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An Evaluation of Covariance and Correlation Analysis in Entropy Method

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Abstract

Objectives: The main aim of this research is to obtain the criterion weight in Multi-Criteria Decision Making (MCDM) with conflicting criteria, as the relevance and impact of the criterion weight alters the outcome of any decision-making. **Methods:** This study proposes a modification to the existing Entropy technique using principal component analysis, which is found to be suitable for all day-to-day real-life problems. **Findings:** A comparative study is done between covariance and correlation analysis to validate the accuracy of the proposed technique. The proposed modification is illustrated with a numerical example. **Novelty**: The inclusion of the concept of covariance analysis in the principal component analysis is a new approach to the determination of the criterion weight in MCDM.

Keywords: Multi-Criteria Decision Making (MCDM); Entropy; Criterion Weight; Covariance; Correlation; Principal Component Analysis (PCA); CRITIC method

1 Introduction

Generally, the weighting techniques are broadly classified into two main categories such as subjective and objective methods. The subjective techniques rely on primary information provided by the analysts based on their expertise and knowledge. The familiar subjective weighting techniques such as Step-wise Weight Assessment Ratio Analysis (SWARA), Decision Making Trial And Evaluation Laboratory (DEMATEL)[1], KEmeny Median Indicator Ranks Accordance (KEMIRA), Full Consistency Method (FUCOM)[2], P-SWING, SIMOS and PIvot Pairwise RElative Criteria Importance Assessment (PIPRECIA) use the idea of pairwise comparisons. Since subjective methods incorporate the expertise of decision-makers, they can introduce bias in the results due to personal beliefs. Additionally, decision-makers[3] may lack complete knowledge or struggle to provide the necessary initial information. Subjective methods can also become complex when dealing with many criteria.

In contrast, the need for initial information from the decision-makers is not essential in objective techniques. The available information in the decision matrix is analyzed to calculate the criteria weights. The techniques aim to reduce the prejudice or favoritism related to subjective evaluation, thereby improving objectivity. The most popular objective techniques include entropy-based methods, IDOCRIW, CRITIC and CILOS.

Among these objective methods, CRiteria Importance Through Inter-criteria Correlation (CRITIC) offers an additional advantage as it deals with both the conflicting relationships and the contrast intensity among the decision criteria. The contrast intensity refers to the variability degree of every criterion's grade. The concept of standard deviation is used to determine the contrast intensity in CRITIC by assigning higher weights to criteria with greater variance. This approach ensures that criteria with more meaningful information receive more attention in the decision-making process[4].

The conflicting relationships among the criteria arise when the alternatives of the decision matrix fail to satisfy all conflicting criteria. For example, it may be challenging to find a new laptop with both a high-end processor and a lower price. CRITIC addresses the concept of conflicting relationships using Karl Pearson's correlation coefficient 'ρ' in the closed interval[1]. A coefficient 'ρ' of 0 indicates independence between two criteria, while a negative coefficient suggests an opposite relationship. When the value of 'ρ' tends to -1, the conflict among the criteria intensifies. Conversely, the high positive correlation value of 'ρ' implies a parallel relationship where the criterion shares redundant information. CRITIC allocates more weights[5] to criterion with a stronger conflict or lower redundancy, as they play an important role in the system.

Gao et al.[6] introduced CRITIC, which is used to determine the criterion weight. The technique considers both the impact and drawbacks while framing the structure of the MCDM. The correlation analysis is used to overcome the conflict among the criteria. The normalized values are evaluated using the standard deviation and pairwise correlation coefficients.

CRITIC assigns higher weights to criteria for those criteria which have maximum conflict degree and contrast intensity compared to other criteria. CRITIC is successfully applied in various real-world problems, sometimes in combination with both subjective or objective techniques for obtaining weights. Although there are a limited number of CRITIC modifications in the literature, researchers have not identified significant issues with its fundamental components that necessitate major modifications.

Gao et al.[6] examined a few pharmaceutical industries using the CRITIC method to study the performance of the company based on fixed common indices. The CRITIC method was used to plan an optimization model for water management in India[7]. The methods such as TOPSIS[8], ELECTRE, SAW, PROMETHEE, VIKOR, and DMATEL are commonly used MCDM[9] techniques used for determining the best alternatives. The techniques like CRITIC, AHP and SWARA[10] are integrated in general with the above-mentioned MCDM to form a hybrid model. AHP[11] method is suitable for subjective criteria, but CRITIC and SWARA shall be used for objective criteria. Kırda and Aytekin[12] applied the CRITIC technique to calculate the variable weight in assessing environmental sustainability performances. Wu et al.[13] compared and examined the criterion weight using CRITIC and entropy[14] with a numerical example. Li et al.[15] integrated fuzzy DEA and AHP to analyze the performance index of the people-oriented urban pedestrian road system.

Luyen and Thanh[16] developed a hybrid method to measure the performance of several logistics firms in which the criterion weight is obtained at the initial stage. Researchers developed an evaluation model with the help of CRITIC and AHP, including all objective and subjective data to obtain the criterion weight. Silva et al.[17] used CRITIC and Grey Relational Analysis (GRA) methods for investment portfolio selection to analyze the financial status of all industries. Şahin[18] employed six weighting and seven MADM approaches that create 42 models that were realized for evaluating dissimilar weighting and multicriteria decision-making approaches. Researchers proposed a logical design using AHP and CRITIC to determine Macau City's sustainability index for the water management board.

Paradowski et al.[19] examined possible objective techniques for criteria weighting together with the application of the method that delivers effective and manageable weight assessments for any decision matrix with the likelihood of matching outcomes of dissimilar weighting methods. Researchers proposed a Non-Traditional Machining Process (NTMP) to determine the relative importance of the criteria using the CRITIC method. Researchers integrated CRITIC and AHP methods and calculated the evaluation scheme index, and assessed the assessment pattern in Suzhou to support the education department. Researchers utilized the grey theory to handle problems that contain imprecise data in order to model and predict decision-making. Further, the original data is analyzed for intrinsic regularities.

However, this study recognizes a shortcoming in the traditional CRITIC technique related to capturing conflicting relationships among each criterion. The method solely relies on the Pearson correlation, which may not always reflect the true relationships between criteria. The Pearson correlation only detects linear relationships, disregarding nonlinear associations. The objective of this work is to study the limitations by proposing an improved version of CRITIC using the concept of principal component analysis[20], which provides a more reliable modeling of conflicting relationships between criteria. Furthermore, the research is validated using numerical examples.

2 Methodology

The modified entropy technique integrating principal component analysis to analyze multiple variables is given below.

(1) Convert the given data into S/N ratios using the formula given below based on the benefit/cost criteria. The calculation of the S/N ratio differs based on the characteristic of the response a) maximum-the better, b) minimum-the better, and c) nominal-the-better, as stated by González-García et al.[21],

$$(a)\ \frac{S}{N} ratio = -10 log\left(\frac{1}{n}\sum_{i=1}^{n}\frac{1}{{y_{ij}}^2}\right); (Max. the\ better)$$

$$(b)\ \frac{S}{N} ratio = -10 log\left(\frac{1}{n}\sum_{i=1}^{n}{y_{ij}}^2\right); (Min. the\ better)$$

$$\text{(c) } \frac{S}{N} \text{ ratio } = 10\log\left(\frac{\bar{y}^2}{s^2}\right); (\text{ Nominal the better })$$

where y_{ij} indicates the position of the i^{th} row and j^{th} column.

$$\bar{y} = \frac{y_1 + y_2 + y_3 + \cdots + y_n}{n}$$

$$s^2 = \frac{\sum\left(y_i - \bar{y}\right)^2}{n-1}$$

(2) The values of S/N ratios are normalized using[22]:

$$Y_{ij} = \frac{y_{ij} - min(y_{ij})}{max(y_{ij}) - min(y_{ij})}$$

(3) Calculate the standard deviation and covariance for the normalized S/N ratios.

(4) Determine the conflict and objective values of the criterion weights.

(5) Calculate eigenvalues and eigenvectors for each principal component.

(6) The principal components which have eigenvalues less than one are ignored as they are insignificant.

(7) The principal components which have eigenvalues one and above are considered for further analysis.

(8) Determine the Multi-Response Performance Index (MRPI) using the formula[23]:

$$Z_1 = P_1Y_1 + P_1Y_2$$

$$Z_2 = P_2Y_1 + P_2Y_2$$

$$MRPI = W_1Z_1 + W_2Z_2$$

where W1 and W2 are the weights of principal components, respectively.

The highest performance index refers to better approximation. The performance index values are correlated to criterion weight.

3 Results and Discussion

Consider an MCDM problem with 5 alternatives (E1, E2, E3, E4 and E5) and 4 four benefit criteria (C1, C2, C3 and C4). The decision matrix values are presented in Table 1.

Table 1. Initial Decision Matrix

	C1	C2	C3	C4
E1	8	4	10	2
E2	7	6	4	6
E3	5	5	6	7
E4	6	6	7	8
E5	5	7	6	6

The Standard deviation for the response is obtained using Step (1) and displayed in Table 2 .

Table 2. Standard Deviation

	C1	C2	C3	C4	AVG	SD	S/N
E1	8	4	10	2	6.00	3.65	-4.32
E2	7	6	4	6	5.75	1.26	-13.20
E3	5	5	6	7	5.75	0.96	-15.57
E4	6	6	7	8	6.75	0.96	-16.96
E5	5	7	6	6	6.00	0.82	-17.32

All the S/N Ratios and their normalized values are calculated using Step (2) and displayed in Table 3 .

Table 3. Normalized S/N Ratio

S/N Ratio					Normalized S/N Ratio			
	C1	**C2**	**C3**	**C4**	**C1**	**C2**	**C3**	**C4**
E1	18.06	12.04	20.00	6.02	1	0	1	0
E2	16.90	15.56	12.04	15.56	0.715	0.724	0	0.792
E3	13.98	13.98	15.56	16.90	0	0.398	0.442	0.903
E4	15.56	15.56	16.90	18.06	0.387	0.724	0.610	1
E5	13.98	16.90	15.56	15.56	0	1	0.442	0.792

Tables 4 and 5 present the correlation matrix and eigenvalue analysis for correlation data, respectively, which are calculated using Minitab 20.3.

Table 4. Correlation Matrix of S/N Ratio

	C1	C2	C3
C2	-0.590		
C3	0.294	-0.652	
C4	-0.727	0.759	-0.660

The Covariance matrix and eigenvalue analysis for Covariance data has been done using Minitab 20.3 and displayed in Tables 6 and 7.

The Multi Response Performance Index (MRPI) for correlation and covariance matrix has been calculated using Step (8) and shown in Table 8.

Table 5 and Table 7 indicate the criteria weight coefficients using the correlation and covariance approach for the given data. The eigenvalue analysis provides enough understanding regarding the subjective data and allows us to make decisions based on the response. The observations indicate that the covariance and correlation influence the criteria in determining the weight of the criterion. Further, there is a notable deviation between covariance and correlation analysis with significant corresponding weight deviation.

Table 5. Eigen Value Analysis of Correlation

	C1	C2	C3	C4
Eigen Value	2.862	0.715	0.270	0.152
Proportion	0.716	0.179	0.068	0.038
Cumulative	0.716	0.894	0.962	1.000
Criteria of Conflict	5.023	3.893	4.660	4.000
Quantity of Data	9.0423	7.2483	13.3577	19.2443
Criteria Weight	**0.185**	**0.148**	**0.272**	**0.394**

Table 6. Covariance Matrix of S/N Ratio

	C1	C2	C3	C4
C1	3.241			
C2	-1.977	3.466		
C3	1.517	-3.482	8.216	
C4	-6.299	6.797	-9.105	23.146

Table 7. Eigen Value Analysis of Covariance

	C1	C2	C3	C4
Eigen Value	31.256	4.511	1.374	0.929
Proportion	0.821	0.118	0.036	0.024
Cumulative	0.821	0.940	0.976	1.000
Criteria of Conflict	7.518	-2.781	4.889	-19.146
Quantity of Data	13.5337	-5.1779	14.0142	-92.1128
Criteria Weight	**0.187**	**0.160**	**0.242**	**0.402**

Table 8. Performance Index based on Correlation

	C1	C2	C3	C4	Z1	MRPI
E1	1.000	0.000	1.000	0.000	0.826	0.590
E2	0.716	0.725	0.000	0.792	1.517	1.084
E3	0.000	0.399	0.443	0.904	2.887	2.064
E4	0.388	0.725	0.611	1.000	2.335	1.669
E5	0.000	1.000	0.443	0.792	2.234	1.598

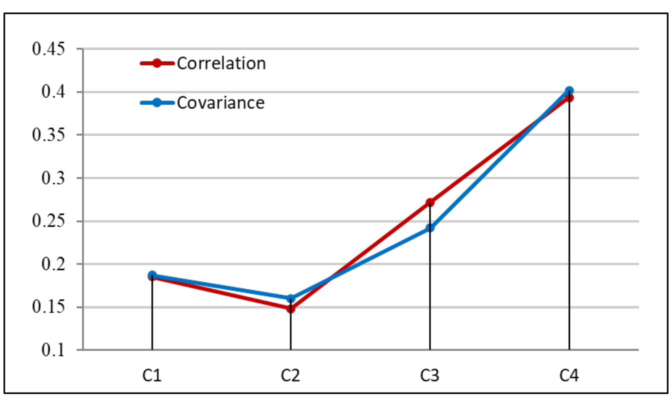

Fig 1. Correlation vs. Covariance

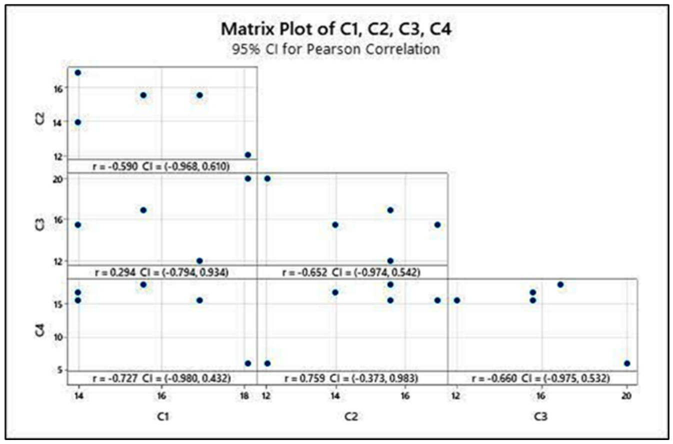

Fig 2. Correlation Analysis

Figure 1 shows the comparison between the correlation and covariance analysis with not much difference in the weight order. Karl Pearson's correlation analysis at a 5% level of significance is shown in Figure 2. There exists a high positive correlation between C2 and C4 with a deviation of 0.759, and similarly between C1 and C4, there is a negative correlation of -0.727. The impact of correlation is reflected in the criterion also. Criteria C4 has the maximum weight in correlation analysis with 0.394. Even in Covariance analysis, criteria C4 has the maximum weight of 0.402, which indicates that the method holds good for all MCDM problems.

4 Conclusion

The nature of decision-making in multi-criteria decision problems sometimes relies on the quantitate measure of the criterion as it directly controls the decision-making. This study proposed a modified weight determination method using covariance and principal components. The normalization to S/N ratio minimizes the standard deviation and that helps to maintain a better objective relationship between the values for the given response. Moreover, the modified covariance approach is used to obtain the criterion weight, which ensures a comprehensive weight for each criterion based on its importance between and within the criteria.

The proposed method aggregates the criterion weight based on the degree of subjectivity of the normalized values. The proposed method eliminates the principal components whose eigenvalues are less than 'one' during the performance index calculation. This eliminates the impact of redundant criteria in weight determination. Criteria C4 has a high positive correlation 'of 0.759' with C2 and a high negative correlation of '-0.727' with C1. The high correlation index of criteria C4 indicates that it will influence the decision-making pattern. Clearly, criteria C4 has the maximum weight in correlation and covariance analysis. The validation of the proposed method proves that the method is more effective for all real-world and MCDM problems.

In the future, a hybrid strategy for MCDM issues that concentrate on the uncertainty and errors of triangular fuzzy numbers shall be developed. Moreover, the development of an integrated MCDM approach for assessing and choosing providers in a sustainable supply chain for critical problems is likewise required. The creation and use of a comparable model using the most recent decision-making technique and an uncertainty theory like grey theory will also be addressed by future research relevant to this study. Future research pertaining to the current study may likewise concentrate on the development of an improved ranking-based decision-making system to address complicated material selection issues.

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