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A Note on Separation Axioms in Kasaj Topological Spaces

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Abstract

Objectives: The separation axioms are about the use of kasaj topological means to distinguish disjoint sets and distinct points and the purpose of this work is to investigate the spaces namely KSgp[KSgsp]-T_0 spaces, KSgp[KSgsp]-T_1 spaces, KSgp[KSgsp]-T_2 spaces by utilizing KSgp[KSgsp]-open sets in Kasaj topological spaces. Also, we discuss their relationship with existing concepts in Kasaj topological spaces. **Methods:** Any two Kasaj topologically distinguishable points must be distinct, and any two separated points must be Kasaj topologically distinguishable. **Finding:** This study consists of three separation axioms namely $KS_{gp}[KS_{gsp}]$-T_0 spaces, $KS_{gp}[KS_{gsp}]$-T_1 spaces, $KS_{gp}[KS_{gsp}]$-T_2 spaces by utilizing $KS_{gp}[KS_{gsp}]$-open sets in Kasaj topological spaces. **Novelty:** This study uses their relationship with existing concepts in Kasaj topological spaces.

AMS Classification: 54A05,54B05,54A99.

Keywords: KSgp-T_0 spaces; KSgsp-T_0 space; KSgp-T_1 spces; KSgsp-T_1 spaces; KSgp-T_2 spaces; KSgsp-T_2 spaces

1 Introduction

In 2021, Abdelwaheb Mhemdi, Tareq M. Al-shami[(1)] introduced the concept of functionally separation axioms on general topology. In 2021 Das & Pramanik[(2)] introduced the concept of ultra neutrosophic set. In 2022, Das and Tripathy[(3)] introduced the concept of separation axioms on spatial topological spaces. In 2022, Das and Tripathy,[(4)] introduced the concept of rough pentapartioned neutrosophic set. In 2022, Das &Tripathy[(5)] introduced the concept of neutrosophic pre-I-open set. In 2022, Das & Tripathy[(6)] introduced the concept of neutrosophic set and systems. In 2020, Kashyap G. Rachchh and Sajeed[(7)] introduced the concept of kasaj topological spaces. Further, the same year auothers[(8)] introduced the concept of Kasaj generalized closed sets in Kasaj topological spaces. In 2022, Sathishmohan[(9)] et.al introduced the new type of closed sets known as KSgp(KSgsp)-closed sets in Kasaj topological spaces and obtained many interesting results. In 2023, Li, PY., Liu, WL., Mou,[(10)] et al. introduced the concept of separation axioms of topological rough groups. In this paper, we shall define KS-T_0 spaces, KS-T_1 spaces, KS-T_2 spaces, KS pre-T_0 spaces, KS pre-T_1 spaces, KS pre-T_2 spaces, KS_α -T_0 spaces KS_α -T_1 spaces, KS_α-T_2 spaces, KS_β-T_0 spaces, KS_β-T_1 spaces, KS_β-T_2 spaces, KS_g $[KS_{gp}, KS_{gsp}]$-T_0 spaces, KS_g $[KS_{gp}, KS_{gsp}]$-T_1 spaces, KS_g $[KS_{gp}, KS_{gsp}]$-T_2 spaces in Kasaj topological spaces.

2 Methodology

In this study, the following methodologies are used.

In KSgp[KSgsp]-T_0 spaces, KSgp[KSgsp]-T_1 spaces, if we take each pair of distinct points and then there exists KSgp[KSgsp]-open sets such that the distinct points are belongs to any one of the KSgp[KSgsp]-open sets in KSgp[KSgsp]-T_0 spaces, KSgp[KSgsp]-T_1 spaces. But in KSgp[KSgsp]-T_2 spaces, if we take each pair of distinct points and then there exists KSgp[KSgsp]-open sets such that the distinct points are belongs to any one of the KSgp[KSgsp]-open sets and so intersection of two KSgp[KSgsp]-open sets are empty.

3 Result and discussion

3.1 Properties of $KS_{gp}[KS_{gsp}]$-T_0 Spaces

The objective of this section is to introduce and investigate the concepts of KS_{gp} $[KS_{gsp}]$-T_0 Spaces in Kasaj Topological Spaces.

Definition 3.1 .1. A Kasaj Topological Spaces U is called

(i) KS-T_0 iff to each pair of distinct points x,y ∈ U, there exists a KS-open set G such that either x ∈ G and y ∉G or x∉G and y ∈ G.

(ii) KS-pre- T_0 iff to each pair of distinct points x,y∈ U, there exists a KS-pre-open set G such that either x ∈ G and y ∉G or x∉G and y ∈ G.

(iii) KS_β-T_0 iff to each pair of distinct points x,y ∈ U, there exists a KS_β-open set G such that either x ∈ G and y ∉G or x∉G and y ∈ G.

(iv) KS_{gp}-T_0 iff to each pair of distinct points x,y ∈ U, there exists a KSgp-open set G such that either x∈ G and y ∉ G or x∉G and y ∈G.

(v) KS_{gsp}-T_0 iff to each pair of distinct points x,y ∈ U, there exists a KSgsp-open set G such that either x ∈ G and y ∉ G or x∉G and y ∈G.

Theorem 3.1.2. In a Kasaj Topological Spaces (U, $\tau_R(X)$, $KS_R(X)$). Then

(i) Every KS-T_0 space is (KS pre-T_0, KS_α-T_0, KS_β-T_0, KS_g-T_0) KS_{gp}-T_0.

(ii) Every KS pre- T_0 space is (KS_β-T_0) KS_{gp}-T_0.

(iii) Every KS_α-T_0 space is (KS pre-T_0, KS_β -T_0) KS_{gp}-T_0

(iv) Every KS_{gp}-T_0 space is KS_{gp}-T_0.

Proof:

(i) Suppose U is KS- T_0 space. Let x and y be two distinct points in U. Since U is KS- T_0, there exists an KS-open set G containing either x ∈ G and y ∉ G or x∉ G and y ∈ G. Since every KS-open set is (KS pre-open, KS_α- open, KS_β- open, KS_g- open) KSgp-open, G is (KS pre-open, KS_α- open, KS_β- open, KS_g- open) KSgp-open. Hence, U is (KS pre-T_0, KS_α-T_0, KS_β-T_0, KS_g-T_0) KSgp- T_0.

Proofof (ii), (iii), (iv) are similar to (i).

Converse of the above the theorem need not be true as shown in the following example.

Example 3.1.3. Let U = {a,b,c,d,e}, with U\R = {b ,d},{c, e},{a}} and X = {a, b}. Then the nano topology, $\tau_R(X)$ = {U, ∅.{a},{a b,d},{b,d}}. S = {b}, S$'$ = {a, c, d, e}. Then x = {b}, y = {e} it is KS_{gp}-T_0-T_0 but not in KS-T_0, KS pre-T_0, KS_β-T_0, KSg-T_0.

Theorem 3.1.4. If U is a KS_{gp}-T_0 space and V is a subspace of U, then V is also KS_{gp}-T_0 space.

Proof:

Let U be a KS_{gp}-T_0 space and V be a subspace of U. To show that V is KS_{gp}-T_0 space. Let x,y ∈ V with x ∉y. Since V ⊆U, we have x,y ∈ U. But U is KS_{gp}-T_0, so there exists an KS_{gp}-open set G such that G contains only one of them, say, x ∈ G and y∉ G. Then V ∩ G is KS_{gp}-open set in V such that x ∈ V ∩ G and y∉ V ∩ G. Hence V is KS_{gp}-T_0 space.

Theorem 3.1.5. A Kasaj Topological Spaces U is KS_{gp}-T_0 space iff the KS_{gp}-closures of distinct points are distinct.

Proof:

Assume that U is a KS_{gp}-T_0 space. Let x,y ∈ U with x ≠ y. Since U is KS_{gp}-T_0, there is an KS_{gp}-open set G such that G contains only one of them, say, x ∈G and y∉ G. Since G is an KS_{gp}-open set containing x and G∩ (y) = ∅, we have x∉KS_{gpcl} ({y}). But x ∈KS_{gpcl}({x}). So KS_{gpcl}({x}) ≠KS_{gpcl}({y}).

Conversely, assume that for x$\neq$ y, KS_{gpcl}({x}) $\neq KS_{gpcl}$({y}). So there exists z $\notin KS_{gpcl}$({x}) such that z$\notin KS_{gpcl}$({y}). Now if x $\in KS_{gpcl}$({y}), then {x}$\subseteq KS_{gpcl}$({y}) this implies that KS_{gpcl}({x})$\subseteq$ KS_{gpcl}({y}) which implied z $\in$ KS_{gpcl}({y}). Since z $\in$ KS_{gpcl} ({x}). This is a contradiction. So x$\in$ KS_{gpcl}({y}) = F, this implies x $\in$ F^c, since y $\in$ F, y $\notin$ F^c. Thus, F^c is an KS_{gp}-open set containing x but not y. Hence, U is KS_{gp}-T_0.

Theorem 3.1.6. In a Kasaj Topological Spaces (U, $\tau_R(X)$, $KS_R(X)$). Then

(i) Every KS-T_0 space is KS_{gsp}-T_0.

(ii) Every KS pre- T_0 space is KS_{gsp}-T_0.

(iii) Every $KS_\alpha - T_0$ space is $KS_{gsp} - T_0$.

(iv) Every KS_β-T_0 space is KS_{gsp}-$T_{0.}$

(v) Every KS_g-T_0 space is KS_{gsp}-T_0.

(vi) Every KS_{gp}-T_0 space is KS_{gsp}-T_0.

Proof:

It is obvious from theorem 3.1.2.

Converse of the above the theorem need not be true as shown in the following example.

Example 3.1.7. Let U = {a,b,c,d,e}, with U\R = {a,e},{c, d},{b}} and X = {b,c}. Then the nano topology, $\tau_R(X)$ = {U, $\varnothing$,{b},{b,c,d},{c,d}}. S = {c}, S' = {a, b, d, e}. Then x = {a}, y = {c} it is KS_{gsp}-T_0 but not in KS-T_0, KS pre-T_0, KS_α-$T_{0,}KS_\beta$-T_0, KSg-T_0, KS_{gp}-$T_{0.}$

Theorem 3.1.8. If U is a KS_{gsp}-T_0 space and V is a subspace of U, then V is also KS_{gsp}-T_0 space.

Proof:

Let U be a KS_{gsp}-T_0 space and V be a subspace of U. To show that V is KS_{gsp}-T_0 space. Let x,y $\in$ V with x $\notin$y. Since V $\subseteq$U, we have x,y $\in$ U. But U is KS_{gsp}-T_0, so there exists an KS_{gsp}-open set G such that G contains only one of them, say, x $\in$ G and y$\notin$ G. Then V $\cap$ G is KS_{gsp}-open set in V such that x $\in$ V $\cap$ G and y$\notin$ V $\cap$ G. Hence V is KS_{gsp}-T_0 space.

Theorem 3.1.9. A Kasaj Topological Spaces U is KS_{gsp}-T_0 space iff the KS_{gsp}-closures of distinct points are distinct.

Proof:

Assume that U is a KS_{gsp}-T_0 space. Let x,y $\in$ U with x $\neq$ y. Since U is KS_{gsp}-T_0, there is an KS_{gsp}-open set G such that G contains only one of them, say, x $\in$ G and y$\notin$ G. Since G is an KS_{gsp}-open set containing x and G$\cap$ (y) = $\varnothing$, we have x$\notin KS_{gspcl}$ ({y}). But x $\in KS_{gspcl}$({x}). So KS_{gspcl}({x}) $\neq KS_{gspcl}$({y}).

Conversely, assume that for x$\neq$y, KS_{gspcl}({x}) $\neq KS_{gspcl}$({y}). So there exists z $\notin KS_{gspcl}$({x}) such that z$\notin KS_{gspcl}$({y}). Now if x $\in KS_{gspcl}$({y}), then {x}$\subseteq KS_{gspcl}$({y}) this implies that KS_{gspcl}({x})$\subseteq$ KS_{gspcl}({y}) which implied z $\in$ KS_{gspcl}({y}). Since z $\in$ KS_{gspcl} ({x}). This is a contradiction. So x$\in$ KS_{gspcl}({y}) = F, this implies x $\in$ F^c, since y $\in$ F, y $\notin$ F^c. Thus, F^c is an KS_{gsp}-open set containing x but not y. Hence U is KS_{gsp}-T_0.

3.2 Properties of KS_{gp}[KS_{gsp}]-T_1 Spaces

Developing and analyzing the concepts of KS_{gp} [KS_{gsp}]-T_1 Spaces in Kasaj Topological Spaces is the purpose of this study.

Definition 3.2.1. A Kasaj Topological Spaces U is called

(i) KS- T_1 space if whenever x and y are distinct points in U, there is an KS-open set F and G such that x$\in$ F and y$\notin$ F, and y $\in$ G and x$\notin$ G.

(ii) KS pre- T_1 space if whenever x and y are distinct points in U, there is an KS-pre-open set F and G such that x $\in$ F and y$\notin$F, and y $\in$ G and x $\notin$G.

(iii) KS_β- T_1 space if whenever x and y are distinct points in U, there is an KS_β-open set F and G such that x $\in$ F and y$\notin$ F, and y $\in$ G and x $\notin$G.

(iv) KS_{gp}-T_1 space if whenever x and y are distinct points in U, there is an KS_{gp}-open set F and G such that x $\in$ F and y$\notin$F, and y$\in$ G and x $\notin$G.

(v) KS_{gsp}-T_1 space if whenever x and y are distinct points in U, there is an KS_{gsp}-open set F and G such that x $\in$ F and y $\notin$F, and y $\in$ G and x $\notin$G.

Theorem 3.2.2. In a Kasaj Topological Spaces (U, $\tau_R(X)$, $KS_R(X)$). Then

(i) Every KS-T_1 space is (KS pre-T_1, KS_α-T_1, KS_β-T_1, KS_g-T_1) KS_{gp}-T_1.

(ii) Every KS pre-T_1 space is (KS_β- T_1) KS_{gp}-T_1.

(iii) Every $KS_\alpha - T_1$ space is $(KS\ pre - T_1,\ KS_\beta - T_1)\ KS_{gp} - T_1$.

(iv) Every $KS_g - T_1\ space\ is\ KS_{gp} - T_1$.

Proof:

(i) Suppose U is KS-T_1 space. Let x and y be two distinct points in U. Since U is KS-T_1, there exists an KS-open sets F and G such that x $\in$ F and y$\notin$ F, and y $\in$ G and x $\notin$ G. Since every KS-open set is (KS-pre-open, KS_α-open, KS_β-open, KS_g-open)

KS_{gp}-open, F and G are KS-open set is (KS-pre-open, KS_{α}-open, KS_{β}-open, KS_{g}-open) KS_{gp}-open. Hence, U is KS_{gp}-T_1.

Proof of (ii), (iii), (iv) are similar to (i).

Converse of the above the theorem need not be true as shown in the following example.

Example 3.2.3. Let U = {a,b, c,d, e}, with U\R = {{d, e} ,{a, b},{c}} and X = {a, c}. Then the nano topology, $\tau_R(X)$ = {U, ∅,{c},{a,b,c},{a,,b}. S = {a}, S' = {b, c, d, e}. Then x = {a}, y = {b} it is KS_{gp}-T_1 but not in KS-T_1, KS pre-T_1, KS_{α}-T_1, KS_{g}-T_1.

Theorem 3.2.4. Let (U, $\tau_R(X)$, $KS_R(X)$) be a Kasaj Topological Space, then for each KS_{gp}-T_1 space is KS_{gp}-T_0 space.

Proof:

Let U be a KS_{gp}-T_1 space and let x and y be two distinct points of U, there exists KS_{gp}-open sets G and H such that x ∈ G, y ∉G and x∉H, y ∈ H. We have x ∈ G, y ∉G. Therefore, U is KS_{gp}-T_0 space.

Theorem 3.2.5. If U is a KS_{gp}-T_1 space and V is a subspace of U, then V is also KS_{gp}-T_1 space.

Proof:

Let (U, $\tau_R(X)$, $KS_R(X)$) be a KS_{gp}-T_1 space and (V, $\tau_R(Y)$, $KS_R(Y)$) be a subspace U. Let x,y ∈ V with x ∉ y. Since V ⊆ U, we have x,y∈ U. Since U is KS_{gp}-T_1, then there exists KSgp-open sets G and H such that x ∈ G, y ∉ G and y ∈ H, x∉ H. Let M = V ∩ G and N = V ∩ H. Then M and N are KSgp-open sets in V. Also x ∈ M, y∉ M and y ∈ N, x ∉ V. Hence, (V, $\tau_R(Y)$, $KS_R(Y)$) is also a KS_{gp}-T_1 space.

Theorem 3.2.6. A Kasaj Topological Space (U, $\tau_R(X)$, $KS_R(X)$) is KS_{gp}-T_1 iff the each one-point set is KSgp-closed.

Proof:

Let (U, $\tau_R(X)$, $KS_R(X)$) be a KS_{gp}-T_1 space. Let x ∈ U, Then for each y ∈ U-{x} there exists a KSgp- open set F such that y ∈ F and x ∉ F, the sets {x} and F are disjoint. That is {x}∩ F = ∅ this implies that F⊆ U-{x}. Thus, y 2∈ F ⊆ U-{x} this implies that U-{x} is an KS_{gp}-open set this implies that {x} is KS_{gp}-closed in U.

Conversely, suppose {x} is KS_{gp}-closed. Let x,y ∈ U with x ∉ y. So U-{x} is an KS_{gp}-open set containing y but not x. Also, U-{y} is an KSgp-open set containing x but not y. Hence, U is a KS_{gp}-T_1 space.

Theorem 3.2.7. A Kasaj Topological Space (U, $\tau_R(X)$, $KS_R(X)$) is KS_{gp}- T_1 iff every finite subset of U is KS_{gp}-closed set in U.

Proof:

Assume that U is KS_{gp}-T_1. Let G be a finite subset of U. Let G = {x_1, x_2,….,x_n}. Then G = {x_1}∪ {x_2}….,∪{x_n} is KSgp-closed, being a finite union of KS_{gp}-closed sets. Conversely, let each finite subset of U is KS_{gp}-closed in U. Then {x} is KS_{gp}-closed, since it is finite. Since each singleton is KS_{gp}-closed, U is KS_{gp}- T_1.

Theorem 3.2.8. A Kasaj Topological Space (U, $\tau_R(X)$, $KS_R(X)$) is KS_{gp}-T_1 iff the intersection of all KS_{gp}-neighbourhoods of any point x in U is the singleton {x}.

Proof:

Assume that U is KS_{gp}-T_1. Let x ∈ U. Let G be the intersection of all KS_{gp}-neighbourhoods of x. Let y be any point in U different from x. Since U is KS_{gp}-T_1, there exists a KS_{gp}-neighbourhood M of x such that y ∉ M. Since y∉ M, we have y ∉ G. Since G is the intersection of all KS_{gp}-neighbourhoods of x. Since y ∉ G, no point different from x in G. Hence G = {x}.

Conversely, assume that intersection of all KS_{gp}-neighbourhoods of x in U is {x}. To prove that U is KS_{gp}-T_1, let x,y ∈ U with x ∉ y.

Let G = ∩ {all KS_{gp}-neighbourhoods of x in U}. Then G = {x}.

Let H = ∩ {all KS_{gp}-neighbourhoods of y in U}. Then H = {y}.

Since y ∉ x, we have y ∉ G then there exists KS_{gp}-neighbourhoods J of x with y ∉ J. Since x ∉ y, x∉ K this implies that KS_{gp}-neighbourhoods y ∈ K with x ∉ K. Hence U is KS_{gp}-T_1.

Theorem 3.2.9. In a Kasaj Topological Spaces (U, $\tau_R(X)$, $KS_R(X)$). Then

(i) Every KS-T_1 space is KS_{gsp}-T_1.

(ii) Every KS pre-T_1 space is KS_{gsp}-T_1.

(iii) Every KS_{α}-T_1 space is KS_{gsp}-T_1.

(iv) Every KS_{β}-T_1 space is KS_{gsp}-T_1.

(v) Every KS_{g}-T_1 space is KS_{gsp}-T_1.

Proof:

It is obvious from theorem 3.2.2.

Converse of the above the theorem need not be true as shown in the following example.

Example 3.2.10. Let U = {a,b c,d,e}, with U\R ={ {a,d},{b,e},{c}} and X = {a,c}. Then the nano topology, $\tau_R(X)$ = {U, ∅, {c},{a,c,d},{a, d}}. S = {a}, S' = {b, c, d, e}. Then x = {a}, y = {b} it is KS_{gsp}-T_1 but not in KS-T_1, KS pre T_1, KS_{α}-T_1, KS_{β}-T_1, KS_{g}-T_1.

Theorem 3.2.11. Let (U, $\tau_R(X)$, $KS_R(X)$) be a Kasaj Topological Space, then for each KS_{gsp}-T_1 space is KS_{gsp}-T_0 space.

Proof:

Let U be a KS_{gsp}-T$_1$ space and let x and y be two distinct points of U, there exists KS_{gsp}-open sets G and H such that x ∈ G, y ∉G and x∉H, y ∈ H. We have x ∈ G, y ∉G. Therefore, U is KS_{gsp}-T$_0$ space.

Theorem 3.2.12. If U is a KS_{gsp}-T$_1$ space and V is a subspace of U, then V is also KS_{gsp}-T$_1$ space.

Proof:

Let (U, $\tau_R(X)$, $KS_R(X)$) be a KS_{gsp}-T$_1$ space and (V, $\tau_R(Y)$, $KS_R(Y)$) be a subspace U. Let x,y ∈ V with x ∉ y. Since V ⊆ U, we have x,y∈ U. Since U is KS_{gsp}-T$_1$, then there exists KS_{gsp}-open sets G and H such that x ∈ G, y ∉ G and y ∈ H, x∉ H. Let M = V ∩ G and N = V ∩ H. Then M and N are KS_{gsp}-open sets in V. Also x ∈ M, y∉ M and y ∈ N, x ∉ V. Hence (V, $\tau_R(Y)$, $KS_R(Y)$) is also a KS_{gsp}-T$_1$ space.

Theorem 3.2.13. A Kasaj Topological Space (U, $\tau_R(X)$, $KS_R(X)$) is KS_{gsp}-T$_1$ iff the each one-point set is KS_{gsp}-closed.

Proof:

Let (U, $\tau_R(X)$, $KS_R(X)$) be a KS_{gsp}-T$_1$ space. Let x ∈ U, Then for each y ∈ U-{x} there exists a KS_{gsp}- open set F such that y ∈ F and x ∉ F, the sets {x} and F are disjoint. That is {x}∩ F = ∅ this implies that F⊆ U-{x}. Thus, y ∈ F ⊆ U-{x} this implies that U-{x} is an KS_{gsp}-open set this implies that {x} is KS_{gsp}-closed in U. Conversely, suppose {x} is KS_{gsp}-closed. Let x,y ∈ U with x ∉ y. So U-{x} is an KS_{gsp}-open set containing y but not x. Also U-{y} is an KS_{gsp}-open set containing x but not y. Hence U is a KS_{gsp}-T$_1$ space.

Theorem 3.2.14. A Kasaj Topological Space (U, $\tau_R(X)$, $KS_R(X)$) is KS_{gsp}-T$_1$ iff every finite subset of U is KS_{gsp}-closed set in U.

Proof:

Assume that U is KS_{gsp}-T$_1$. Let G be a finite subset of U. Let G = {x$_1$, x$_2$,…,x$_n$}. Then G = {x$_1$}∪ {x$_2$}…,∪{x$_n$} is KS_{gsp}-closed, being a finite union of KS_{gsp}closed sets.

Conversely, let each finite subset of U is KS_{gp}-closed in U. Then {x} is KS_{gsp}-closed, since it is finite. Since each singleton is KS_{gsp}-closed, U is KS_{gsp}- T$_1$.

Theorem 3.2.15. A Kasaj Topological Space (U, $\tau_R(X)$, $KS_R(X)$) is KS_{gsp}-T$_1$ iff the intersection of all KS_{gsp}-neighbourhoods of any point x in U is the singleton {x}.

Proof:

Assume that U is KS_{gsp}-T$_1$. Let x ∈ U. Let G be the intersection of all KS_{gsp}-neighbourhoods of x. Let y be any point in U different from x. Since U is KS_{gsp}-T$_1$, there exists a KS_{gsp}-neighbourhood M of x such that y ∉ M. Since y∉ M, we have y ∉ G. Since G is the intersection of all KS_{gsp}-neighbourhoods of x. Since y ∉ G, no point different from x in G. Hence G = {x}.

Conversely, assume that intersection of all KS_{gsp}-neighbourhoods of x in U is {x}. To prove that U is KS_{gsp}-T$_1$, let x,y ∈ U with x ∉ y.

Let G = ∩ {all KS_{gsp}-neighbourhoods of x in U}. Then G = {x}.

Let H = ∩ {all KSs_{gp}-neighbourhoods of y in U}. Then H = {y}.

Since y ∉ x, we have y ∉ G then there exists KS_{gsp}-neighbourhoods J of x with y ∉ J. Since x ∉ y, x∉ K this implies that KS_{gsp}-neighbourhoods y ∈ K with x ∉ K. Hence, U is KS_{gsp}-T$_1$.

3.3 Properties of KS_{gp}[KS_{gsp}]-T$_2$ Spaces

Describing and researching the notations of KS_{gp} [KS_{gsp}]-T$_2$ Spaces in Kasaj Topological Spaces is the motto of this section.

Definition 3.3.1. A Kasaj topological spaces U is called

(i) KS-T$_2$ space if for each pair of distinct points x,y in U, there exists an KS-open set F and G such that x ∈ F, y∈ G and F ∩ G = ∅.

(ii) KS pre-T$_2$ space if for each pair of distinct points x,y in U, there exists an KS pre-open set F and G such that x ∈ F, y ∈ G and F ∩ G = ∅.

(iii) KS_β-T$_2$ space if for each pair of distinct points x,y in U, there exists an KS_β-open set F and G such that x ∈ F, y∈ G and F ∩ G = ∅.

(iv) KS_{gp}-T$_2$ space if for each pair of distinct points x,y in U, there exists an KS_{gp}-open set F and G such that x ∈ F, y ∈ G and F ∩ G = ∅.

(v) $KS_{\beta gsp}$-T$_2$ space if for each pair of distinct points x,y in U, there exists an KS_{gsp}-open set F and G such that x ∈ F, y ∈ G and F ∩ G = ∅.

Theorem 3.3.2. In a Kasaj Topological Spaces (U, $\tau_R(X)$, $KS_R(X)$). Then

(i) Every KS-T$_2$ space is (KS pre-T$_2$, KS_α-T$_2$, KS_β-T$_2$, KS_g-T$_2$) KS_{gp}-T$_2$.

(ii) Every KS pre-T$_2$ space is (KS_β-T$_2$) KS_{gp}-T$_2$.

(iii) Every KS_α-T$_2$ space is (KS pre-T2, KS_β-T$_2$) KS_{gp}-T$_2$.

(iv) Every KS_g-T$_2$ space is KS_{gp}-T$_2$.

Proof:

(i) Suppose U is KS-T$_2$ space. Let x and y be two distinct points in U. Since U is KS-T$_2$, there exists an KS-open sets F and G such that x ∈ F and y∈ G. Since every KS-open set is (KS-pre-open, KS_α-open, KS_β-open, KS_g-open) KS_{gp}-open, F and G are distinct (KS-pre open, KS_α-open, KS_β-open, KS_g-open) KS_{gp}-open sets such that x ∈ F and y ∈ G. Hence U is KS_{gp}-T$_2$.

Proof of (ii), (iii), (iv) are similar to (i).

Converse of the above the theorem need not be true as shown in the following example.

Example 3.3.3. Let U = {a,b,c,d,e}, with U\R = {b,d},{c,e}, {a} and X = {a, c}. Then the nanotopology, $\tau_R(X)$ = {U, ∅, {a, c,e}, {c, e}}. S = {b}, S$^{'}$= {a, b, d, e}. Then x ={a}, y = {b} it is KS_{gp}-T$_2$ but not in KS-T$_2$, KS pre-T$_2$, KS_α-T$_2$, KS_g-T$_2$.

Theorem 3.3.4. Let (U, $\tau_R(X)$, $KS_R(X)$) be a Kasaj Topological Space, then for each KS_{gp}-T$_2$ space is KS_{gp}-T$_0$ space.

Proof:

Let (U, $\tau_R(X)$, $KS_R(X)$) be a KS_{gp}-T$_2$ space and let x,y ∈ U, x ∉ y, then there exists a two distinct KSgp-open sets G,H ⊆ U such that x ∈ G, y ∈ H, G ∩ H = ∅. Since G ∩ H = ∅ , x ∈ G, y ∉ G and x∉ H, y ∈ H. Hence U is KS_{gp}-T$_0$ space.

Theorem 3.3.5. Let (U, $\tau_R(X)$, $KS_R(X)$) be a Kasaj Topological Space, then for each KS_{gp}-T$_2$ space is KS_{gp}-T$_1$ space.

Proof:

Let (U, $\tau_R(X)$, $KS_R(X)$) be a KS_{gp}-T$_2$ space and let x,y∈ U, x ∉ y, then there exists a two distinct KS_{gp}-open sets G,H such that x ∈ G, y ∉ G and y ∈ H, x∉ H, G ∩ H = ∅. Therefore, U is KS_{gp}-T$_1$ space.

Theorem 3.3.6. If U is a KS_{gp}-T$_2$ space and V is a subspace of U, then V is also KS_{gp}-T$_2$ space.

Proof:

Let (U, $\tau_R(X)$, $KS_R(X)$) be a KS_{gp}-T$_2$ space and (V, $\tau_R(Y)$, $KS_R(Y)$) be a subspace of U. Let x,y ∈ V with x ∉ y. Then x,y ∈ U. Since U is KS_{gp}-T$_2$, there exists KS_{gp}-open sets F and G such that x ∈ F, y ∈ G and F ∩ G = ∅. Thus, we have F ∩ V, G ∩ V are KS_{gp}-open set in V, (F ∩ V) ∩ (G ∩ V) = ∅., x ∈ F ∩ V, y ∈ G ∩ V. Hence (V, $\tau_R(Y)$, $KS_R(Y)$) is KS_{gp}-T$_2$ space.

Theorem 3.3.7. In a Kasaj Topological Spaces (U, $\tau_R(X)$, $KS_R(X)$). Then

(i) Every KS-T$_2$ space is KS_{gsp}-T$_2$.

(ii) Every KS pre-T$_2$ space is KS_{gsp}-T$_2$.

(iii) Every KS_α-T$_2$ space is KS_{gsp}-T$_2$.

(iv) Every KS_β -T$_2$ space is KS_{gsp}-T$_2$.

(v) Every KS_g-T$_2$ space is KS_{gsp}-T$_2$.

Proof:

It is obvious from theorem 3.3.2.

Converse of the above the theorem need not be true as shown in the following example.

Example 3.3.8. Let U = {a,b, c, d, e}, with U\R = {{a,b},{c,d},{e}} and X = {a,e}. Then the nano topology, $\tau_R(X)$ = {U, ∅, {e}, {a, b, e}, a,b}}. S = {a}, S'= {b, c, d,e}. Then x = {c}, y = {d} it is KS_{gsp}-T$_2$ but not in KS-T$_2$, KS pre-T$_2$, KS_α-T$_2$, KS_β-T$_2$, KS_g-T$_2$.

Theorem 3.3.9. Let (U, $\tau_R(X)$, $KS_R(X)$) be a Kasaj Topological Space, then for each KS_{gsp}-T$_2$ space is KS_{gsp}-T$_0$ space.

Proof:

Let (U, $\tau_R(X)$, $KS_R(X)$) be a KS_{gsp}-T$_2$ space and let x,y ∈ U, x ∉ y, then there exists a two distinct KS_{gsp}-open sets G,H ⊆ U such that x ∈ G, y ∈ H, G ∩ H = ∅ . Since G ∩ H = ∅, x ∈ G, y ∉ G and x∉ H, y ∈ H. Hence, U is KS_{gsp}-T$_0$ space.

Theorem 3.3.10. Let (U, $\tau_R(X)$, $KS_R(X)$) be a Kasaj Topological Space, then for each KS_{gsp}-T$_2$ space is KS_{gsp}-T$_1$ space.

Proof:

Let (U, $\tau_R(X)$, $KS_R(X)$) be a KS_{gsp}-T$_2$ space and let x,y∈ U, x ∉ y, then there exists a two distinct KS_{gsp}-open sets G, H such that x ∈ G, y ∉ G and y ∈ H, x∉ H, G ∩ H = ∅. Therefore, U is KS_{gsp}-T$_1$ space.

Theorem 3.3.11. If U is a KS_{gsp}-T$_2$ space and V is a subspace of U, then V is also KS_{gsp}-T$_2$ space.

Proof:

Let (U, $\tau_R(X)$, $KS_R(X)$) be a KS_{gsp}-T$_2$ space and (V, $\tau_R(Y)$, $KS_R(Y)$) be a subspace of U. Let x,y ∈ V with x ∉ y. Then x,y ∈ U. Since U is KS_{gsp}-T$_2$, there exists KS_{gsp}-open sets F and G such that x ∈ F, y ∈ G and F ∩ G = ∅. Thus, we have F ∩ V, G ∩ V are KS_{gsp}-open set in V, (F ∩ V) ∩ (G ∩ V) = ∅, x ∈ F ∩ V , y ∈ G ∩ V. Hence (V, $\tau_R(Y)$, $KS_R(Y)$) is KS_{gsp}-T$_2$ space.

The result yields from the above section was an application of optimal choices using the idea of $(KS_{gp})KS_{gsp}$-T$_i$ (i = 0,1,2) on the content of these Kasaj topological structure. The idea of this application is based on personality characteristics of the applicants.

4 Conclusion

The class of KSgp(KSgsp)-open and KSgp(KSgsp)-closed sets has an important role to examine the separation axiom in Kasaj topological space. In this work, we studied new types of separation axioms namely, KSgp-Ti, (i=0,1,2) spaces; KSgsp-Ti, (i=0,1,2) spaces and the relation between spaces are discussed in new form. Several characterizations and the relation between properties of the above sets of separation axioms are discussed and proved. Furthermore, useful results are investigated by comparing KSgp(KSgsp)-T0 spaces, KSgp(KSgsp)-T1, KSgp(KSgsp)-T2 spaces in the context of these new concepts.

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