f^{-1}(F_{Q_j}) \subseteq F_{P_k}$

(iv) \Rightarrow (iii) Let F_{Q_j} be any Soft regular closed set in F_Q containing $f(F_{P_i})$. Then $F_Q - F_{Q_i}$ is soft regular open in F_Q containing $f(F_{P_i})$. By (iv), there exists a soft $g * *$ -Closed set F_{P_k} in F_P not containing F_{P_i} such that $f^{-1}(F_Q - F_{Q_i}) \subseteq F_{P_k}$. Then $f^{-1}(F_Q) - f^{-1}(F_{Q_i}) \subseteq F_{P_k}$ which implies $F_P - f^{-1}(F_{Q_i}) \subseteq F_{P_k} \Rightarrow F_P - F_{P_k} \subseteq f^{-1}(F_{Q_i})$. Hence $f(F_P - F_{P_k}) \subseteq F_{Q_i}$. Take $F_{P_j} = F_P - F_{P_k}$. Then F_{P_j} is a soft $g * *$ -open set in F_P containing F_{P_i} such that $f(F_{P_j}) \subseteq F_{Q_i}$

Theorem: 4.5

Let $f : (F_P, \tilde{\tau}) \rightarrow (F_Q, \tilde{\sigma})$ is an almost Contra Soft $g * *$ -Continuous function and F_{P_i} is a Soft open subset of $(F_A, \tilde{\tau})$ then the restriction $f/F_{P_i} : F_{P_i} \rightarrow (F_Q, \tilde{\sigma})$ is almost contra soft $g * *$ -Continuous”.

Proof:

Let $f : (F_P, \tilde{\tau}) \rightarrow (F_Q, \tilde{\sigma})$ is an almost Contra soft $g * *$ -Continuous function and F_{Q_i} be a soft regular closed set in $(F_Q, \tilde{\sigma})$. (ie) $F_{Q_i} \in SRC(F_Q)$. Since f is almost Contra soft $g * *$ -continuous, $f^{-1}(F_{Q_i}) \in Sg * * O(F_P)$. Since F_{P_i} is soft open. $\Rightarrow (f/F_{P_i})^{-1}(F_{Q_i}) = F_{P_i} \cap f^{-1}(F_{Q_i}) \in Sg * * O(F_{P_i})$. Therefore f/F_{P_i} is an almost Contra Soft $g * *$ -Continuous function.

4 Conclusion

In this study, we proposed Contra Soft $g * *$ -Continuous functions and virtually Contra Soft $g * *$ -Continuous functions and analysed some of their characteristics. The next step of this study will involve the introduction of new notions such as ”Soft Connectedness,” ”Compactness,” ”Separation Axioms,” and so on.

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