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Varieties of Linguistic Intuitionistic Fuzzy Distance Measures for Linguistic Intuitionistic Fuzzy TOPSIS Method

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Abstract

Objective: In this paper we propose various Linguistic Intuitionistic Fuzzy Distance Measures (LIFDMs) for Linguistic Intuitionistic Fuzzy Sets (LIFSs) which are then utilized in the Linguistic Intuitionistic Fuzzy-Technique of Order Preference by Similarity to Ideal Solution (LIF-TOPSIS) method of Decision Support Systems (DSS). **Methods:** Some novel distance measures including membership, non-membership degrees and the linguistic index and distance measures based on Hausdorff metric are proposed in this work and related theorems are proved. **Findings:** The proposed distance measures are used to find the weights involved in the TOPSIS method. Numerical illustration is presented for the LIF-TOPSIS method and comparisons are made with existing ranking method and the ranking methods obtained from the different distance measures. The comparison study reveals the consistency of the ranking of the best alternative from the final ranking of the alternatives through the proposed distance measures. **Novelty:** Most of the researchers have done decision making with Linguistic Intuitionistic Fuzzy Sets, where the best alternatives are chosen using traditional distance measures involving only the intuitionistic fuzzy number or using some other calculations. In this paper we have proposed varieties of distance measures involving intuitionistic characterization and the linguistic characterization and proved that those distance measures are metrices. Using these different metrices we have derived different weight vectors for LIF-TOPSIS and the results give consistent decision for the discussed numerical illustration.

Keywords: Linguistic Intuitionistic Fuzzy Sets (LIFSs); LIF-TOPSIS; Decision Support System (DSS); MAGDM; Linguistic Distance Measures

1 Introduction

The TOPSIS method is one of the DSS which operates with the idea of ranking the best alternative out of the available ones in any decision system whenever attributes with difference of opinion are involved. The TOPSIS method is where the decision making problem will concentrate its methodology based on the ranking methods done by measuring the closeness to the positive or negative ideal solution[1–4]. In recent days, linguistic intuitionistic fuzzy data has gained the attention of researchers to a large extent[5–11]. There are various aggregation operators proposed by researchers where a few can be mentioned[12–17]. Fuzzy metric and distance measures are extremely important in Fuzzy Decision-Making situations[18–20]. In this work we have proposed some distance measures for Linguistic Intuitionistic Fuzzy Sets (LIFSs) and utilized them in attribute weight determination and also ranking of the alternatives in the Linguistic Intuitionistic Fuzzy TOPSIS algorithm. Different computations are performed with the proposed distance measures and comparisons are made with an existing decision algorithm. The study reveals that our new distance measures are less sensitive to the changes allowed in the weight vectors derived from the varieties of proposed distance measures for LIFNs.

2 Methodology

2.1 Linguistic Intuitionistic Fuzzy Numbers (LIFNs)

The basic idea of LIFNs is discussed in this section, and several new arithmetic operations for LIFNs are proposed and used in decision making situations.

Definition 2.1.1:[6] Let $L = \{l_1, l_2, \ldots, l_h\}$ be a finite totally ordered set. When h=5: $L = \{l_1, l_2, \ldots, l_5\}$ == {poor, fair, good, very good, excellent}. Any linguistic set L must include the following other qualities:

1) The set S is an ordered set: That is $l_i \prec l_j$, if and only if $i < j$;
2) The inverting operator exists and is given as: $inv(l_i) = l_{h-i}$;
3) The utmost operator is defined as follows: $mos(l_i, l_j) = l_i,\ if\ \ i \geq j$;
4) The lowest operator is as follows: $low(l_i, l_j) = l_i,\ if\ \ i \leq j$;
5) If the linguistic information has to be preserved, then the set $L = \{l_1, l_2, \ldots, l_h\}$ should be stretched to a continuous linguistic set $\overline{L} = \{l_\alpha | \alpha \in R\}$ which is observed to satisfy the above four conditions.

The preceding are some of the arithmetic operations:

i. $\beta l_i = l_{\beta \times i}$ ii. $l_i \oplus l_j = l_{i+j}$ iii. $l_i / l_j = l_{i/j}$
iv. $(l_i)^n = l_{i^n}$ v. $\lambda(l_i \oplus l_j) = \lambda l_i \oplus \lambda l_j$ vi. $(\lambda_1 + \lambda_2) l_i = \lambda_1 l_i \oplus \lambda_2 l_i$

2.2 The Linguistic Intuitionistic Fuzzy Set (LIFS)

Definition 2.2.1: [7] The Linguistic Intuitionistic Fuzzy Set (LIFS) is defined as follows:

Let $A = \{< v[l_{\theta(v)}, (\alpha_A(v), \gamma_A(v))] >: v \in X\}$, where $I_{\theta(v)} \in \bar{S}, \alpha_A(v) : X \to [0,1] \gamma_A(v) : X \to [0,1]$, $\alpha_A(v)$ and $\gamma_A(v)$ satisfying $0 \leq \alpha_A(v) + \gamma_A(v) \leq 1, \forall v \in X$.

The numbers $\alpha_A(v)$ is the grade of membership and $\gamma_A(v)$ is the grade of non-membership of the element v to the linguistic index $l_{\theta(v)}$.

In X, for every LIFS A, indeterminacy of v to the linguistic index $l_{\theta(v)}$, is given as $\eta(v) = 1 - \alpha_A(v) - \gamma_A(v), \forall v \in X, 0 \leq \eta(v) \leq 1, \forall v \in X$.

Definition 2.2. 3:[7] Let$A = \{< v[l_{\theta(v)}, (\alpha_A(v), \gamma_A(v))] >: v \in X\}$ be a LIFS. The ternary group $< l_{\theta(v)}, (\alpha_A(v), \gamma_A(v)) >$ is then referred to as a linguistic intuitionistic Fuzzy Number (LIFN), and A can be thought of as a collection of LIFNs.

Definition 2.2.4:[7] Let $\widetilde{\sigma}_1 =< l_{\theta(\sigma_1)}, (\alpha(\sigma_1), \gamma(\sigma_1)) >$ and$\widetilde{\sigma}_2 =< l_{\theta(\sigma_2)}, (\alpha(\sigma_2), \gamma(\sigma_2)) >$ be two LIFNs and$\lambda \geq 0$.

Then the operations of LIFNs are defined as:

$\widetilde{\sigma}_1 + \widetilde{\sigma}_2 =< l_{\theta(\sigma_1)+\theta(\sigma_2)}, (\alpha(\sigma_1) + \alpha(\sigma_2) - \alpha(\sigma_1)\alpha(\sigma_2), \gamma(\sigma_1)\gamma(\sigma_2)) >$,
$\widetilde{\sigma}_1 \otimes \widetilde{\sigma}_2 =< l_{\theta(\sigma_1)\times\theta(\sigma_2)}, (\alpha(\sigma_1)\alpha(\sigma_2), \gamma(\sigma_1) + \gamma(\sigma_2) - \gamma(\sigma_1)\gamma(\sigma_2)) >$,
$\lambda \widetilde{\sigma}_1 =< l_{\lambda \times \theta(\sigma_1)}, (1 - (1 - \alpha(\sigma_1))^\lambda, (\gamma(\sigma_1))^\lambda) >$, and $\widetilde{\sigma}_1^\lambda =< l_{\theta(\sigma_1)^\lambda}, (\alpha(\sigma_1))^\lambda, 1 - (1 - (\gamma(\sigma_1))^\lambda) >$.

Theorem 2.2.1:[4] For any two LIFNs $\widetilde{\sigma}_1 =< l_{\theta(\sigma_1)}, (\alpha(\sigma_1), \gamma(\sigma_1)) >$ and$\widetilde{\sigma}_2 =< l_{\theta(\sigma_2)}, (\alpha(\sigma_2), \gamma(\sigma_2)) >$, the computational rules are given as follows:

i)$\widetilde{\sigma}_1 + \widetilde{\sigma}_2 = \widetilde{\sigma}_2 + \widetilde{\sigma}_1$, ii)$\widetilde{\sigma}_1 \otimes \widetilde{\sigma}_2 = \widetilde{\sigma}_2 \otimes \widetilde{\sigma}_1$,
iii)$\lambda(\widetilde{\sigma}_1 + \widetilde{\sigma}_2) = \lambda \widetilde{\sigma}_2 + \lambda \widetilde{\sigma}_1, \lambda \geq 0$,
iv)$\lambda_1 \widetilde{\sigma}_1 + \lambda_2 \widetilde{\sigma}_1 = (\lambda_1 + \lambda_2)\widetilde{\sigma}_1, \lambda_1, \lambda_2 \geq 0$,
v)$\widetilde{\sigma}_1^{\lambda_1} \otimes \widetilde{\sigma}_1^{\lambda_2} = (\widetilde{\sigma}_1)^{\lambda_1+\lambda_2}, \lambda_1, \lambda_2 \geq 0$, and
vi) $\widetilde{\sigma}_1^{\lambda_1} \otimes \widetilde{\sigma}_2^{\lambda_1} = (\widetilde{\sigma}_1 \otimes \widetilde{\sigma}_2)^{\lambda_1}, \lambda_1 \geq 0$.

2.3 The Distance between two Intuitionistic Linguistic Fuzzy numbers

Definition 2.3.1: [4] Let $\widetilde{\sigma}_1 =< l_{\theta(\sigma_1)}, (\alpha(\sigma_1), \gamma(\sigma_1)) >$ and $\widetilde{\sigma}_2 =< l_{\theta(\sigma_2)}, (\alpha(\sigma_2), \gamma(\sigma_2)) >$ be Linguistic Intuitionistic Fuzzy numbers. Then$d(\widetilde{\sigma}_1, \widetilde{\sigma}_2)$ is called the distance between $\widetilde{\sigma}_1$and$\widetilde{\sigma}_2$:

$$d(\widetilde{\sigma}_1, \widetilde{\sigma}_2) = |\theta(\sigma_1)\alpha(\sigma_1) - \theta(\sigma_2)\alpha(\sigma_2)| + |\theta(\sigma_1)\gamma(\sigma_1) - \theta(\sigma_2)\gamma(\sigma_2)| + |\theta(\sigma_1)\eta(\sigma_1) - \theta(\sigma_2)\eta(\sigma_2)| \tag{1}$$

Theorem 2.3.1: [4] For any Linguistic Intuitionistic Fuzzy numbers:$\widetilde{\sigma}_1, \widetilde{\sigma}_2$and $\widetilde{\sigma}_3$:
$d(\widetilde{\sigma}_1, \widetilde{\sigma}_1) \geq 0; d(\widetilde{\sigma}_1, \widetilde{\sigma}_1) = 0;$
$d(\widetilde{\sigma}_1, \widetilde{\sigma}_2) = d(\widetilde{\sigma}_2, \widetilde{\sigma}_1);$
$d(\widetilde{\sigma}_1, \widetilde{\sigma}_2) + d(\widetilde{\sigma}_2, \widetilde{\sigma}_3) \geq d(\widetilde{\sigma}_1, \widetilde{\sigma}_3).$

2.4 New Distance Measures for Linguistic Intuitionistic Fuzzy Number

In the following, we propose varieties of new distance measures for Linguistic Intuitionistic Fuzzy Numbers (LIFNs).

Definition 2.4.1 : If M, N are two Linguistic Intuitionistic Fuzzy Sets of the universal set X, the distance $d_\theta(M,N)$ is:

$$d_\theta(M,N) = \sum_{i=1}^{n} [|\theta_M(v_i) - \theta_N(v_i)| + |\alpha_M(v_i) - \alpha_N(v_i)| + |\gamma_M(v_i) - \gamma_N(v_i)|] \tag{2}$$

Definition 2.4.2 : If M, N are two Linguistic Intuitionistic Fuzzy Sets of the universal set X, the Hamming distance $d'_\theta(M,N)$ is:

$$d'_\theta(M,N) = \frac{1}{2}\sum_{i=1}^{n} [|\theta_M(v_i) - \theta_N(v_i)| + |\alpha_M(v_i) - \alpha_N(v_i)| + |\gamma_M(v_i) - \gamma_N(v_i)|] \tag{3}$$

Definition 2.4.3 : If M, N are two Linguistic Intuitionistic Fuzzy Sets of the universal set X, the Normalized Hamming distance ${l_\theta}'(M,N)$ is:

$$l'_\theta(M,N) = \frac{1}{2n}\sum_{i=1}^{n} [|\theta_M(v_i) - \theta_N(v_i)| + |\alpha_M(v_i) - \alpha_N(v_i)| + |\gamma_M(v_i) - \gamma_N(v_i)|]. \tag{4}$$

Definition 2.4.4 : If M, N are two Linguistic Intuitionistic Fuzzy Sets of the universal set X, the Euclidean distance ${e_\theta}'(M,N)$ is:

$$\mathrm{e}'_\theta(M,N) = \sqrt{\frac{1}{2}\sum_{i=1}^{n}\left[\left((\theta_M(v_i) - \theta_N(v_i))^2 + (\alpha_i) - \gamma_N(v_i)\right)^2\right]} \tag{5}$$

Definition 2.4.5 : If M, N are two Linguistic Intuitionistic Fuzzy Sets of the universal set X, the Normalized Euclidean distance ${q_\theta}'(M,N)$ is:

$$\mathrm{q}'_\theta(M,N) = \sqrt{\frac{1}{2n}\sum_{i=1}^{n}\left[\left((\theta_M(v_i) - \theta_N(v_i))^2 + (\alpha_i) - \gamma_N(v_i)\right)^2\right]}. \tag{6}$$

Now we propose to define the following distance measures by considering the four parameters that categorizes linguistic intuitionistic fuzzy sets as: the degree of membership, non-membership, indeterminacy, and the linguistic index.

Definition 2.4.6 : If M, N are two Linguistic Intuitionistic Fuzzy Sets of the universal set X, the distance with intuition degree $d_{\theta\pi}(M,N)$ is:

$$d_{\theta_\pi}(M,N) = \sum_{i=1}^{n} [|\theta_M(v_i) - \theta_N(v_i)| + |\alpha_M(v_i) - \alpha_N(v_i)| + |\gamma_M(v_i) - \gamma_N(v_i)| + |\eta_M(v_i) - \eta_N(v_i)|] \tag{7}$$

Definition 2.4.7 : If M, N are two Linguistic Intuitionistic Fuzzy Sets of the universal set X, the Hamming distance with intuition degree $d''_\theta(M,N)$ is:

$${d_\theta}''(M,N) = \frac{1}{2}\sum_{i=1}^{n} [|\theta_M(v_i) - \theta_N(v_i)| + |\alpha_M(v_i) - \alpha_N(v_i)| + |\gamma_M(v_i) - \gamma_N(v_i)| + |\eta_M(v_i) - \eta_N(v_i)|]. \tag{8}$$

Definition 2.4.7 : If M, N are two Linguistic Intuitionistic Fuzzy Sets of the universal set X, the Normalized Hamming distance with intuition degree $l_\theta{}''(M,N)$ is:

$$1''_\theta(M,N) = \frac{1}{2n}\sum_{i=1}^{n}\left[\left|\theta_M(v_i)-\theta_N(v_i)\right| + \left|\alpha_M(v_i)-\alpha_N(v_i)\right| + \left|\gamma_M(v_i)-\gamma_N(v_i)\right| + \left|\eta_M(v_i)-\eta_N(v_i)\right|\right] \tag{9}$$

Definition 2.4.8. : If M, N are two Linguistic Intuitionistic Fuzzy Sets of the universal set X, the Euclidean distance with intuition degree $e_\theta{}''(M,N)$ is:

$$e_\theta{}''(M,N) = \sqrt{\frac{1}{2}\sum_{i=1}^{n}\left[(\theta_M(v_i)-\theta_M(v_i))^2 + \left((\alpha_N(v_i)-\alpha_N(v_i))^2 + (\gamma_M(v_i)-\gamma_N(v_i))^2\right.\right.} \left. + (\eta_M(v_i)-\eta_N(v_i))^2\right] \tag{10}$$

Definition 2.4.9 : If M, N are two Linguistic Intuitionistic Fuzzy Sets of the universal set X, the Normalized Euclidean distance with intuition degree $q_\theta{}''(M,N)$ is:

$$\mathrm{q}_\theta{}''(M,N) = \sqrt{\begin{array}{l}\frac{1}{2n}\sum_{i=1}^{n}\left[(\theta_M(v_i)-\theta_M(v_i))^2 + \left((\alpha_N(v_i)-\alpha_N(v_i))^2 + \right.\right. \\ (\gamma_M(v_i)-\gamma_N(v_i))^2 + (\eta_M(v_i)-\eta_N(v_i))^2\Big]\end{array}} \tag{11}$$

Now we propose the distance measures based on the Hausdorff metric:

Definition 2.4.10 : If M, N are two Linguistic Intuitionistic Fuzzy Sets of the universal set X, the Hamming distance based on Hausdorff metric$d_{h'_\theta}(M,N)$ is

$$d_{h_{\theta'}}(M,N) = \sum_{i=1}^{n}\max\left\{\left|\theta_M(v_i)-\theta_N(v_i)\right|, \left|\alpha_M(v_i)-\alpha_N(v_i)\right| \; \left|\gamma_M(v_i)-\gamma_N(v_i)\right|, \left|\eta_M(v_i)-\eta_N(v_i)\right|\right\} \tag{12}$$

Definition 2.4.11 : If M, N are two Linguistic Intuitionistic Fuzzy Sets of the universal set X, the normalized Hamming distance based on Hausdorff metric $l_{h'_\theta}(M,N)$ is

$$l_{h_{\theta'}}(M,N) = \frac{1}{n}\sum_{i=1}^{n}\max\left\{\left|\theta_M(v_i)-\theta_N(v_i)\right|, \left|\alpha_M(v_i)-\alpha_N(v_i)\right|, \; \left|\gamma_M(v_i)-\gamma_N(v_i)\right|, \left|\eta_M(v_i)-\eta_N(v_i)\right|\right\}. \tag{13}$$

Definition 2.4.12 : If M, N are two Linguistic Intuitionistic Fuzzy Sets of the universal set X, the Euclidean distance $e_{h'_\theta}(M,N)$ is:

$$e_{h_{\theta'}}(M,N) = \sqrt{\sum_{i=1}^{n}\max\left\{(\theta_M(v_i)-\theta_N(v_i))^2, (\alpha_M(v_i)-\alpha_N(v_i))^2 \; (\gamma_M(v_i)-\gamma_N(v_i))^2, (\eta_M(v_i)-\eta_N(v_i))^2\right\}}. \tag{14}$$

Definition 2.4.13 : If M, N are two Linguistic Intuitionistic Fuzzy Sets of the universal set X, the Normalized Euclidean distance $q_{h'_\theta}(M,N)$ is:

$$q_{h'}(M,N) = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\max\left\{(\theta_M(v_i)-\theta_N(v_i))^2, (\alpha_M(v_i)-\alpha_N(v_i))^2 \; (\gamma_M(v_i)-\gamma_N(v_i))^2, (\eta_M(v_i)-\eta_N(v_i))^2\right\}}. \tag{15}$$

2.5 Properties of proposed Distance Measures:

In this section we have proved the properties of proposed distance measures.

Lemma 2.5. 1: Let X denote a finite universe of discourse.

Then the functions

$d'_\theta(M,N)$,$l_\theta{}'(M,N)$,$e_\theta{}'(M,N)$,$q_\theta{}'(M,N)$,$d''_\theta(M,N)$,$l_\theta{}''(M,N)$,$e_\theta{}''(M,N)$,$q_\theta{}''(M,N)$,$d_{h'_\theta}(M,N)$,$l_{h'_\theta}(M,N)$,$e_{h'_\theta}(M,N)$,$q_{h'_\theta}(M,N)$: $LIFS(X) \to R^+ \cup \{0\}$, are metrices.

Proof.

$(i) d'_\theta(M,N) \geq 0.$

$$d'_\theta(M,N) = \frac{1}{2}\sum_{i=1}^{n}\left[(\theta_M(v_i)-\theta_N(v_i)\right| + (\alpha_M(v_i)-\alpha_N(v_i)| + (\gamma_M(v_i)-\gamma_N(v_i)|\right].$$

Clearly, $d'_{\theta}(M,N) \geq 0$.

$(ii) d_{\theta}{}'(M,N) = 0 \Leftrightarrow M = N$.

$$d'_{\theta}(M,N) = \frac{1}{2}\sum_{i=1}^{n} \left[(\theta_M(v_i) - \theta_N(v_i)| + (\alpha_M(v_i) - \alpha_N(v_i)| + (\gamma_M(v_i) - \gamma_N(v_i)| \right] = 0.$$

$$\Leftrightarrow |\theta_M(v_i) - \theta_N(v_i)|, |\alpha_M(v_i) - \alpha_N(v_i)| \, and \, |\gamma_M(v_i) - \gamma_N(v_i)| = 0 \forall i$$

$$\Leftrightarrow \theta_M(v_i) = \theta_N(v_i), \alpha_M(v_i) = \alpha_N(v_i) \text{ and } \gamma_M(v_i) = \gamma_N(v_i) \forall i \Leftrightarrow M = N.$$

(iii) $d_{\theta}{}'(M,N) = d_{\theta}{}'(N,M)$

$$d'_{\theta}(M,N) = \frac{1}{2}\sum_{i=1}^{n} \left[(\theta_M(v_i) - \theta_N(v_i)| + (\alpha_M(v_i) - \alpha_N(v_i)| + (\gamma_M(v_i) - \gamma_N(v_i)| \right].$$

$$\leq \frac{1}{2}\sum_{i=1}^{n} \left[|\theta_M(v_i) - \theta_W(v_i)| + |\theta_W(v_i) - \theta_N(v_i)| + |\alpha_M(v_i) - \alpha_W(v_i)| \right.$$
$$\left. + |-(\gamma_N(v_i) - \gamma_M(v_i))| \right]$$
$$d'_{\theta}(M,N) = d'_{\theta}(N,M)$$

(iv) $d'_{\theta}(M,N) \leq d'_{\theta}(M,W) + d'_{\theta}(W,N)$

$$d'_{\theta}(M,N) = \frac{1}{2}\sum_{i=1}^{n} \left[|\theta_M(v_i) - \theta_N(v_i)| + |\alpha_M(v_i) - \alpha_N(v_i)| + |\gamma_M(v_i) \right.$$
$$\left. - \gamma_N(v_i)| \right].$$

$$\leq \frac{1}{2}\sum_{i=1}^{n} \left[|\theta_M(v_i) - \theta_W(v_i)| + |\theta_W(v_i) - \theta_N(v_i)| + |\alpha_M(v_i) - \alpha_W(v_i)| \right.$$
$$\left. + |\alpha_W(v_i) - \alpha_N(v_i)| + |\gamma_M(v_i) - \gamma_W(v_i)| + |\gamma_W(v_i) - \gamma_N(v_i)| \right]$$
$$d'_{\theta}(M,N) \leq d'_{\theta}(M,W) + d'_{\theta}(W,N)$$

Hence $d'_{\theta}(M,N)$ is a metric. Similarly, we can prove that all other distances mentioned above are metrices.

Lemma 2.5.2: For any two Linguistic Intuitionistic Fuzzy subsets $A = \{< v[l_{\theta(v)}, (\alpha_A(v), \gamma_A(v), \eta_A(v))] > | v \in X\}$and $B = \{< v[l_{\theta(v)}, (\alpha_B(v), \gamma_B(v), \eta_B(v))] > | v \in X\}$of the universe of discourse$X = \{v_1, v_2, ..., v_n\}$, the following inequalities hold:

$d_{h_{\theta'}}(M,N) \leq n,m; l_{h_{\theta'}}(M,N) \leq m; e_{h_{\theta'}}(M,N) \leq m\sqrt{n}; q_{h_{\theta'}}(M,N) \leq m$

Where m is the largest index in the linguistic label.

Proof. Given $\max\{\theta_A, \theta_B\} = m$,

$$i) \; d_{h_{\theta'}}(M,N) \leq \sum_{i=1}^{n} \max\{|\theta_M(v_i) - \theta_N(v_i), |\alpha_M(v_i) - \alpha_N(v_i)|, |\gamma_M(v_i) - \gamma_N(v_i)|$$
$$|\eta_M(v_i) - \eta_N(v_i)|\}$$

$$\text{clearly, } d_{h_{\theta'}}(M,N) \leq \sum_{i=1}^{n} m [\text{ Since } |\theta_M(v_i) - \theta_N(v_i)| \leq m, |\alpha_M(v_i) - \alpha_N(v_i)| \leq 1,$$
$$|\gamma_M(v_i) - \gamma_N(v_i)| \leq 1, |\eta_M(v_i) - \eta_N(v_i)| \leq 1]$$
$$d_{h_{\theta'}}(M,N) \leq nm.$$

(ii) $l_{h_{\theta'}}(M,N) \leq m$

$$l_{h_{\theta'}}(M,N) = \frac{1}{n}\sum_{i=1}^{n} \max\{|\theta_M(v_i) - \theta_N(v_i)|, |\alpha_M(v_i) - \alpha_N(v_i)|,$$

$$|\gamma_M(v_i) - \gamma_N(v_i)|, |\eta_M(v_i) - \eta_N(v_i)|\} \leq \frac{1}{n}\sum_{i=1}^{n} m,$$

$$l_{h_{\theta'}}(M,N) \leq m.$$

$(iii) e_{h_\theta(M,N) \leq m\sqrt{n}}$

$$e_{h_{\theta'}}(M,N) = \sqrt{\begin{array}{c}\sum_{i=1}^{n} \max\left\{(\theta_M(v_i) - \theta_N(v_i))^2, (\alpha_M(v_i) - \alpha_N(v_i))^2\right. \\ \left.(\gamma_M(v_i) - \gamma_N(v_i))^2, (\eta_M(v_i) - \eta_N(v_i))^2\right\}\end{array}}$$

$$\leq \sqrt{\sum_{i=1}^{n} m^2}$$

$$e_{h'_\theta}(M,N) \leq m\sqrt{n}.$$

$(iv) q_{h_\theta}(M,N) \leq m$

$$q_{h_{\theta'}}(M,N) = \sqrt{\begin{array}{l}\frac{1}{n}\sum_{i=1}^{n} \max\left\{(\theta_M(v_i) - \theta_N(v_i))^2, (\alpha_M(v_i) - \alpha_N(v_i))^2\right. \\ \left.(\gamma_M(v_i) - \gamma_N(v_i))^2, (\eta_M(v_i) - \eta_N(v_i))^2\right\}\end{array}} \leq \sqrt{\frac{1}{n}\sum_{i=1}^{n} m^2}$$

$$q_{h'_\theta}(M,N) \leq m.$$

Lemma 2.5. 3: For any two Linguistic Intuitionistic Fuzzy Subsets $A = \{< v[l_{\theta(v)}, (\alpha_A(v), \gamma_A(v), \eta_A(v))] > | v \in X\}andB = \{< v[l_{\theta(v)}, (\alpha_B(v), \gamma_B(v), \eta_B(v))] > | v \in X\}$of the universe of discourse $X = \{v_1, v_2, ..., v_n\}$, the following inequalities hold:

$$d'_\theta(M,N) \leq d''_\theta(M,N); d'_\theta(M,N) \leq d_{h'_\theta}(M,N); l'_\theta(M,N) \leq l''_\theta(M,N);$$

$$l'_\theta(M,N) \leq l_{h'_\theta}(M,N); e_\theta{}'(M,N) \leq e_\theta{}''(M,N); e_\theta{}'(M,N) \leq e_{h'_\theta}(M,N);$$

$$q_\theta{}'(M,N) \leq q_\theta{}''(M,N); q_\theta{}'(M,N) \leq q_{h'_\theta}(M,N);$$

Proof. Now we present the proof only for $d'_\theta and\ d_{h'_\theta}$,for any two numbers m and n we have $\frac{1}{2}(m+n+p) \leq max\{m,n,p\}$,since $n,p \leq 1$.

Hence $d'_\theta(M,N) \leq d_{h'_\theta}(M,N)$;

And $\frac{1}{2}(m+n+p) \leq \frac{1}{2}(m+n+p+s)$, since $n,p,s \leq 1$.

Hence $d'_\theta(M,N) \leq d''_\theta(M,N)$;

In the same manner we can prove all the other relations.

2.6 The Linguistic Intuitionistic Fuzzy TOPSIS method

Let E=$\{\varepsilon_1, \varepsilon_2, ...\varepsilon_p\}$ be a set of experts,$L = (L_1, L_2, ...L_m)$ be a discrete set of alternatives, $C = (C_1, C_2, ...C_n)$be the set of attributes, and$W = (\omega_1, \omega_2, ...\omega_n)^T$be the weighting vector of the attributes, $\sum_{j=1}^{n} \omega_j = 1, \omega_j \geq 0$.

Let $\lambda = (\lambda_1, \lambda_2, ...\lambda_p)$ be the expert's weighting vector,$\sum_{k=1}^{p} \lambda_k = 1$. Suppose that $\widetilde{R}^k = \left[\widetilde{t}_{ij}^k\right]_{mXn}$ is the decision matrix, where $\tilde{t}^{k}{}_{ij} = \left\langle a_{ij}^{k}\left(\alpha_{ijk}, \gamma_{ijk}\right)\right\rangle$ takes the form of the Linguistic Intuitionistic number, given by the decision maker ε_k, for alternative L_i with respect to attribute C_j.

Rank the alternatives by using the steps below:

Step 1: Make the integrated matrix

Integrate the matrix $\widetilde{R}^k = \left[\widetilde{t}_{ij}^k\right]_{mXn}$ given by decision maker ε_k into the integrated matrix

$\widetilde{R}^k = \left[\widetilde{t}_{ij}^k\right]_{mXn}$:$\widetilde{t}_{ij} = \sum_{k=1}^{p} \lambda_k \widetilde{t}_{ij}^k$ where, $\tilde{t}_{ij} = \left\langle a_{ij}\left(\alpha_{ij}, \gamma_{ij}\right)\right\rangle$.

Step 2: Evaluate the attribute weights

For the attributeC_j, the deviation values of alternative L_i to all the other alternatives can be defined as $D_{ij}(\omega_j) = \sum_{l=1}^{m} d(\widetilde{t}_{ij}, \widetilde{t}_{lj})\omega_j$,

where$D_j(\omega_j) = \sum_{i=1}^{m} D_{ij}(\omega_j) = \sum_{i=1}^{m}\sum_{l=1}^{m} d(\widetilde{t}_{ij}, \widetilde{t}_{lj})\omega_j$indicates the total deviation values of all alternatives to the other alternatives for the attributeC_j.

$D(\omega_j) = \sum_{j=1}^{n} D_j(\omega_j) = \sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{l=1}^{m} d(\widetilde{t}_{ij}, \widetilde{t}_{lj})\omega_j$represents the deviation of all attributes to all alternatives. The optimum model is built as follows:

$$\begin{cases} \max D(\omega_j) = \sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{l=1}^{m} d\left(\tilde{t}_{ij}, \tilde{t}_{lj}\right)\omega_j \\ \text{s.t } \sum_{j=1}^{n}\omega_j^2 = 1, \omega_j \geq 0, j = 1, 2, \ldots n \end{cases} \tag{16}$$

We can get: $\omega_j = \frac{\sum_{i=1}^{m}\sum_{l=1}^{m} d(\widetilde{t}_{ij}, \widetilde{t}_{lj})}{\sqrt{\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{l=1}^{m} d^2(\widetilde{t}_{ij}, \widetilde{t}_{lj})}}$.

Furthermore, the normalized attribute weights are:$\omega_j = \frac{\sum_{i=1}^{m}\sum_{l=1}^{m} d(\widetilde{t}_{ij}, \widetilde{t}_{lj})}{\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{l=1}^{m} d(\widetilde{t}_{ij}, \widetilde{t}_{lj})}$.

Step 3: To rank the alternatives, use the TOPSIS method.

The fundamental principle of TOPSIS is that the chosen alternative ought to be closest to the positive ideal solution and most far away from the negative optimal solution.

1) Construct the weighted matrix

$$\tilde{P} = \left(\tilde{p}_{ij}\right)_{m\times n} = \begin{bmatrix} \omega_1\tilde{t}_{11} & \omega_2\tilde{t}_{12} & \ldots & \omega_n\tilde{t}_{1n} \\ \omega_1\tilde{t}_{21} & \omega_2\tilde{t}_{22} & \ldots & \omega_n\tilde{t}_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ \omega_1\tilde{t}_{m1} & \omega_2\tilde{t}_{m2} & \ldots & \omega_n\tilde{t}_{mn} \end{bmatrix}$$

$where \tilde{p}_{ij} = \left\langle b_{ij}, (\alpha_{ij}, \gamma_{ij})\right\rangle$.

2) Decide the positive and negative ideal solution:

$$\begin{cases} \tilde{p}^{+}{}_{j} = \left\langle l_{h,:}(1,0)\right\rangle \\ \tilde{p}^{-}{}_{j} = \left\langle l_{1,}(0,1)\right\rangle \end{cases} \quad j = 1, 2, \ldots, n$$

3) Compute the distance between the alternative and the positive/negative optimal solution as follows:

$d_i^+ = \sum_{j=1}^{n} d\left(\tilde{p}_{ij}, \tilde{p}_j^+\right) and d_i^- = \sum_{j=1}^{n} d\left(\tilde{p}_{ij}, \tilde{p}_j^-\right), i = 1, 2, \ldots, m$

4) Compute the relative closeness coefficient as follows: $O_i = \frac{d_i{}^+}{d_i{}^+ + d_i{}^-}(i = 1, 2, ..., m)$

5) To rank the alternatives, apply the relative closeness coefficient. The smallerO_i, the better is the alternative.

3 Results and Discussion

Assume there are four industries (alternatives) $\{L_1, L_2, L_3, L_4\}$ to be weighed against certain criteria. Evaluate industries in terms of their technological innovation capability, evaluating 'factors' such as resource ability for digitalization(C_1), organizational innovation(C_2), Innovation Centers(C_3), and Innovative products(C_4). Consider a group of experts whose weights are given as$\lambda = (0.4, 0.32, 0.28)$. The Experts assessment of the four industries are listed in Tables 1, 2, and 3.

Make the integrated matrix as mentioned in the algorithm.

$$\tilde{R} =$$

$$\begin{bmatrix} \langle l_{4.68}, (0.169, 0.670)\rangle & \langle l_{2.6}, (0.313, 0.658)\rangle & \langle l_{4.08}, (0.388, 0.581)\rangle & \langle l_{4.24}, (0.270, 0.591)\rangle \\ \langle l_{4.32}, (0.374, 0.591)\rangle & \langle l_{4.36}, (0.342, 0.558)\rangle & \langle l_{3.04}, (0.133, 0.730)\rangle & \langle l_{3.4}, (0.388, 0.586)\rangle \\ \langle l_{3.6}, (0.200, 0.666)\rangle & \langle l_{4.28}, (0.229, 0.670)\rangle & \langle l_{2.52}, (0.285, 0.692)\rangle & \langle l_{4.08}, (0.233, 0.700)\rangle \\ \langle\langle l_{4.84}, (0.365, 0.533)\rangle & \langle l_{2.92}, (0.246, 0.663)\rangle & \langle l_{2.96}, (0.262, 0.600)\rangle & \langle l_{3.88}, (0.300, 0.570)\rangle \end{bmatrix}$$

Calculate the attribute weights using the distance measures proposed in the paper and the model mentioned in the algorithm. Hence $\omega = (0.245925, 0.308033, 0.279993, 0.166049)^T$.

To rank the alternatives, use the TOPSIS method: Compute the distance (The Hamming distance) between the Alternative and the positive/negative ideal solution as follows:

1. Make the weighted matrix:

$\tilde{P} =$

$$\begin{bmatrix} \langle l_{1.151},(0.045,0.906)\rangle & \langle l_{0.809},(0.109,0.879)\rangle & \langle l_{1.142},(0.129,0.859)\rangle & \langle l_{0.704},(0.051,0.916)\rangle \\ \langle l_{1.062},(0.109,0.879)\rangle & \langle l_{1.343},(0.121,0.835)\rangle & \langle l_{0.851},(0.039,0.916)\rangle & \langle l_{0.565},(0.078,0.915)\rangle \\ \langle l_{0.885},(0.053,0.905)\rangle & \langle l_{1.318},(0.077,0.884)\rangle & \langle l_{0.706},(0.090,0.902)\rangle & \langle l_{0.677},(0.043,0.942)\rangle \\ \langle l_{1.190},(0.106,0.857)\rangle & \langle l_{0.899},(0.083,0.881)\rangle & \langle l_{0.828},(0.081,0.8867\rangle & \langle l_{0.644},(0.058,0.911)\rangle \end{bmatrix}$$

Compute the distance (The Hamming distance) between the alternative and the positive/negative ideal solution:

$$d_1{}^{+} = 15.715; d_2{}^{+} = 15.688; d_3{}^{+} = 15.892; d_4{}^{+} = 15.813;$$

$$d_1{}^{-} = 0.781; d_2{}^{-} = 0.896; d_3{}^{-} = 0.840; d_4{}^{-} = 0.815.$$

The relative closeness coefficient is calculated as follows:

$$O_1 = 0.9527; O_2 = 0.9460; O_3 = 0.9498; O_4 = 0.9510.$$

Hence we conclude that the ranking of the best alternative is $O_1 > O_4 > O_3 > O_2$. Based on the order of ranking, $L_2$is observed to be the best alternative. Then calculating the attribute weights from using the proposed distance measures and the ranking of the alternatives are recorded in Table 4.

For solving the MAGDM problem where the values are LIFN, we have proposed various distance measures, and have proved that all the proposed distance measures are metrices. The new LIF- TOPSIS method is proposed based on the different distance measures and the best alternative for the decision making problem is identified in an effective way. Using the different distance measures, we have derived different weight vectors which is utilized in the LIF-TOPSIS algorithm. The positive and negative ideal solution in the LIF-TOPSIS algorithm is also derived using the distance measures proposed, and the best alternative is chosen based on the relative closeness coefficient. The decision making based on the distance measure proposed in [4], which is not a metric is compared with the decision making based on the varieties of distance measures newly proposed in this paper, which are metrices. The methods proposed in this paper are effective and novel since all the distance measures are metrices.

Table 1. Decision Matrix I

Industries	**Digitalization**C_1	**Organizational innovation**C_2	**Innovation Centers**C_3	**Innovative products**C_4
L_1	$\langle l_5,(0.2,0.7)\rangle$	$\langle l_2,(0.4,0.6)\rangle$	$\langle l_5,(0.5,0.5)\rangle$	$\langle l_3,(0.2,0.6)$
L_2	$\langle l_4,(0.4,0.6)\rangle$	$\langle l_5,(0.4,0.5)\rangle$	$\langle l_3,(0.1,0.8)\rangle$	$\langle l_4,(0.5,0.5)$
L_3	$\langle l_3,(0.2,0.7)\rangle$	$\langle l_4,(0.2,0.7)\rangle$	$\langle l_4,(0.3,0.7)\rangle$	$\langle l_5,(0.2,0.7)$
L_4	$\langle l_6,(0.5,0.4)\rangle$	$\langle l_2,(0.2,0.8)\rangle$	$\langle l_3,(0.2,0.6)\rangle$	$\langle l_3,(0.3,0.6)$

Table 2. Decision Matrix II

Industries	**Digitalization**C_1	**Organizational innovation**C_2	**Innovation Centers**C_3	**Innovative products**C_4
L_1	$\langle l_4,(0.1,0.7)\rangle$	$\langle l_3,(0.2,0.7)\rangle$	$\langle l_3,(0.2,0.8)\rangle$	$\langle l_6,(0.4,0.5)$
L_2	$\langle l_5,(0.4,0.5)\rangle$	$\langle l_3,(0.3,0.6)\rangle$	$\langle l_4,(0.2,0.6)\rangle$	$\langle l_3,(0.2,0.7)$
L_3	$\langle l_3,(0.2,0.7)\rangle$	$\langle l_4,(0.2,0.7)\rangle$	$\langle l_2,(0.4,0.6)\rangle$	$\langle l_3,(0.3,0.7)$
L_4	$\langle l_5,(0.3,0.6)\rangle$	$\langle l_4,(0.4,0.5)\rangle$	$\langle l_2,(0.3,0.6)\rangle$	$\langle l_4,(0.2,0.6)$

Table 3. Decision Matrix III

Industries	DigitalizationC_1	Organizational innovationC_2	Innovation CentersC_3	Innovative productsC_4
L_1	$\langle l_5,(0.2,0.6)\rangle$	$\langle l_3,(0.3,0.7)\rangle$	$\langle l_4,(0.4,0.5)\rangle$	$\langle l_4,(0.2,0.7)$
L_2	$\langle l_4,(0.3,0.7)\rangle$	$\langle l_5,(0.3,0.6)\rangle$	$\langle l_2,(0.1,0.8)\rangle$	$\langle l_3,(0.4,0.6)$
L_3	$\langle l_4,(0.2,0.7)\rangle$	$\langle l_5,(0.3,0.6)\rangle$	$\langle l_1,(0.1,0.8)\rangle$	$\langle l_4,(0.2,0.7)$
L_4	$\langle l_3,(0.2,0.7)\rangle$	$\langle l_3,(0.1,0.7)\rangle$	$\langle l_4,(0.3,0.6)\rangle$	$\langle l_5,(0.4,0.5)$

Table 4. Weight Vector obtained from different Distance Measures and the ranking of alternatives.

Sl.No	Distance Measures	Weights using Distance measures	Ranking of Alternatives
1	Distance measure [4]	$\omega_1=0.27857;\omega_2=0.26822;\omega_3=0.25339;\omega_4=0.19982.$	$L_4\succ L_1\succ L_2\succ L_3$
2	$d_\theta(Y,Z)$	$\omega_1=0.24593;\omega_2=0.30803;\omega_3=0.27999;\omega_4=0.16605.$	$L_2\succ L_3\succ L_4\succ L_1$
3	$d_{\theta_\pi}(Y,Z)$	$\omega_1=0.24773;\omega_2=0.29785;\omega_3=0.28163;\omega_4=0.17279.$	$L_2\succ L_4\succ L_1\succ L_3$
4	$d'_\theta(Y,Z)$	$\omega_1=0.24593;\omega_2=0.30803;\omega_3=0.27999;\omega_4=0.16605.$	$L_2\succ L_3\succ L_4\succ L_1$
5	$l_\theta{}'(Y,Z)$	$\omega_1=0.24593;\omega_2=0.30803;\omega_3=0.27999;\omega_4=0.16605.$	$L_2\succ L_3\succ L_4\succ L_1$
6	$e_\theta{}'(Y,Z)$	$\omega_1=0.23697;\omega_2=0.33373;\omega_3=0.27281;\omega_4=0.15649.$	$L_2\succ L_3\succ L_4\succ L_1$
7	$q_\theta{}'(Y,Z)$	$\omega_1=0.23697;\omega_2=0.33373;\omega_3=0.27281;\omega_4=0.15649.$	$L_2\succ L_3\succ L_4\succ L_1$
8	$d''_\theta(Y,Z)$	$\omega_1=0.24773;\omega_2=0.29785;\omega_3=0.28163;\omega_4=0.17279.$	$L_2\succ L_4\succ L_1\succ L_3$
9	$l_\theta{}''(Y,Z)$	$\omega_1=0.24773;\omega_2=0.29785;\omega_3=0.28163;\omega_4=0.17279.$	$L_2\succ L_4\succ L_1\succ L_3$
10	$e_\theta{}''(Y,Z)$	$\omega_1=0.23732;\omega_2=0.33201;\omega_3=0.27301;\omega_4=0.15766.$	$L_2\succ L_3\succ L_4\succ L_1$
11	$q_\theta{}''(Y,Z)$	$\omega_1=0.23732;\omega_2=0.33201;\omega_3=0.27301;\omega_4=0.15766.$	$L_2\succ L_3\succ L_4\succ L_1$
12	$d_{h'_\theta}(Y,Z)$	$\omega_1=0.23333;\omega_2=0.33976;\omega_3=0.27269;\omega_4=0.15422.$	$L_3\succ L_2\succ L_4\succ L_1$
13	$l_{h'_\theta}(Y,Z)$	$\omega_1=0.23333;\omega_2=0.33976;\omega_3=0.27269;\omega_4=0.15422.$	$L_3\succ L_2\succ L_4\succ L_1$
14	$e_{h'_\theta}(Y,Z)$	$\omega_1=0.23333;\omega_2=0.33976;\omega_3=0.27269;\omega_4=0.15422.$	$L_3\succ L_2\succ L_4\succ L_1$
15	$q_{h'_\theta}(Y,Z)$	$\omega_1=0.23333;\omega_2=0.33976;\omega_3=0.27269;\omega_4=0.15422.$	$L_3\succ L_2\succ L_4\succ L_1$

4 Conclusion

Since many real-world problems under Linguistic Intuitionistic Fuzzy sets are complex in nature, the proposed methodology will definitely relieve the biased role of the decision makers involved. Varieties of distance measures for LIFNs are proposed and some distance measures based on Hausdorff metric are also proposed and are utilized to calculate attribute weights involved in the decision problem. All the proposed distance measures are also used to find the closeness coefficient in the TOPSIS method of decision making. Some basic theorems and lemmas are proved for the proposed distance measures in order to ensure the distance measures' stability. Numerical illustration is provided with LIFN decision data and comparison of the proposed methods of distance measures is made with an existing method of finding weights through distance measures. Comparison between the different computational methods proposed are also highlighted at the end of the work revealing their consistency.

5 Declaration

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References

1) Herrera F, Herrera-Viedma E. Linguistic decision analysis: steps for solving decision problems under linguistic information. *Fuzzy Sets and Systems*. 2000;115(1):67–82. Available from: https://doi.org/10.1016/S0165-0114(99)00024-X.
2) Kumar K, Chen SM. Group decision making based on weighted distance measure of linguistic intuitionistic fuzzy sets and the TOPSIS method. *Information Sciences*. 2022;611:660–676. Available from: https://doi.org/10.1016/j.ins.2022.07.184.
3) Liu P, Diao H, Zou L, Deng A. Uncertain multi-attribute group decision making based on linguistic-valued intuitionistic fuzzy preference relations. *Information Sciences*. 2020;508:293–308. Available from: https://doi.org/10.1016/j.ins.2019.08.076.
4) Du Y, Zuo J. An extended TOPSIS method for the multiple attribute group decision making problems based on intuitionistic linguistic numbers. *Academic Journals*. 2011;6(19):4125–4132. Available from: https://academicjournals.org/journal/SRE/article-full-text-pdf/DC1596527403.

5) Bao J, Zhou Y, Li R. Competitive advantage assessment for container shipping liners using a novel hybrid method with intuitionistic fuzzy linguistic variables. *Neural Computing and Applications*. 2023;35(17):12261–12269. Available from: https://doi.org/10.1007/s00521-021-05718-z.
6) Bhaumik A, Roy SK, Weber GW. Hesitant interval-valued intuitionistic fuzzy-linguistic term set approach in Prisoners' dilemma game theory using TOPSIS: a case study on Human-trafficking. *Central European Journal of Operations Research*. 2020;28(2):797–816. Available from: https://doi.org/10.1007/s10100-019-00638-9.
7) Xu Z. On generalized induced linguistic aggregation operators. *International Journal of General Systems*. 2006;35(1):17–28. Available from: https://doi.org/10.1080/03081070500422836.
8) Kumar K, Chen SMM. Multiple attribute group decision making based on advanced linguistic intuitionistic fuzzy weighted averaging aggregation operator of linguistic intuitionistic fuzzy numbers. *Information Sciences*. 2022;587:813–824. Available from: https://doi.org/10.1016/j.ins.2021.11.014.
9) Liu J, Mai J, Li H, Huang B, Liu Y. On three perspectives for deriving three-way decision with linguistic intuitionistic fuzzy information. *Information Sciences*. 2022;588:350–380. Available from: https://doi.org/10.1016/j.ins.2021.12.072.
10) Liu Y, Wei G, Liu H, Xu L. Group decision making for internet public opinion emergency based upon linguistic intuitionistic fuzzy information. *International Journal of Machine Learning and Cybernetics*. 2022;13(3):579–594. Available from: https://doi.org/10.1007/s13042-020-01262-9.
11) Leonishiya A, Robinson JP. Linguistic Intuitionistic Fuzzy VIKOR Method for MAGDM problems with sensitivity analysis. *Advances and Applications in Mathematical Sciences*. 2022;21:6765–6781. Available from: https://www.mililink.com/upload/article/426701116aams_vol_2112_october_2022_a13_p6765-6781_a._leonishiya_and_p._joohn_robinson.pdf.
12) Arora R, Garg H. Group decision-making method based on prioritized linguistic intuitionistic fuzzy aggregation operators and its fundamental properties. *Computational and Applied Mathematics*. 2019;38(2):1–32. Available from: https://doi.org/10.1007/s40314-019-0764-1.
13) Faizi S, Sałabun W, Nawaz S, Rehman AU, Wątróbski J. Best-Worst method and Hamacher aggregation operations for intuitionistic 2-tuple linguistic sets. *Expert Systems with Applications*. 2021;181:115088. Available from: https://doi.org/10.1016/j.eswa.2021.115088.
14) Qiyas M, Naeem M, Abdullah L, Riaz M, Khan N. Decision Support System Based on Complex Fractional Orthotriple Fuzzy 2-Tuple Linguistic Aggregation Operator. *Symmetry*. 2023;15(1):251–251. Available from: https://doi.org/10.3390/sym15010251.
15) Xian S, Guo H, Chai J. Intuitionistic fuzzy linguistic induced generalized hybrid weighted averaging operator and its application to take targeted measures in poverty alleviation. *Computational and Applied Mathematics*. 2019;38(3):1–21. Available from: https://doi.org/10.1007/s40314-019-0902-9.
16) Robinson PJ, Li DFF, Nirmalsingh SS. An Automated Decision Support Systems Miner for Intuitionistic Trapezoidal Fuzzy Multiple Attribute Group Decision-Making Modeling with Constraint Matrix Games. *Artificial Intelligence and Technologies*. 2022;806:343–351. Available from: https://doi.org/10.1007/978-981-16-6448-9_35.
17) Yuan R, Tang J, Meng F. Linguistic Intuitionistic Fuzzy Group Decision Making Based on Aggregation Operators. *International Journal of Fuzzy Systems*. 2019;21(2):407–420. Available from: https://doi.org/10.1007/s40815-018-0582-4.
18) Jeevaraj PSE, Shanmugavadivu P. Fuzzy logic and distance measure based adaptive fixed value impulse noise filter. *The Journal of Analysis*. 2019;27(1):89–102. Available from: https://doi.org/10.1007/s41478-018-0105-6.
19) Priyadharsini J, Balasubramaniam P. Multi-criteria decision making method based on interval-valued intuitionistic fuzzy sets. *The Journal of Analysis*. 2019;27(1):259–276. Available from: https://doi.org/10.1007/s41478-018-0122-5.
20) Roopkumar R, Vembu R. Actual metric representing a fuzzy metric. *The Journal of Analysis*. 2020;28(4):973–985. Available from: https://doi.org/10.1007/s41478-020-00228-y.