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M/M(a,b)/1 Model Of Interdependent Queueing With Controllable Arrival Rates

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Abstract

Objectives: Instead of only providing individualized one-on-one assistance, some studies in the literature on queueing theory describe systems that provide services in batches. This study introduces controllable arrival rates and interdependency in such a system's service and arrival processes and then obtains the queueing system's probabilities and characteristics. It also verified the obtained results numerically. **Methods:** Controlling the arrival rates by faster and slower arrival rates are expected for the input, with Poisson (each time Poisson occurrence has one arrival) being the default assumption. The general bulk service rule dictates that the service be delivered in batches. Service begins only when the count of customers in the queue approaches or surpasses a and the capacity b ($\geq$a $\geq$ 1). For brevity, a batch's service time distribution is assumed to be exponential and is not dependent on the batch size. Then, all the steady-state probabilities are derived using a recursive approach. **Findings:** We used M/M(a,b)/1 as the notation. For this model, steady-state solutions & characteristics are derived and explored. All the probabilities are expressed in terms of $P_{0,0}(0)$. The expected count of customers and waiting time depends on the interdependency, service rate, faster arrival rate, and slower arrival rate. According to each parameter, all the results are verified. **Novelty:** There are works related to bulk service in queuing theory, but this is a new approach to give a bridge between bulk service and controllable arrival rates along with interdependency in the arrival and service process.

Keywords: M/M(a; b)/1 Queueing System; Bulk Service; Controllable Arrival Rates; Steady States; Interdependent Model; Stochastic Processes

1 Introduction

In several real-world settings, queueing models serve as a foundation for the efficient design and analysis of diverse technological systems and estimations of system behavior, such as customer waiting time, the estimated number of customers, etc. Bailey was the first to propose bulk service (1954)[(1)]. Over time, the bulk service literature has expanded. These concepts are applicable in various contexts, including the transportation industry, where mass transit cars, elevators, and carriers are allnatural batch servers. Numerous real-life systems, such as those for telecommunication services, voice or data transfer, production, etc., may confront queuing issues. Messages to be conveyed over computer communication networks may consist of an arbitrary number of packets. Now this model is applicable in situations where arrivals are regulated by nonmanual robotic systems, similar to the scenarios encountered in bulk services.

When customers arrive, most queuing models immediately begin providing service. However, here in bulk service, service starts only when the count of customers reaches number a. Nevertheless, service will not commence until the number of customers surpasses a. The single-server bulk service models have prompted queue theorists to examine the performance characteristics of queuing systems in which a particular distribution does not constrain the service pattern. The extant literature on bulk service queuing theory encompasses investigations into both customer behaviour within the queue and the behaviour and attributes of the system's servers [1–7]. Presently, the challenge lies in devising a novel approach to the arrival rate that can be controlled in accordance with the rate's speed.

Numerous academics have significantly contributed to the bulk service queueing models due to their widespread applications. In queueing theory, several researchers deal with different types of services with other parameters and various arrivals in different models. Here it is a combined model as bulk service with control in arrivals. In previous studies, J. Medhi (2002) discussed bulk service systems [8]. Neuts M. F contributed to the idea of bulk queues in the literature in 1967 [9]. A. Srinivasan and M. Thiagarajan (2006) researched the controllable arrival rates in various queueing models [10] in that study, discussing the concept of the speed of arrival rates in some queueing models. Various studies have been conducted on queueing systems to elucidate the concept of bulk service. These investigations provide a comprehensive understanding of queueing systems, such as k-stage bulk service, heterogeneous bulk service, group service for impatient customers, performance analysis of dependent bulk service queues with server breakdowns, multiple vacation transient behaviours of bulk service queueing systems with standby servers, and others [11–17]. Additionally, Anyue Chen, Xiaohan Wu & Jing Zhang (2020) proposed "Markovian bulk-arrival and bulk-service queues with general state-dependent control" [18]. Finally, the present study deals with controllable arrivals by dividing the faster and slower rates in the bulk service queuing system, and it connects or makes a bridge between the bulk service to controlled arrivals. So all the probabilities can split according to the speed of arrivals with this concept. This interdependent model can apply to model the real-world situation to new queueing models. This model has controls for both service and arrival.

This study made an effort to examine the M/M(a,b)/1 model of interdependent queuing with controllable arrival rates. We defined the model and steady-state equations and derived model properties. We produced numerical data for system performance metrics to conform to the analytical conclusions and facilitate sensitivity analysis. Following is a summary of the queueing model research. This study describes the model and then the steady-states, formulation, and notation. After that, it covers the properties of the models and then provides illustrative findings for system performance indicators to conform to the analytical conclusions and simplify the sensitivity analysis.

2 Methodology

2.1 Model Description

This queuing system constitutes a single server and a limitless waiting space. The arrival and service completion process $\{X_1(t)\}$ and $\{X_2(t)\}$ of the system follow a bivariate Poisson process and are correlated, given that,

$P(X_1(t) = x_1, X_2(t) = x_2)$

$$= e^{-(\lambda_j+\mu-2\varepsilon)} \sum_{s=0}^{\min\{x_1,x_2\}} (\varepsilon t)^s \left[(\lambda_j-\varepsilon)t\right]^{x_1-s} \left[(\mu-\varepsilon)t\right]^{x_2-s} \frac{1}{s!\,(x_1-s)!\,(x_2-s)!} \tag{2.1}$$

where x_i= 0, 1, 2,…(values of i is 1 and 2); $\lambda_j > 0$; j = 0,1; $\mu > 0$; $0 \leq \varepsilon < \min(\lambda_j, \mu)$; j = 0, 1.

The parameters $\mu,\ \varepsilon,\ \lambda_0, and\ \lambda_1$ describe

μ = the mean service rate.

ε = the mean dependence rate (the covariance of $\{X_1(t)\}$ & $\{X_2(t)\}$).

λ_0 = the mean faster arrival rate.

λ_1 = the mean slower arrival rate.

FCFS is the queue discipline. Based on size [a, b], services are provided in batches. When the queue length reaches or surpasses a, and the capacity is b (≥a≥1), does service begin. A batch's service time distribution is considered exponential with parameter μ. The states of the system are denoted by (j, n), with n is the number of units in the queue, and j = 1 indicates the server in this system is busy serving a batch of size m (a≥ m ≥b), and j = 0 shows idle server. We consider the system's states.

Denote, $P_{j,n}(t)$ = Pr[the state of the system (j, n) at t (time)].

$P_{j,n}(t)$ is non zero Only for j = 1, n $\geq$0, and j = 0; 0 $\leq$ n $\leq$a -1.

The movement in the system specified in the arrival rate describes,

(i) As the system size increases from below F, the arrival rate, which was λ_0 until F- 1, goes downhill to λ_1 and stays there for the next up jump of the system size.

(ii) As the size of the system falloff to f($0 \leqslant f \leqslant F$) from above, the rate of arrival, which was λ_1 until f + 1, is more significant to λ_0 and remains λ_0 throughout the subsequent falling to 0 and rising to F-1. This procedure is repeated again and again.

Now, the steady-state probabilities (SSP) are as follows.

Let $P_{0,n}(0)$, describe the SSP that there are queued n customers when idle server and the system is at a faster arrival rate.

Let $P_{0,n}(1)$, describe the SSP that there are queued n customers when idle server and the system is at a slower arrival rate.

Let $P_{1,n}(0)$, describe the SSP that there are queued n customers when the busy server and system are at a faster arrival rate.

Let $P_{1,n}(1)$, describe the SSP that there are queued n customers when the busy server and system are at a slower arrival rate.

Clearly, the process N(t); t $\geq$ 0, where N(t) is the system size at time t, is a Markov chain with state space

{0,1, 2,…,f -1 f, f + 1,f+ 2,…,F -2,F -1,F,F+ 1,…} and F < a < b

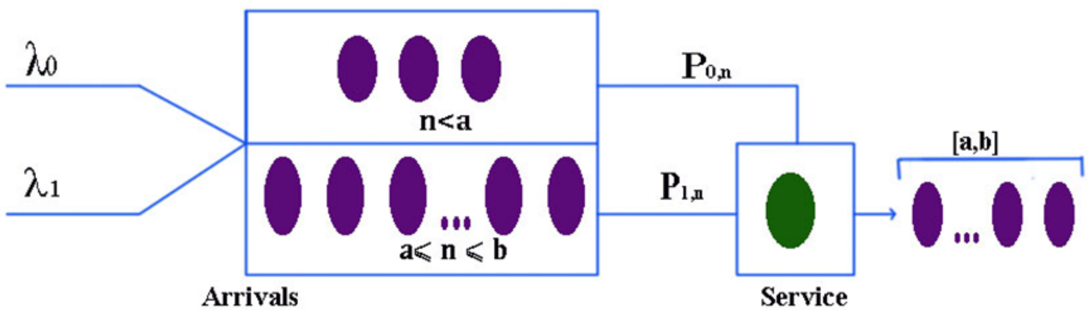

Fig 1. The schematic representation of the model

3 Results and Discussion

3.1 Steady-State Equations

The steady state means the states of any queuing system at the probability of the count of the clients in any queueing system being independent of time t. In "Fundamentals of queueing theory," Donald Gross and Carl M. Harris explained steady-state equations and illustrated them in some models[11]. Here we can see that $P_{j,n}(0)$ exists only when n = 0,1, 2, f-1, f; $P_{j,n}(1)$ exists only when n = F, F + 1,…∞: both $P_{j,n}(0)$ & $P_{j,n}(1)$ exists elsewhere where j = 0, 1 .

Assume that the steady state exists.

Let, $P_{i,n} = \lim_{t\to\infty} P_{i,n}(t)$

The steady-state equations become

$$-(\lambda_0+\mu-2\varepsilon)P_{1,n}(0)+(\lambda_0-\varepsilon)P_{1,n-1}(0)=0$$

$$(n=1,2,3\ldots f-1) \tag{3.1}$$

$$-(\lambda o+\mu-2\varepsilon)P_{1,f}(0)+(\lambda_0-\varepsilon)P_{1,f-1}(0)+(\mu-\varepsilon)P_{1,f+b}(1)=0 \tag{3.2}$$

$$-(\lambda_0+\mu-2\varepsilon)P_{1,n}(0)+(\lambda_0-\varepsilon)P_{1,n-1}(0)=0$$

$$(n=f+1,f+2,\ldots,F-1) \tag{3.3}$$

$$-(\lambda_1+\mu-2\varepsilon)P_{1,f+1}(1)+(\mu-\varepsilon)P_{1,f+1+b}(1)=0 \tag{3.4}$$

$$-(\lambda_1+\mu-2\varepsilon)P_{1,n}(1)+(\lambda_1-\varepsilon)P_{1,n-1}(1)+(\mu-\varepsilon)P_{1,n+b}(1)=0$$

$$(n=f+2,f+3,\ldots F-1) \tag{3.5}$$

$$-(\lambda_1+\mu-2\varepsilon)P_{1,F}(1)+(\lambda_1-\varepsilon)P_{1,F-1}(1)+(\lambda_0-\varepsilon)P_{1,F-1}(0)+(\mu-\varepsilon)P_{1,F+b}(1)=0 \tag{3.6}$$

$$-(\lambda_1-\mu+2\varepsilon)P_{1,n}(1)+(\lambda_1-\varepsilon)P_{1,n-1}(1)+(\mu-\varepsilon)P_{1,n+b}(1)=0$$
$$(n=F+1,F+2\ldots) \tag{3.7}$$

$$-(\lambda_0+\mu-2\varepsilon)P_{1,0}(0)+(\lambda_0-\varepsilon)P_{0,a-1}(0)=0 \tag{3.8}$$

$$-(\lambda_1+\mu-2\varepsilon)P_{1,0}(1)+(\lambda_1-\varepsilon)P_{0,a-1}(1)+(\mu-\varepsilon)\textstyle\sum_{j=a}^{b}P_{1,j}(1)=0 \tag{3.9}$$

$$-(\lambda_0-\varepsilon)P_{0,0}(0)+(\mu-\varepsilon)P_{1,0}(0)=0 \tag{3.10}$$

$$-(\lambda_0-\varepsilon)P_{1,f+1}(0)+(\mu-\varepsilon)P_{1,f+1}(1)=0 \tag{3.11}$$

$$-(\lambda_0-\varepsilon)P_{0,n}(0)+(\lambda_0-\varepsilon)P_{0,n-1}(0)+(\mu-\varepsilon)P_{1,n}(0)=0$$
$$(\mathrm{n}=1,2,3,\ldots,\mathrm{f}-1) \tag{3.12}$$

$$-(\lambda_0-\varepsilon)P_{0,f}(0)+(\lambda_0-\varepsilon)P_{0,f-1}(0)+(\mu-\varepsilon)P_{1,f}(0)+(\mu-\varepsilon)P_{1,f}(1)=0 \tag{3.13}$$

$$-(\lambda_0-\varepsilon)P_{0,n}(0)+(\lambda_0-\varepsilon)P_{0,n-1}(0)+(\mu-\varepsilon)P_{1,n}(0)=0$$
$$(\mathrm{n}=\mathrm{f}+1,\mathrm{f}+2,\ldots,\mathrm{F}-2) \tag{3.14}$$

$$-(\lambda_0-\varepsilon)P_{0,F-1}(0)+(\lambda_0-\varepsilon)P_{0,F-2}(0)=0 \tag{3.15}$$

$$-(\lambda_1-\varepsilon)P_{0,f+1}(1)+(\mu-\varepsilon)P_{1,f+1}(0)=0 \tag{3.16}$$

$$-(\lambda_1-\varepsilon)P_{0,n}(1)+(\lambda_1-\varepsilon)P_{0,n-1}(1)+(\mu-\varepsilon)P_{1,n}(1)=0$$

$$(n=f+2,f+3,\ldots,F-1) \tag{3.17}$$

$$-(\lambda_1-\varepsilon)P_{0,F}(1)+(\lambda_1-\varepsilon)P_{0,F-1}(1)+(\lambda_0-\varepsilon)P_{0,F-1}(0)+(\mu-\varepsilon)P_{1,F}(1)=0 \tag{3.18}$$

$$-(\lambda_1-\varepsilon)P_{0,n}(1)+(\lambda_1-\varepsilon)P_{0,n-1}(1)+(\mu-\varepsilon)P_{1,n}(1)=0$$
$$(\mathrm{n}=\mathrm{F}+1,\ \mathrm{F}+2,\ldots,\mathrm{a}-1) \tag{3.19}$$

3.2 Computation Of Steady-State Solutions

Let $\frac{\lambda_0-\varepsilon}{\mu-\varepsilon}=\rho_0,\ \ \frac{\lambda_1-\varepsilon}{\mu-\varepsilon}=\rho_1$ and both ρ_0 and ρ_1 less than 1 then all the steady states exist.

Now, from equation (3.11)

$$P_{1,0}(0)=\rho_0 P_{0,0}(0) \tag{3.2.1}$$

Recursively using equation (3.2.1) in equations (3.1),(3.2) and (3.3), we get

$$P_{1,n}(0)=\frac{{\rho_0}^{n+1}}{(\rho_0+1)^n}P_{0,0}(0)$$
$$(\mathrm{n}=1,2,3,\ldots,\mathrm{F}-1) \tag{3.2.2}$$

Let E denote the displacement operator and it is given by

$$EP_{i,j}(x)=P_{i,j+1}(x)$$

From equation (3.6),(3.7) and (3.8)

$$-(\rho_1+1)EP_{1,n-1}(1)+\rho_1 P_{1,n-1}(1)+E^{b+1}P_{1,n-1}(1)=0$$

$$(\mathrm{n}=\mathrm{f}+1,\mathrm{f}+2,\mathrm{f}+3\ldots)$$

Or $A(EP_{1,n}(1))=0$

With characteristic equation,

$$A(z)\equiv z^{b+1}-(\rho_1+1)z+\rho_1=0$$

Now, let $A_1(z)=-(\rho_1+1)z\ \ and\ \ A_2(z)=z^{b+1}+\rho_1$

From the circle $|z|=1-\xi$ such that ξ is arbitrarily small. Now, $z=(1-\xi)e^{i\theta}$It could be delineated as in the contour of the here-mentioned circle $|A_2(z)_2(z)<(A_1(z)|$ Then by Rouche's theorem, $A_1(z)$ and $A_1(z)$ + $A_2(z)$, the total count of zeroes will be the same inside of the circle $|z|=1-\xi$ Here $A_1(z)$ is only one zero within the circle. So the number of zeros of $A(z)\equiv A_1(z)$ + $A_2(z)$ also only one within $|z|=1-\xi$ .This zero of A(z) will be unique and real, implies and implied by $\rho=\frac{1}{b}\rho_1<1$, and this is denoted by $\omega(0<\omega<1)$ and the other roots are denoted by $\omega_1,\omega_2,\omega_3\ldots\omega_b\quad|\omega_i|\geq 1$, then ω satisfies the equation

$$b\rho=\rho_1=\frac{\omega\left(1-\omega^b\right)}{1-\omega}=\omega+\omega^2+\omega^3+\cdots+\omega^b$$

When $0<\rho<1$, we have $\rho\leq\omega\leq\rho^{\frac{2}{b+1}}$ It will lead to finding the value for ω.

Thus $P_{1,n}(1)=\alpha\omega^n+\sum_{i=1}^{b}\alpha_i\omega_i^n$ for n= 0,1,2… where $\alpha_i's$ are constants.

Again $\sum_{n=0}^{\infty}P_{1,n}(1)<1$ We must have $\alpha_i=0\forall i$ which implies $P_{1,n}(1)=\alpha\omega^n$

From equation (3.11)

$$P_{1,r+1}(1)=\frac{{\rho_0}^{f+3}}{(\rho_0+1)^{f+1}}P_{0,0}(0) \tag{3.2.3}$$

Now,

$$P_{1,n}(1)=\left(\frac{\rho_0^2}{\omega(\rho_0+1)}\right)^{f+1}\omega^n P_{0,0}(0)$$
$$(n=f+1,f+2,\ldots)$$

From equation (3.2.2) and (3.2.4)

$$P_{1,n}=P_{1,n}(0)+P_{1,n}(1)$$

$$\sum_{n=0}^{\infty} P_{1,n} = \sum_{n=1}^{F-1} P_{1,n}(0) + \sum_{n=f+1}^{\infty} P_{1,n}(1)$$

From equation (3.8), we have,

$$P_{0,a-1}(0) = (\rho_0 + 1) P_{0,0}(0) \tag{3.2.5}$$

From equation (3.12) n= a-1,a-2,…,1 recursively using (3.2.5) we get,

$$P_{0,n}(0) = \left((\rho_0+1) - (\rho_0+1)\left(\left(\frac{\rho_0}{(\rho_0+1)}\right)^F + \left(\frac{\rho_0}{(\rho_0+1)}\right)^{n+1} \right) \right\} P_{0,0}(0)$$

$$(\mathrm{n} = 1,2,3,\ldots,\mathrm{F}-1) \tag{3.2.6}$$

Using equation (3.9) in (3.17), (3.18), and (3.19), we obtain,

$$P_{0,n}(1) = \frac{1}{\rho_1}\left(\frac{{\rho_0}^2}{\omega(\rho_0+1)}\right)^{f+1} \left((\rho_1+1) - \frac{\omega^{n+1}\left(1-\omega^{b-n}\right)}{1-\omega} \right) P_{0,0}(1)$$

$$(n = f+1, f+2, \ldots, a-1) \tag{3.2.7}$$

Now, $P_{0,n} = P_{0,n}(0) + P_{0,n}(1)$

$$\sum_{n=1}^{a-1} P_{0,n} = \sum_{n=1}^{F-1} P_{0,n}(0) + \sum_{n=f+1}^{\infty} P_{0,n}(1)$$

We observed that every SSP of the system is defined by $P_{0,0}(0)$ values.

3.3 The Model's Characteristics

Here expected and analytical results are derived for the system.

Now,$P_{1,n} + P_{0,n} = 1$

$$\begin{aligned} &\left[\sum_{n=1}^{F-1} \frac{{\rho_0}^{n+1}}{(\rho_0+1)^n} + \sum_{n=f}^{\infty}\left(\frac{{\rho_0}^2}{\omega(\rho_0+1)}\right)^{f+1}\omega^n \right.\\ &+\sum_{n=1}^{F-1}\left\{(\rho_0+1)-(\rho_0+1)\left(\left(\frac{\rho_0}{(\rho_0+1)}\right)^F+\left(\frac{\rho_0}{(\rho_0+1)}\right)^{n+1}\right)\right\}\\ &\left.+\sum_{n=f+1}^{a-1}\frac{1}{\rho_1}\left(\frac{\rho_0^2}{\omega(\rho_0+1)}\right)^{f+1}\left((\rho_1+1)-\frac{\omega^{n+1}\left(1-\omega^{b-n}\right)}{1-\omega}\right)\right] P_{0,0}(0) = 1\\ &\Rightarrow MP_{0,0}(0) = 1 \end{aligned} \tag{3.3.1}$$

Where,

M =

$$\begin{aligned} &\left[\sum_{n=1}^{F-1} \frac{{\rho_0}^{n+1}}{(\rho_0+1)^n} + \sum_{n=f}^{\infty}\left(\frac{{\rho_0}^2}{\omega(\rho_0+1)}\right)^{f+1}\omega^n \right.\\ &+\sum_{n=1}^{F-1}\left\{(\rho_0+1)-(\rho_0+1)\left(\left(\frac{\rho_0}{(\rho_0+1)}\right)^F+\left(\frac{\rho_0}{(\rho_0+1)}\right)^{n+1}\right)\right\} \end{aligned}$$

$$+\sum_{n=f+1}^{a-1}\frac{1}{\rho_1}\left(\frac{\rho_0^2}{\omega(\rho_0+1)}\right)^{f+1}\left((\rho_1+1)-\frac{\omega^{n+1}(1-\omega^{b-n})}{1-\omega}\right)\Bigg]$$

Hence,

$P_{0,0}(0)=M^{-1}$

$P_\lambda(0)$ represents the probability that this system will have a faster arrival rate. And it is given by

$$P_\lambda(0)=\sum_{n=0}^{F-1}(P_{0,n}(0)+P_{1,n}(0))=$$

$$\left\{\sum_{n=1}^{F-1}\left\{(1+\rho_0)-(1+\rho_0)\left(\left(\frac{\rho_0}{(1+\rho_0)}\right)^F+\left(\frac{\rho_0}{(1+\rho_0)}\right)^{n+1}\right)\right\}+\sum_{n=1}^{F-1}\frac{{\rho_0}^{n+1}}{(\rho_0+1)^n}\right\}P_{0,0}(0) \tag{3.3.2}$$

$P_\lambda(1)$ represents the probability that this system will have a slower arrival rate. And it is given by

$$\begin{aligned}P_\lambda(1)&=\sum_{n=f+1}^{\infty}(P_{0,n}(1)+P_{1,n}(1))\\&=\Bigg[\sum_{n=f+1}^{a-1}\frac{1}{\rho_1}\left(\frac{\rho_0^2}{\omega(\rho_0+1)}\right)^{f+1}\left((\rho_1+1)-\frac{\omega^{n+1}(1-\omega^{b-n})}{1-\omega}\right)\\&\quad+\sum_{n=f}^{\infty}\left(\frac{\rho_0^2}{\omega(\rho_0+1)}\right)^{f+1}\omega^n\Bigg]P_{0,0}(0)\end{aligned} \tag{3.3.3}$$

Now, the probability that the count of units in the system between f and a - 1 can be expressed as

$$P(f\le n\le a-1)=\sum_{n=f}^{F-1}P_{1,n}(0)+\sum_{n=f+1}^{a-1}P_{1,n}(1)+\sum_{n=f}^{F-1}P_{0,n}(0)+\sum_{n=f+1}^{a-1}P_{0,n}(1)$$

$$P(f\le n\le a-1)=TP_{0,0}(0) \tag{3.3.4}$$

Where,

$$\begin{aligned}T=&\Bigg[\sum_{n=1}^{F-1}\frac{{\rho_0}^{n+1}}{(\rho_0+1)^n}+\sum_{n=f}^{a-1}\left(\frac{{\rho_0}^2}{\omega(\rho_0+1)}\right)^{f+1}\omega^n\\&+\sum_{n=1}^{F-1}\left\{(\rho_0+1)-(\rho_0+1)\left(\left(\frac{\rho_0}{(\rho_0+1)}\right)^F+\left(\frac{\rho_0}{(\rho_0+1)}\right)^{n+1}\right)\right\}\\&+\sum_{n=f+1}^{a-1}\frac{1}{\rho_1}\left(\frac{\rho_0^2}{\omega(\rho_0+1)}\right)^{f+1}\left((\rho_1+1)-\frac{\omega^{n+1}(1-\omega^{b-n})}{1-\omega}\right)\Bigg]\end{aligned}$$

Now,

Conditional probability $P(0\mid f\le n\le a-1)$ that this system is in a faster arrival rate when the size of the system lies between f and a - 1 is given by

$$P(0\mid f\le n\le a-1)=\frac{\sum_{n=f}^{a-1}(P_{0,n}(0)+p_{1,n}(0))}{TP_{0,0}(0)} \tag{3.3.5}$$

Now,

Conditional probability $P(1\mid f\le n\le a-1)$ that this system is in a slower arrival rate when the size of the system lies between f and a -1 is given by

$$P(1\mid f\le n\le a-1)=\frac{\sum_{n=f}^{a-1}(P_{0,n}(1)+P_{1,n}(1))}{TP_{0,0}(0)} \tag{3.3.6}$$

The expected count of consumers utilizing this system L_s is indicated by the sum L_{s_0} - The expected count of units or customers utilizing this system when the rate of arrivals is faster and L_{s_1} - The expected count of units or customers utilizing this system when the rate of arrivals is slower.

$$L_s = L_{s_0} + L_{s_1}$$

Where,

$$L_{s_0} = \sum_{n=0}^{F-1} n\left(P_{0,n}(0) + P_{1,n}(0)\right\} \tag{3.3.7}$$

$$L_{s_1} = \sum_{n=0}^{\infty} n\left(P_{1,n}(1) + P_{0,n}(1)\right\} \tag{3.3.8}$$

Now, from Little's formula to this model, the expected waiting time of the consumer utilizing the system can be computed by

$$w_s = \frac{L_s}{\bar{\lambda}} \tag{3.3.9}$$

Where, $\bar{\lambda} = \lambda_0\left(P_{0,n}(0) + P_{1,n}(0)\right) + \lambda_1\left(P_{0,n}(1) + P_{1,n}(1)\right)$

When λ_0 approaches to λ_1, the $\bar{\lambda}$ becomes λ .

3.4 Numerical Illustrations

In this section, the queuing system is numerically and graphically illustrated with the values of $P_{0,0}(0)$, $P_\lambda(0)$, $P_\lambda(1)$, L_s *and* W_s for various values of λ_0, λ_1, μ *and* ε. Using the above-obtained equations for each value.

Let f = 4, F = 8, a = 10 and b = 15

Table 1. Numerical Illustrations

λ_0	λ_1	μ	ε	$P_{0,0}(0)$	$P_\lambda(0)$	$P_\lambda(1)$	L_s	W_s
8	6	10	0.5	0.0799	0.7254	0.27455	61.9062	0.6182
8	6	12	0.5	0.0865	0.9311	0.062299	54.8724	0.5933
8	6	14	0.5	0.0919	0.9832	0.016718	50.1233	0.5755
8	5	12	0	0.0858	0.9571	0.042864	56.0607	0.6009
8	5	12	0.25	0.0861	0.9633	0.036681	55.7444	0.6001
8	5	12	0.75	0.0869	0.9741	0.025826	55.0734	0.5983
8	5	12	1	0.0873	0.9788	0.021158	54.7139	0.5973
7	5	10	0.25	0.0845	0.9126	0.087318	57.0325	0.6881
6	4	10	0.5	0.0905	0.9858	0.014193	51.5485	0.7775
5	3	9	0.5	0.0934	0.996	0.003946	49.3151	0.9214
8	6	9	0.5	0.076	0.37655	0.62345	66.8228	0.6346

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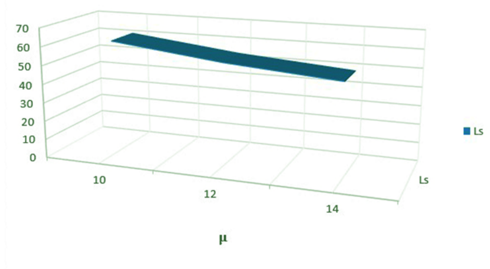

Fig 2. L_s by varying service rates, μalso, other parameters are unaltered

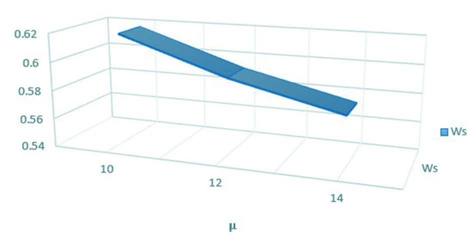

Fig 3. W_s by varying service rates, μ also, other parameters are unaltered.

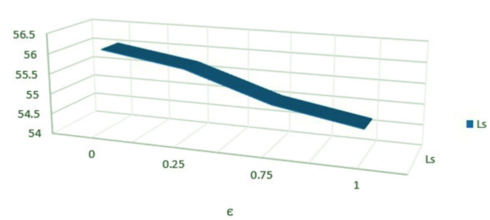

Fig 4. L_s by varying dependence rate, ε also, other parameters are unaltered

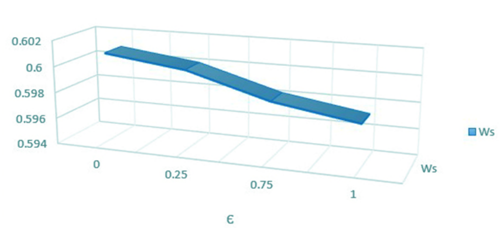

Fig 5. W_s by varying dependence rate, ε also, other parameters are unaltered

4 Conclusion

The present research presents a novel methodology to handle controllable arrival rates, which encompasses both faster and slower rates in bulk service as well as interdependence in the arrival and service processes. Probabilistic outcomes and associated traits are determined based on faster and slower arrival rates, thereby providing valuable insights for future research endeavors. As a potential avenue for future research, this model exhibits the capacity to engage in mathematical modeling of real-world scenarios. This particular model serves as a fundamental framework for the purpose of evaluating and advancing comparable queuing models of this kind. This model encompasses the previous iterations as specific instances. For instance, when the value of b becomes 1 with finite capacity this model is reduced to the M/M/1/K Interdependent queueing model with controllable arrival rate by M. Thiagarajan and A. Srinivasan also when $\lambda_0$approaches λ_1 And ε equals zero, this model refers to the traditional M/M(a,b)/1 model described by J Medhi (2006) in Stochastic Models in Queueing Theory. The numerical illustration demonstrates that. L_s and W_s decreases as the service rate increases while all other parameters remain unaltered. The average dependency rate increases; others are unaltered, L_s and W_s drop. Simply this study is a bridge between bulk service to controllable arrival rates. It is capable of further modifications.

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